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#### Abstract

Blob shape recognition is used in various application fields such as industrial vision systems, parts recognition, positioning, inspection, etc. This is based on numerical signature which is an effective image processing technique for shape recognition.

This paper describe a new fast algorithm for pattern recognition, shapes are being coded by using two vectors, the first one is contour distances and the second one is of the surface histogram, these are working in relative manner. The principle coding is reduced to a product between the mask and the binary image of the shape.

The first merit of this algorithm is the high-speed image processing, instead of using images it does operate on vectors. The second merit is the precise recognition of known geometries shapes even for arbitrary or complexes ones.


Keywords: Distance mask, Histogram equality, Numerical signature, linear approximation, Stationary series, Pattern recognition

## 1. Introduction

The pattern recognition technique is a very important task and very required in various industrials systems and in vision systems; such as in positioning, automated visual inspection and many other applications.

In these applications the processing time is very important and high speeds are much sought for the industrial requirements. But the image has a large numerical data which means a conventional basic processing is too slow. In order to speed up the processing there are two types of solutions. The first type consists of subdividing the processing into tasks which will be given to parallel processors [1]. Other solution consist of reducing the quantity of information and instead of working on the whole data of the image; we use a compressed or reduced quantity [2].

Our work is based on the second type of solution, because the image immediately coded in two vectors, the first vector having the dimension of the contour pixels number of the object in question, the second vector having the dimension of the circle radius generating this object, then all the operations are deduced from the manipulations of the two vectors. In addition, the operation of information compression is very fast as we are going to present, this has been leading to inexpensive method in time and very reliable. Thus, this will find implementation in visual recognition real time process.

In this paper, we will present first the theoretical tools used. Then, we will use these tools to present the information compression principal and the image coding. Later this presentation will be used in detailed manner to expand our steps of recognition. Finally we will present the obtained experimental results over several samples of images.

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## 2. Theoretical Tools

From the viewpoint of shape recognition, the shape plane, this can be described by the couple of information; its surface $S$ and its contour $C$. This information can be given by the Cartesian coordinate as follows:

1. A set of Cartesian coordinate point $(x, y)$ such that $(x, y) \in S$.
2. An analytical function $f$ describing $C$ such that $y=f(x)$.
with $x \in D$
$D$ : The definition domain of the shape,
$x$ : The departure set of points of $D$,
$y$ : The arrival set of points.
The original image data is two dimensional matrix therefore this shape will be presented by a set of finite couple. If the contour $C$ is constituted of $n$ pixels the a couple vector of ( $x_{i}, y_{i}$ ) with $i=1 . . n$ of dimension $n$ sufficient to represent it $C=\left\{\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{i}, y_{i}\right), \ldots,\left(x_{n}, y_{n}\right)\right\}$.

Or in the coordinate system $(d, \theta), C=\left\{\left(d_{1}, \theta_{1}\right),\left(d_{2}, \theta_{2}\right), \ldots,\left(d_{i}, \theta_{i}\right), \ldots,\left(d_{n}, \theta_{n}\right)\right\}$. Thus, $C$ is represented by two sets:

1. The set of distances

$$
\begin{equation*}
V=\left\{d_{1}, d_{2}, \ldots d_{i}, \ldots d_{n}\right\} \tag{1}
\end{equation*}
$$

2. The set of angles $\left\{\theta_{1}, \theta_{2}, \ldots \theta_{i}, \ldots \theta_{n}\right\}$ which will be replaced by a set pixels forming $C$.

This representation of shapes offers two possibilities of exploitation. Handling these distances to sort an analytical expression $y=f(x)$ modeling the contour $C$, which is valid for known geometries shapes. But for arbitrary or complexes shapes the modeling becomes difficult, thus we were interested to working directly from this numerical representation of shapes without going through the analytical expression.

A planer shape can undergo translations, rotations and homothety zoom. If we denote by $S_{\text {before }}, C_{\text {before }}$ the surface and the contour before transformation and $S_{a f t e r}, C_{a f t e r}$ after then,

$$
\begin{equation*}
S_{a f t e r}=k^{2} . S_{\text {before }} \cdots \text { and } \cdots C_{\text {after }}=k . C_{\text {before }} \tag{2}
\end{equation*}
$$

with $k=1$ for translations and rotations and $\neq 1$ for homothety.
We start by coding the contour by the distances vector (1) $C_{\text {before }}$ by $V$ and $C_{a f t e r}$ by $W$, therefore for the translations and rotations we have

$$
\begin{equation*}
V=W \Longrightarrow\|V\| \simeq\|W\| \tag{3}
\end{equation*}
$$

Whereas for homothety of $k$ ratio:

$$
\begin{equation*}
\|V\| \simeq k *\|W\| \tag{4}
\end{equation*}
$$

For $k<1, V$ and $W$ are of the type:

$$
\begin{equation*}
V=\left\{d_{1}, d_{2}, \ldots d_{i}, \ldots d_{n}\right\} \ldots W=\left\{d_{11}, \ldots d_{1 k}, d_{21}, \ldots d_{2 k}, \ldots \ldots \ldots d_{n 1}, \ldots d_{n k}\right\} \tag{5}
\end{equation*}
$$

Such as,

$$
\begin{equation*}
d_{1}=k * d_{11}, d_{2}=k * d_{21}, \ldots \ldots d_{n}=k * d_{n 1} \tag{6}
\end{equation*}
$$

Inversely if $k>1$.
At this stage, we must point out the following problems:

1. The ratio $k$ is not always an integer which gives a number of pixels $d_{i 1} d_{i 2} \ldots d_{i k}$ not necessarily an integer and which should be an integer due to the digital mesh?
2. The pixels $d_{i 1} d_{i 2} \ldots d_{i k}$ of $W$ have only one image which is $d_{i}$ in $V$ that we have associated $d_{i 1}$, then how about the other points $d_{i 2} \ldots d_{i k}$ ?

For the first problem when we are on the element $d_{i}$ of rank $i$ in $V$, we take the rounded product $j=\operatorname{round}(k * i)$ to determine its counterpart for $d_{j}$ in $W$. This allow us some stationary, hence a frequency of repetition more or less regularly. Concerning the second problem, this consists of giving estimation of missing distances $d_{i 2} \ldots d_{i k}$ that have not images in $V$, we have opted for a linear approximation of first order, for example having;

$$
\begin{equation*}
d_{1}=k * d_{11} \wedge d_{2}=k * d_{21} \tag{7}
\end{equation*}
$$

We estimate $d_{12} \ldots d_{1 k}$ data distances by the right segment linking the points $d_{11}$ and $d_{21}$. Finally, to check (4)...(7) it is necessary to find particulars pixels that can serve as reference and as departure points in and, while working as relative manner.

For the surfaces, the pixels forming $S_{b e f o r}$ and $S_{a f t e r}$ are coded in distances to give images of gray level, if we denote by $H_{\text {befor }}$ and $H_{\text {after }}$ their histograms [3], then for the translations and the rotations we have;

$$
\begin{equation*}
S_{b e f o r}=S_{a f t e r} \Rightarrow H_{b e f o r}=H_{a f t e r} \tag{8}
\end{equation*}
$$

Whereas, for homothety of $k$ ratio $S_{\text {after }}=k^{2}$. $S_{\text {before }}$.
Having the level $i$ in $S_{\text {before }}$, it will be affected by the level $k . i$ and which repeat $H_{\text {after }}(k . i)$ so that

$$
\begin{equation*}
H_{\text {after }}(k . i)=k * H_{\text {before }}(i) \tag{9}
\end{equation*}
$$

Let $H_{\text {before }}=\{h(1), h(2), \ldots, h(i), \ldots, h(n)\}$ and $H_{\text {after }}=\{f(1), f(2), \ldots, f(k .1), \ldots, f(k .2), \ldots, f(k . i), \ldots, f(k . n)\}$.
The two histograms must verify (10) and (11) first their sum;

$$
\begin{equation*}
\sum_{j=1 . . k n} f(j)=k^{2} . \sum_{i=1 . . n} h(i) \tag{10}
\end{equation*}
$$

And their compounds are related by:

$$
\begin{equation*}
f(k . i)=k * h(i) \tag{11}
\end{equation*}
$$

Practically, (11) means that the transition from the histogram $H_{\text {before }}$ to the histogram $H_{\text {after }}$ is performed by a simple linear transformation.

Now, we go to the information reduction step of the image, by doing its code by two vectors, the first is of contour distances and the second is of the surface histograms.

## 3. Procedure of representations

### 3.1. Principle of coding

This representation is very delicate if we proceed with theoretical coding of distances as the ultimate erode [4], furthermore it is very computationally prohibitive task. To remedy to these problems we have develop a simple, fast and effective; it consists of creating a mask image $M$, of gray level formed by concentric circles, where the pixel take as
gray level the distances which does separate them from the center as it shown by figure 1 [5] [6]. Later, this mask image is used in the representation as follow:

1. From the binary image that represent the shape, its gravity center is calculated then its contour of thickness 1 pixel.


Figure $1 M$ : Distance mask
2. By simple translation of the mask, we coincide the mask center $M$ with the gravity center of $C$ the shape, the product point by point is done to give two images of gray level, the first contain the contour $C$ where the pixels have as level the distance which separate them from the gravity center of the shape and the second image is of the shape where each pixel is coded in distance.

### 3.2. Departure pixel choice

For each shape, we have to do a particular and wise choice of the followed departure pixel. For example, for this shape we have chosen the most far pixel regarding the gravity center; which correspond to the maximum of distances. Let $E$ be the set of pixels:

$$
\begin{equation*}
E=\{(i, j) / C(i, j)=\operatorname{Max}(C)\} \tag{12}
\end{equation*}
$$

If $|E|>1$, which means several pixels verify this condition, we will see later that we have to pass them one after the other so that we converge to the one which will verify our recognition criteria.

### 3.3. Loading of representation vectors

According to what is presented in 3.1 and 3.2; for each shape and in relative manner we determine;

1. The reference pixels, which is its gravity center and departure pixel.
2. Using this data and by basic operation of the contour tracking $C$, the distances met are loaded systematically to the vector distance $V$ defined by (1)(5).
3. Finally the image distances gives the histogram vector $H$.

## 4. Recognition Algorithm

Having an object beside the camera, its image is taken then we apply the representation procedure given in paragraph 3. There are two pairs of representation vectors, $\left(V_{\text {model }}, H_{\text {model }}\right)$ for the model and ( $V_{\text {object }}, H_{\text {object }}$ ) for the object candidate. The recognition procedure uses the following steps:

Step 1: Calculation of the two forms ratio: A global ratio between the two forms is calculated based on the dimension ratio of the two vectors

$$
k=\left\|V_{\text {object }}\right\| /\left\|V_{\text {model }}\right\|
$$

or reverse so to have $k>1$. In the same time, ratio between the two surfaces is been calculated according to (9) (10)

$$
k^{2}=S_{\text {object }} / S_{\text {model }}=\sum_{i} H_{\text {object }}(i) / \sum_{j} H_{\text {model }}(j)
$$

these two ratios should coincide. We have to point out that the surface ratio is more precise than the contour one because this latter is not always smoothed.

Step 2: Histograms Comparison: According to (11) the histograms are related and we can switch from one to the other by simple linear transformation. Knowing that the ratio $k$ is calculated form step 1 , if $k>1$ we transform $H_{o b j e c t}$ if not we transform $H_{\text {model }}$ according to (11). The histogram obtained by transformation s compared to the non transformed one. For example for $k>1 H_{\text {object }}$ is transformed onto $H_{o b j e c t}^{T}$ this latter is been compared to $H_{\text {model }}$. Thus, the two histograms have to be equal. If so we follow the process to step 3 , if not we stop the comparison and the samples are different.

Step 3: Contours Comparison: This step is divided into three elementary stages these are as follow:
3.1: Determinations of departure point and the tracking contour direction in each contour: With respect to the application, a particular point such the most far one or the nearest of the gravity center, is chosen as departure point for the two contours. If $V_{\text {object }}$ contains $n$ pixels and $V_{\text {model }}$ contains $m$ pixels checking this particularity then we combine $n$ and $m$ pixels, and for each pair of pixel, we try to maximize a topologic similarity function furthermore by combining the incremental angle between positive and negative in each contour, all the possibilities of tracking direction are taken into account. The conditions allowing this maximization are retained. These conditions correspond to departure pixels and the tracking direction in each distance contour.
3.2: Synchronous reading and partial ratios calculation: Two reading operations are simultaneously triggered in $C_{\text {object }}$ and $C_{\text {model }}$, their distances are loaded respectively in $V_{\text {object }}$ and $V_{\text {model }}$. With an increment $i=i+1$ for the small vector and $k$ for the great vector $j=\operatorname{round}(k * i)$ the ratios of these distances $V_{\text {object }}(j), V_{\text {model }}(i)$ is calculated then stored in $R(i)$ with $R$ the partial ratios vector.
3.3: Evolution analysis of the partial ratios: Following the loading of vector $R$, for two identical shapes, the partial ratios have to be close to the global ratio $k$ [7]; then a second decision is taken;

If (the series $R$ is stationary with mean value $k$ ) Then
Follow the process to step 4,
Else stop the comparison and
Shapes are different.
EndIf
Step 4: Refine the comparison: This step is necessary only for ratios $k \gg 1$ or $k \ll 1$. It consists of doing a linear approximation of the first order to estimate the points $V_{\text {object }}$ that have not an image in $V_{\text {model }}(11)$. Then, calculate the error as a gap between the reel distances and their estimated counterparts by approximation, these gaps are stored in the vector $\varepsilon$.

If
(mean $(\varepsilon)$ and variance $(\varepsilon)$ are weak) Then
the shapes are similar
of proportionality factor equal to $k$.
Else
shapes are different.
EndIf

## 5. Experimental results

### 5.1. The algorithm illustration:

In what follows, we will present the algorithm progress upon two objects showing the obtained results in each step. The two objects are presented by distances as it is shown by figure 2 .

Step 1: Ratios Calculation between the two shapes The vector $V_{\text {object }}$ and $V_{\text {model }}$ are calculated $\left\|V_{\text {object }}\right\|=330$ and $\left\|V_{\text {model }}\right\|=249$, hence the global ratio from the contours is $k=1.3253$. And from histograms the surfaces are $S_{\text {object }}=7986$ and $S_{\text {model }}=4721$. Using (9) $k^{2}=1.6916$ where the calculated ratio from the surfaces is $\sqrt{k^{2}}=$ 1.3006. the ratios are close then we go to step 2 .

Step 2: Histograms comparison The images histograms coded in distances are presented in figure 3.a, as we can notice these histograms are different. By using (11), we have kept the model histogram ( $H_{\text {model }}$ blue) not changed but the object one ( $H_{\text {object }}$ red) we apply upon it the transformation $H_{\text {object }}^{T}(i)=\frac{1}{1.3006} H_{\text {object }}(1.3006 * i)$. In the figure 3.b we have presented the histogram unchanged of the model ( $H_{\text {model }}$ blue) and the transformed histogram of the object ( $H_{o b j e c t}^{T}$ red). As we can notice the two histograms coincide perfectly, and to quantify the degree of similarity, we have used the


Figure 2 a-object, b-model
criteria of Swain [8]:

$$
\begin{align*}
& \text { Dist }_{\text {Swain }}^{\min }=\frac{\sum_{i=1}^{256} \min \left(H_{\text {object }}^{T}(i), H_{\text {model }}(i)\right)}{\sum_{i=1}^{256} H_{\text {model }}(i)}  \tag{13}\\
& \text { Dist }_{\text {Swain }}^{\max }=\frac{\sum_{i=1}^{256} \max \left(H_{\text {object }}^{T}(i), H_{\text {model }}(i)\right)}{\sum_{i=1}^{256} H_{\text {model }}(i)} \tag{14}
\end{align*}
$$

These distances are close to unity for identical histograms, whereas for different histograms Dist $t_{\text {Swain }}^{\min }$ is weak and Dist $t_{\text {Swain }}^{\max }$ is great. In our publication, Dist $_{\text {Swain }}^{\max }=1.048$ and Dist $_{S w a i n}^{\min }=0.9506$, which show effectively that the two histograms coincide perfectly because the two criteria are $\simeq 1$ and we go to step 3 .

## Step 3: Contour comparison

1. The departure pixel as well the direction tracking contour are calculated for the object and for the model they are presented in figure 4.


Figure 3 a-pre-histograms, b-histograms after


Figure 4 Left: object, Right: model
2. The synchronous reading of the two shapes has allowed the marked red points in figure 4 , and the partial ratios vector $R$ is loaded. We notice that still there are white points in $V_{\text {object }}$ that not images in $V_{\text {model }}$.
3. The variation of partial ratios stored in $R$, its average $R_{\text {average }}=1.3018$ its standard deviation $\sigma=0.0932$ and the accuracy $\frac{\left|R_{\text {average }}-k\right|}{k}=0.0009=0.09 \%$ this confirm that the shapes are related and that one is the image of the other by simple homothety.

Finale decision: The two shapes are identical and they have a ratio of proportionality of $k=1.3006$.

### 5.2. Recognition of random shapes:

We have constituted two lots of images, the first contains images presenting the same object, only the acquisition conditions are different (camera setting, camera distance, object orientation), and the second constituted of images of different object. Its on these two lots that we have developed a comparative study of the following parameters:

### 5.2.1. Variations of the three ratios

For the same test, we have calculated three ratios, the one of contour or global $k$, the one of surfaces and finally the average of partial ratios, obtained in the following of $R_{\text {average }}$; results of first lot are presented in figure 5.a and the those of the second lot in figure 5.b.


Figure 5 Variations of the three Ratios a-lot1 b-lot2

### 5.2.2. The accuracy variations

We have marked the accuracy evolution $\frac{\left|R_{\text {average }}-k\right|}{k}$, the results of the first lot in figure 6.a, the accuracy is less that $5 \%$ whereas in the second lot of figure $6 . b$ the accuracy is greater than $27 \%$.


Figure 6 Accuracy variations a:lot1 b:lot2

### 5.2.3. Histograms equality

The figure 7.a summarizes the variation of Dist $t_{S w a i n}^{\min }(13)$, and the Dist $t_{\text {Swain }}^{\max }(14)$ during the different tests of the first lot, and figure $7 . b$ those of the second lot.


Figure 7 Histograms equality a-lot1 b-lot2

### 5.3. Summary

According to our comparative study, the following table summarizes the different parameters.

| The parameter | Identical samples | Different samples |
| :---: | :---: | :---: |
| Contour ratios, <br> Surface, <br> Average | Equals | Different |
| Partial ratios <br> Variations | Weak | Great |
| Accuracy | $<5 \%$ | Not stationary |
| Histograms | Equals | $>5 \%$ |
| Dist $_{\text {Swain }}^{\text {min }}$ | $\simeq 1$ | Different |
| Dist $_{\text {Swain }}^{\text {max }}$ | $\simeq 1$ | $\ll 1$ |

## Conclusion

We have presented here, a novel and very simple technique of shapes representation as two vectors, the first is of contour distances and the second is the distances histogram of its surface. The boring step of calculus is accelerated enormously by the use of image mask of distances; which is generated previously outside the recognition procedure.

Thus, the coding is reduced to a product point by point between the mask and the binary image of the shape, and the shape itself becomes a vector of weak dimension, so less memory used, and simple to use, for this reason it is called numerical signature.

As we can notice, our method has two strengths. The first is its simplicity of manipulation, instead of using images it does operate on vectors, so it does manipulate shapes quickly. The second is its working in relative manner; it does manipulate random shapes and without previous setting of the acquisition chain neither for precise positioning.

This last strong point has permitted a great request in industry. In addition with its linear approximation, this technique could compare shapes with different scales, this is very promising as a recognition operations in uncontrolled conditions.

Finally, with determined parameters of recognition, the obtained results are very promising and testify its effectiveness.

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