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# Hybrid genetic algorithm to federal government capital budgeting

*D. Hakimi, D. O. Oyewola , K.R.Adeboye, R.O.Olayiwola*

Department of Mathematics,School of Physical and Applied Sciences, Federal University of Technology, Minna, Niger State, Nigeria

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**Abstract:** Multidimensional Knapsack problem is a NP-hard problem. The problem has been studied extensively in the literature. In this paper we provides a solution to a capital budgeting problem of Federal Government Budget in Nigeria using Hybrid Genetic Algorithms proposed by [32].We use also used the approach of [33] to model federal government budget by dividing the capital project into four groups which includes : The Economic sector $(x_1)$ , The Social Service sector $(x_2)$, The Environmental/Regional Development sector $(x_3)$, and The Administration sector $(x_4)$. Using MATLAB software for the analysis, it was observed that the optimal solution was optimal solution value is 277.64 billion naira. The result shows that the first and the second sector will be selected $(x_1, x_2 = 1; x_3, x_4 = 0)$.

**Keywords:** Capital budgeting, Knapsack problem, capital projects, Genetic Algorithms, Population, Survival of the fittest.

## 1 Introduction

In the Multidimensional Knapsack Problem (MKP), each of the item $x_j$ has a profit $p_j$.However, instead of having a single knapsack to fill, we have a number $m$ of knapsack of capacity $C_i$ $(i = 1, \ldots, m)$. Each $x_j$ has a weight $w_j$ that depends of the knapsack $j$ . The main objective in the Multidimensional Knapsack Problem is to find a subset of items that maximize the total profit without exceeding the capacity of all dimensions of the knapsack. The equation (1) below is the formulation of Multidimensional Knapsack Problem(MKP) :

$$Max_x \sum_{j=1}^{n} p_j x_j$$
$$S.t \quad \sum_{j=1}^{n} w_j x_j \leq C_i \quad i = 1, \ldots, m \tag{1}$$

Different approaches have been proposed to solve it by approximate dynamic programming [13]. Dimitris Bertsimas proposed an Approximate Dynamic Programming approach for the multidimensional knapsack problem. In their paper they approximate the value function using parametric and non-parametric methods and using a base-heuristic. Vincent Boyer, Didier El Baz and Moussa Elkihel proposed an exact cooperative method for the solution of the Multidimensional Knapsack Problem ( MKP) which combines dynamic programming and branch and bound. Their method makes cooperate a dynamic programming heuristics based on surrogate relaxation and a branch and bound procedure [30]. On the other hand, Jason Deane and Anurag Agarwal presented a Neural approach, a Genetic Algorithms approach and a Neurogenetic approach, which is a hybrid of the Neural and the Genetic Algorithms approach. In their findings Neurogenetic approach performs better than genetic algorithms and neural approach [31]. Recently,genetic algorithms has been used to solve



* Corresponding author e-mail: davidakaprof01@yahoo.com

Multidimensional Knapsack problem. This paper will present a Hybrid Genetic Algorithm proposed by [32]. The new method will be apply to Federal Government Budget in Nigeria.

## 2 Theoretical Background of Hybrid Genetic Algorithms

A hybrid Genetic Algorithm developed by [32] was used. They modified the classical Genetic Algorithm in such a way that more problem specific knowledge is considered in their constraint handling. The steps of modified Genetic Algorithm for the Multidimensional Knapsack problem is as follows.

Step 1. Representation and fitness function: The population is represented as n-bit binary string, where n is the number of variables in the Multidimensional Knapsack problem, a value of 0 or 1 at the $jth$ position implies that $x_j$ is either 0 or 1 in the Multidimensional knapsack problem solution, respectively. The individual's chromosome is illustrated in figure 1.

| $j$ | 1 | 2 | 3 | 4 | 5 | ... | $n-1$ | $n$ |
|---|---|---|---|---|---|---|---|---|
| $s[j]$ | 0 | 1 | 0 | 0 | 1 | ... | 0 | 1 |

**Fig. 1:** Binary representation of a multidimensional Knapsack problem.

The fitness function is given as follows.

$$fitn(S) = \sum_{j=1}^{n} p_j s[j] Pen[j] \tag{2}$$

Where Pen is a penalty function for an infeasible individual.

Step 2: Parent selection: The parent selection involves generating two parents who will later combine to form children. A stochastic universal sampling(SUS) to generate the parent.

Table 1 below shows the selection probability for 11 individuals. From the table individual with the largest interval is the most fit, but individual with the smallest interval on the line is the second least fit i.e individual 10 etc.

**Table 1:** Selection probability and fitness value

| Number of Individual | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Fitness value | 2.0 | 1.8 | 1.6 | 1.4 | 1.2 | 1.0 | 0.8 | 0.6 | 0.4 | 0.2 | 0.0 |
| Selection Probability | 0.18 | 0.16 | 0.15 | 0.13 | 0.11 | 0.09 | 0.07 | 0.06 | 0.03 | 0.02 | 0.0 |

for number 6 individual to be selected, we used$1/NPointer$ as the number of individual to be selected. We have 1/6 = 0.167, then a random number in the range [0.167] is selected.

Step 3: Creating Initial population: In order to create an initial population, we used dantzig algorithm by sorting the items according to non-increasing profit to weight ratios, and using a greedy algorithm for filling the knapsack. We used the equation below.

$$\frac{p_1}{w_1} \geq \frac{p_2}{w_2} \geq \ldots \frac{p_n}{w_n} \tag{3}$$

Step 4: Crossover and Mutation: Crossover involves two mating Chromosomes are cut once at corresponding points and the sections after the cuts exchanged. Here, we used a two point crossover which involves two crossover points are chosen and the contents between these points are exchanged between two mated parents as shown below.


The following mutation rate was adopted.

$$Mutationrate = (0.6 \times Iter^{10}) \times (\frac{0.1}{(popsize)^{10}} + 0.01) \tag{4}$$

where $Iter$ = iteration number, $popsize$ = number of population size.

Step 5: Penalty functions: Let $S$ be an arbitrary binary chromosome, that is,
$S = s_1, \ldots, s_n$ where$s_i \varepsilon 0, 1$ for $i = 0, \ldots, n$. We define

$$I = \{i | \sum_{j=1}^{n} w_{ij} s_j\} > C_i, \quad i = 1, \ldots, m \tag{5}$$

$I$ is the set of indices of the corresponding constraint $i$ is violated. We also define $sum_j$ as follows.

$$sum_j = \sum_{i \varepsilon I} w_{ij}, j = 1, \ldots, n \tag{6}$$

where $sum_j$ is the sum of entries in column $j$ of the coefficient matrix whose index is in $I$. The penalty functions is given by the equation (7) below

$$SUM = \sum_{j \varepsilon J} sum_j \tag{7}$$

Also the penalty function is given below

$$Pen(S) = \frac{1}{SUM} \tag{8}$$

**The outline of the algorithm is as follows:**
Input: $W[m \times n], P[1 \times n], C[m \times 1]$;
set gen = 0; set MAXGEN = 2000; set $C_r$ = 0.95; set N = 5 $\times$ $n$; initialize P(t)=$S_1, \ldots, S_N, S_i \varepsilon 0, 1^n$;
**while**$(gen \leq MAXGEN || Gap > 0)$

evaluate $FitnV = fint(S_1), \ldots, fitn(S_N)$;

Bestsol = select best feasible solution;
select individual from the population;
Recombine selected individuals (crossover);
Mutate offspring;
Insert offspring in population;
Gap = $((Optsolution - HGAsolution)/Optsolution) \times 100$;
gen = gen + 1;

**end**

The following Hybrid Genetic Algorithm(HGA) will be apply for federal government capital budget span from 2005-2014.

## 3 Modelling Of Federal Government Budget To Multidimensional Knapsack Problem

In this paper we shall be modelling Nigeria budget using HGA we had developed in section 2 above. The statistical data for this study will be from National Bureau of Statistics for the period of 2005 - 2014. Capital budgeting models select a maximum value collection of project, investment and so on, subject to limitations on budgets or other resources consumed.

We shall be considering the following ministries:

1. Agriculture, Commerce, Tourism, Works, Transport, Power and Steel, Petroleum Resources, Communication, Manufacturing, Solid Minerals, Finance and Aviation.
2. Education, Science and Technology, Health, Information, Culture, Youth and Sports, Water resources, Defence, Police Affairs and Police formation, ICPC, EFCC.
3. Land, Housing and Urban Development.
4. General Administration (Presidency), Judiciary, National Assembly.

The aim of this research work is to selecting at least a sector from each year in such a way that the total cost of executing projects in a certain year does not exceed the total allocation or assigning to capital budget for that year such that the value of that sector is maximixed. The capital projects are summarized in four groups which include:

1. The Economic Sector will represent $(x_1)$
2. The Social Service Sector will represent $(x_2)$
3. The Environmental/Regional Development will represent $(x_3)$
4. The Administration Sector will represent $(x_4)$

We shall make use of five constraints representing the five years considered and also solve for the remaining five years to make a total of ten years and four non basic variables representing the four sectors of the capital projects budgetting as shown below.

1. Sector 1 $(x_1)$ is made up of Agriculture, Commerce, Tourism, Works, Transport, Power and Steel, Petroleum Resources, Communication, Manufacturing, Solid Minerals, Finance and Aviation.
2. Sector 2 $(x_2)$ is made up of Education, Science and Technology, Health, Information, Culture, Youth and Sports, Water resources, Defence, Police Affairs and Police formation, ICPC, EFCC.
3. Sector 3 $(x_3)$ is made up of Land, Housing and Urban Development
4. Sector 4 $(x_4)$ is made up of General Administration (Presidency), Judiciary, National Assembly.

## 4 Results

Table2 below shows four sectors that are being evaluated over a ten year planning horizon as well as their respective return. The various budgets represent each year from 2005-2014 will form the coefficients of the constraints while the return will form the coefficients of the objective function .

**Table 2:** Nigeria budget for the period of ten years spanning from 2005-2014

| | Budget(N billion)/year | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Return (N billion) |
| $x_1$ | 144.1 | 154.46 | 179.67 | 160.69 | 184.54 | 195.97 | 187.67 | 97.85 | 212.39 | 236.7 | 175.41 |
| $x_2$ | 85.6 | 87.5 | 91.27 | 94.63 | 112.2 | 124.48 | 130.54 | 88.87 | 117.51 | 89.96 | 102.23 |
| $x_3$ | 14.49 | 14.05 | 12.68 | 16.67 | 13.57 | 16.61 | 19.07 | 23.02 | 16.5 | 36.5 | 18.31 |
| $x_4$ | 46.43 | 60.76 | 76.82 | 89.65 | 112.25 | 124.35 | 107.88 | 115.65 | 63 | 70.94 | 86.77 |
| Available fund (N billion) | 253.7 | 245.96 | 275.95 | 280.32 | 296.74 | 337.06 | 340.28 | 209.92 | 376.4 | 363.2 | |

## 5 Computational Procedure

A MATLAB code which was developed for HGA was used . The code was run on a Dell Inspiron Windows vista.

## 6 Conclusions

The (1) Multidimensional knapsack problem plays an important role in real-life applications. In this paper, a Hybrid genetic algorithms was apply to solve federal government budget in Nigeria.

The solution set ($x_1$, $x_2$, $x_3$,$x_4$) with the optimal value is (1 1 0 0). The output shows that sector 1 and sector 2 will be selected. The selected sector which include sector 1 comprises of Agriculture, Commerce, Tourism, Works, Transport, Power and Steel, Petroleum Resources, Communication, Manufacturing, Solid Minerals, Finance and Aviation. Sector 2 is made up of Education, Science and Technology, Health, Information, Culture, Youth and Sports, Water resources, Defence, Police Affairs and Police formation, ICPC, EFCC. The optimal solution value is 277.64 billion naira. It shows that it is 277.64 billion naira is required to execute the projects.

## Competing interests

The authors declare that they have no competing interests.

## Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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