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On some Hermite-Hadamard type inequalities for strongly *s*-convex functions

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Abstract: In this paper, we establish some new results related to the left-hand of the Hermite-Hadamard type inequalities for the class of functions whose second derivatives are strongly *s*-convex functions in the second sense. Some previous results are also recaptured as a special case.

Keywords: Hermite-Hadamard type inequalities, Hölder inequality, strongly s-convex functions.

1 Introduction

The following definition is well-known in the literature a functions $f: I \to \mathbb{R}$, $\varnothing \neq I \subset \mathbb{R}$, is said to be convex on I if the inequality

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y) \tag{1}$$

holds for all $x.y \in I$ and $t \in [0,1]$.

Theorem 1. [10] *Let $f: I \subseteq R \to R$ be a convex function on the interval I of real numbers and $a,b \in I$ with $a < b$. If f is a convex function then the following double inequality, which is well known in the literature as the Hermite–Hadamard inequality, holds*

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \tag{2}$$

In [5], ,Hudzik and Maligranda considered the class of functions which are *s*-convex in the second sense. This class is defined in the following way: a funciton $f: [0,\infty] \to \mathbb{R}$ is said to be s-convex in the second sense if

$$f(tx+(1-t)y) \leq t^s f(x)+(1-t)^s f(y) \tag{3}$$

holds for $x,y \in [0,\infty]$, $t \in [0,1]$ and for some fixed $s \in [0,1]$. This class of s-convex functions in the second sense is usually by K_S^2.

It can be easily see that for $s=1$ s-convexity reduces to the ordinary convexity of functions defined on $[0,\infty)$.

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Inequality (2) has studied a huge amount of interest over the years. We just remember the recent studies in ([2]-[4],[9],[13],[15]).

Definition 1. [11] *A function $f : I \to \mathbb{R}$ is called strongly s-convex with modulus c if*

$$f(tx+(1-t)y) \le t^s f(x) + (1-t)^s f(y) - ct(1-t)(b-a)^2. \tag{4}$$

In [1], Angulo et al. proved the following Hermite-Hadamard type inequality for strongly h-convex function:

Theorem 2. *Let $h : (0,1) \to (0,\infty)$ be a given function. If a function $f : I \subset \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable and strongly h-convex with modulus $c > 0$, then*

$$\frac{1}{h\left(\frac{1}{2}\right)}\left[f\left(\frac{a+b}{2}\right)+\frac{c}{12}(b-a)^2\right] \le \frac{1}{b-a}\int_a^b f(x)\,dx \le (f(a)+f(b))\int_0^1 h(t)dt - \frac{c}{6}(b-a)^2 \tag{5}$$

for all $a,b \in I$, $a<b$.

Corollary 1. *Suppose that $f : [0,\infty] \to \mathbb{R}$ is a strongly $s-$convex function in the second sense with modulus $c > 0$, where $s \in (0,1)$ (i.e $h(t) = t^s$ in (5)), then following inequalities hold;*

$$2^{s-1}\left[f\left(\frac{a+b}{2}\right)+\frac{c}{12}(b-a)^2\right] \le \frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{f(a)+f(b)}{s+1} - \frac{c}{6}(b-a)^2. \tag{6}$$

For more information and recent developments on inequalities for srongly convex function, please refer to ([1],[6],[7],[8],[12],[14],[16],[17]).

The aim of the paper is to establish some new Hermite-Hadamard inequalities for function whose second derivatives in absolute value are strongly s-convex.

2 Main results

To prove our main results, we consider the following Lemma given by Ozdemir et al. in [9].

Lemma 1. *Let $f : I \to \mathbb{R}$ be a differantiable mapping on I^0 where $a,b \in I$ with $a<b$. If $f'' \in L[a,b]$, then the following equality holds*

$$\frac{1}{b-a}\int_a^b f(x)\,dx - f\left(\frac{a+b}{2}\right) = \frac{(b-a)^2}{16}\left(\int_0^1 t^2\left(f''t\frac{a+b}{2}+(1-t)a\right)dt + \int_0^1 (1-t)^2 f''\left(tb+(1-t)\frac{a+b}{2}\right)d\right)t. \tag{7}$$

Theorem 3. *Suppose that $f : I \subset [0,\infty] \to \mathbb{R}$ be a twice differantiable mapping on I^0 such that $f'' \in L([a,b])$, where $a,b \in I$ with $a<b$. If $|f''|$ is strongly s-convex on $[a,b]$, for some $s \in (0,1]$ with modulus $c > 0$, thenfollowing inequalities hold*

$$\left|f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{8(s+1)(s+2)(s+3)}\left\{|f''(a)| + (s+1)(s+2)\left|f''\left(\frac{a+b}{2}\right)\right| + |f''(b)|\right\} - \frac{c}{160}(b-a)^2 \tag{8}$$
$$\le \frac{(b-a)^2}{8[(s+1)(s+2)(s+3)]} \times \left\{\left[1+(s+2)2^{1-s}\right]\left[|f''(a)|+|f''(b)|\right] - \frac{\left[1+(s+1)(s+2)2^{1-s}\right]c}{12}(b-a)^2\right\} - \frac{c}{160}(b-a)^2.$$

Proof. Taking modulus on both sides of (7), we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\int_0^1 t^2\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|dt+\frac{(b-a)^2}{16}\int_0^1 (1-t)^2\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|dt. \quad (9)$$

Since $|f''|$ is strongly s-convex, we get

$$\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right| \le t^s\left|f''\left(\frac{a+b}{2}\right)\right|+(1-t)^s\left|f''(a)\right|-ct(1-t)(b-a)^2 \quad (10)$$

and

$$\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right| \le t^s\left|f''(b)\right|+(1-t)^s\left|f''\left(\frac{a+b}{2}\right)\right|-ct(1-t)(b-a)^2. \quad (11)$$

If we substitute the inequalities (10) and (11) in (9), then we obtain

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\int_0^1 t^2\left[t^s\left|f''\left(\frac{a+b}{2}\right)\right|+(1-t)^s\left|f''(a)\right|-ct(1-t)(b-a)^2\right]dt \quad (12)$$
$$+\frac{(b-a)^2}{16}\int_0^1 (1-t)^2\left[t^s\left|f''(b)\right|+(1-t)^s\left|f''\left(\frac{a+b}{2}\right)\right|-ct(1-t)(b-a)^2\right]dt.$$

Using the facts that

$$\int_0^1 t^{s+2}dt=\int_0^1 (1-t)^{s+2}=\frac{1}{s+3} \quad (13)$$

and

$$\int_0^1 t^2(1-t)^s\,dt=\int_0^1 t^s(1-t)^2\,dt=\frac{2}{(s+1)(s+2)(s+3)} \quad (14)$$

in (12), we obtain

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\left[\frac{1}{s+3}\left|f''\left(\frac{a+b}{2}\right)\right|+\frac{2\left|f''(a)\right|}{(s+1)(s+2)(s+3)}-\frac{c}{20}(b-a)^2\right] \quad (15)$$
$$+\frac{(b-a)^2}{16}\left[\frac{2\left|f''(b)\right|}{(s+1)(s+2)(s+3)}+\frac{1}{s+3}\left|f''\left(\frac{a+b}{2}\right)\right|-\frac{c}{20}(b-a)^2\right]$$
$$=\frac{(b-a)^2}{8\left[(s+1)(s+2)(s+3)\right]}\left\{\left|f''(a)\right|+\left|f''(b)\right|+(s+1)(s+2)\left|f''\left(\frac{a+b}{2}\right)\right|\right\}-\frac{c}{160}(b-a)^2.$$

This completes the proof of the first inequality in (8).

For the proof of the second inequality in (8), since $|f''|$ is strongly s-convex, by (6), we have

$$\left|f''\left(\frac{a+b}{2}\right)\right| \le 2^{1-s}\left[\frac{\left|f''(a)\right|+\left|f''(b)\right|}{s+1}-\frac{c}{6}(b-a)^2\right]-\frac{c}{12}(b-a)^2 \quad (16)$$

Combining (15) and (16), we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)^2}{8\left[(s+1)(s+2)(s+3)\right]}$$
$$\times\left\{\left|f''(a)\right|+\left|f''(b)\right|+(s+1)(s+2)\left\{2^{1-s}\left[\frac{\left|f''(a)\right|+\left|f''(b)\right|}{s+1}-\frac{c}{6}(b-a)^2\right]-\frac{c}{12}(b-a)^2\right\}\right\}-\frac{c}{160}(b-a)^2$$
$$=\frac{(b-a)^2}{8\left[(s+1)(s+2)(s+3)\right]}\times\left\{\left[1+(s+2)2^{1-s}\right]\left[\left|f''(a)\right|+\left|f''(b)\right|\right]-\frac{\left[1+(s+1)(s+2)2^{2-s}\right]c}{12}(b-a)^2\right\}$$
$$-\frac{c}{160}(b-a)^2$$

which completes the proof of the second inequality in (8), and thus the proof is completed.

Corollary 2. *In Theorem 3, if we choose $s=1$, we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)^2}{192}\left\{\left|f''(a)\right|+6\left|f''\left(\frac{a+b}{2}\right)\right|+\left|f''(b)\right|\right\}-\frac{c}{160}(b-a)^2 \tag{17}$$
$$\leq \frac{(b-a)^2}{48}\left[\left[\left|f''(a)\right|+\left|f''(b)\right|\right]\right]-\frac{13c}{2304}(b-a)^4-\frac{c}{160}(b-a)^2.$$

Theorem 4. *Let $f: I\subset[0,\infty)\to\mathbb{R}$ be a twice differantiable mapping on I^0 such that $f''\in L[a,b]$, where $a,b\in I$ with $a<b$. If $|f''|^q$ is strongly s-convex on $[a,b]$ with modulus $c>0$, for some fixed $s\in(0,1]$, then the following inequality holds:*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{\frac{1}{p}}\left[\frac{1}{s+1}\left(\left|f''\left(\frac{a+b}{2}\right)\right|^q+\left|f''(a)\right|^q-\frac{c}{6}(b-a)^2\right)^{\frac{1}{q}}\right. \tag{18}$$
$$\left.+\frac{1}{s+1}\left(\left|f''\left(\frac{a+b}{2}\right)\right|^q+\left|f''(b)\right|^q-\frac{c}{6}(b-a)^2\right)^{\frac{1}{q}}\right]$$

where $q>1$ with $\frac{1}{p}+\frac{1}{q}=1$.

Proof. From Lemma 1 and using Hölder inequality, we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)dt\right| \leq \frac{(b-a)^2}{16}\left[\int_0^1 |t|^2\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|dt+\int_0^1 |t-1|^2\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|dt\right] \tag{19}$$
$$\leq \frac{(b-a)^2}{16}\left[\left(\int_0^1 |t|^{2p}dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|^q dt\right)^{\frac{1}{q}}+\left(\int_0^1 |t-1|^{2p}\right)^{\frac{1}{p}}\left(\int_0^1\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|^q dt\right)^{\frac{1}{q}}\right].$$

Using strongly s-convexity of $|f''|^q$, we have

$$\int_0^1\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|^q dt \leq \frac{1}{s+1}\left[\left|f''\left(\frac{a+b}{2}\right)\right|^q+\left|f''(a)\right|^q\right]-\frac{c}{6}(b-a)^2 \tag{20}$$

and

$$\int_0^1 \left| f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|^q dt \le \frac{1}{s+1}\left[\left|f''(b)\right|^q + \left|f''\left(\frac{a+b}{2}\right)\right|^q\right] - \frac{c}{6}(b-a)^2. \tag{21}$$

By a simple computation, we have

$$\int_0^1 t^{2p}dt = \int_0^1 (1-t)^{2p}dt = \frac{1}{2p+1}. \tag{22}$$

If we put (20)-(22) in (19), we obtain

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(t)dt\right| \le \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{\frac{1}{p}}\left[\left(\frac{1}{s+1}\left[\left|f''\left(\frac{a+b}{2}\right)\right|^q + \left|f''(a)\right|^q\right] - \frac{c}{6}(b-a)^2\right)^{\frac{1}{q}} + \left(\frac{1}{s+1}\left[\left|f''(b)\right|^q + \left|f''\left(\frac{a+b}{2}\right)\right|^q\right] - \frac{c}{6}(b-a)^2\right)^{\frac{1}{q}}\right] \tag{23}$$

which completes the proof.

Corollary 3. *Under the assumption of Theorem 4 with $s=1$, we have*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{\frac{1}{p}}\left[\frac{1}{2}\left(\left|f''\left(\frac{a+b}{2}\right)\right|^q + \left|f''(a)\right|^q - \frac{c}{6}(b-a)^2\right)^{\frac{1}{q}} + \left(\frac{1}{2}\left[\left|f''\left(\frac{a+b}{2}\right)\right|^q + \left|f''(b)\right|^q\right] - \frac{c}{6}(b-a)^2\right)^{\frac{1}{q}}\right].$$

Theorem 5. *Let $f: I \subset [0,\infty) \to \mathbb{R}$ be a twice differantiable mapping on I^0 such that $f'' \in L[a,b]$, where $a,b \in I$ with $a<b$. If $|f''|^q$ is strongly s-convex on $[a,b]$ with modulus $c>0$, for some fixed $s\in(0,1]$, and $q\ge 1$, then the following inequality holds*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\left(\frac{1}{3}\right)^{\frac{1}{p}}\left[\left(\frac{1}{s+3}\left|f''\left(\frac{a+b}{2}\right)\right|^q + \frac{2}{(s+1)(s+2)(s+3)}\left|f''(a)\right|^q - \frac{c}{20}(b-a)^2\right)^{\frac{1}{q}} + \left(\frac{2}{(s+1)(s+2)(s+3)}\left|f''(b)\right|^q + \frac{1}{s+3}\left|f''\left(\frac{a+b}{2}\right)\right|^q - \frac{c}{20}(b-a)^2\right)^{\frac{1}{q}}\right]. \tag{24}$$

Proof. From Lemma 1 and using the power mean inequality, we have

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(t)dt\right| \le \frac{(b-a)^2}{16}\left[\int_0^1 t^2\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right| + \int_0^1 (t-1)^2\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|dt\right] \tag{25}$$

$$\le \frac{(b-a)^2}{16}\left[\left(\int_0^1 t^2dt\right)^{\frac{1}{p}}\left(\int_0^1 t^2\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|^q dt\right)^{\frac{1}{q}} + \left(\int_0^1 (t-1)^2\right)^{\frac{1}{p}}\left(\int_0^1 (t-1)^2\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|^q dt\right)^{\frac{1}{q}}\right].$$

Since $|f''|^q$ is strongly s-convex, by a simple computations, we obtain

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)dt\right| \le \frac{(b-a)^2}{16}\left[\left(\int_0^1 t^2 dt\right)^{\frac{1}{p}}\left(\int_0^1 t^2\left[t^s\left|f''\left(\frac{a+b}{2}\right)\right|^q\right.\right.\right.$$
$$\left.\left.+(t-1)^s\left|f''(a)\right|^q - ct(1-t)(b-a)^2\right]dt\right)^{\frac{1}{q}}$$
$$\left.+\left(\int_0^1 (t-1)^2\right)^{\frac{1}{p}}\left(\int_0^1 (t-1)^2\left[t^s\left|f''(b)\right|^q+(t-1)^s\left|f''\left(\frac{a+b}{2}\right)\right|^q-ct(1-t)(b-a)^2\right]dt\right)^{\frac{1}{q}}\right]$$
$$\le \frac{(b-a)^2}{16}\left(\frac{1}{3}\right)^{\frac{1}{p}}\left[\left(\frac{1}{s+3}\left|f''\left(\frac{a+b}{2}\right)\right|^q+\frac{2}{(s+1)(s+2)(s+3)}\left|f''(a)\right|^q-\frac{c}{20}(b-a)^2\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\frac{2}{(s+1)(s+2)(s+3)}\left|f''(b)\right|^q+\frac{1}{s+3}\left|f''\left(\frac{a+b}{2}\right)\right|^q-\frac{c}{20}(b-a)^2\right)^{\frac{1}{q}}\right].$$

This completes the proof.

Corollary 4. *In Theorem 5, if we choose $s=1$, we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{48}\left(\frac{3}{4}\right)^{\frac{1}{q}}\left[\left(\left|f''\left(\frac{a+b}{2}\right)\right|^q+\frac{1}{3}\left|f''(a)\right|^q-\frac{c}{5}(b-a)^2\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\frac{1}{3}\left|f''(b)\right|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q-\frac{c}{5}(b-a)^2\right)^{\frac{1}{q}}\right].$$

Now, we give the following Hadamard-type inequality for strongly s-concave mappings.

Theorem 6. *Let $f: I\subset[0,\infty)\to\mathbb{R}$, be a twice differentiable mapping on I^0 such that $f''\in L[a,b]$, where $a,b\in I$ with $a<b$. If $|f''|^q$ is strongly s-concave on $[a,b]$ with the modulus $c>0$, for some fixed $s\in(0,1]$, then the following inequality holds*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)dt\right| \le \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{\frac{1}{p}}2^{\frac{s-1}{q}}\left[\left[\left|f''\left(\frac{3a+b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]^{\frac{1}{q}}\right.$$
$$\left.+\left[\left|f''\left(\frac{a+3b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]^{\frac{1}{q}}\right].$$

where $q>1$ and $\frac{1}{p}+\frac{1}{q}=1$.

Proof. From Lemma 1 and using Hölder inequality for $q>1$ and $\frac{1}{p}+\frac{1}{q}=1$, we obtain

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)dt\right| \le \frac{(b-a)^2}{16}\left[\left(\int_0^1 |t|^{2p}dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|^q dt\right)^{\frac{1}{q}}\right. \tag{26}$$
$$\left.+\left(\int_0^1 |t-1|^{2p}\right)^{\frac{1}{p}}\left(\int_0^1\left|f''\left(tb+(1-t)\frac{a+b}{2}\right)\right|^q dt\right)^{\frac{1}{q}}\right].$$

Since $|f''|^q$ is strongly s-concave, we have

$$\int_0^1 \left| f''\left(t\frac{a+b}{2}+(1-t)a\right)\right|^q dt \leq 2^{s-1}\left[\left|f''\left(\frac{3a+b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right] \tag{27}$$

and

$$\int_0^1 \left| f''\left(tb+(1-t)\frac{a+b}{2}\right)\right| dt \leq 2^{s-1}\left[\left|f''\left(\frac{a+3b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]. \tag{28}$$

From (27) and (28), we get

$$\left| f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)dt\right| \leq \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{\frac{1}{p}} 2^{\frac{s-1}{q}}\left[\left[\left|f''\left(\frac{3a+b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]^{\frac{1}{q}} + \left[\left|f''\left(\frac{a+3b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]^{\frac{1}{q}}\right]$$

which completes the proof.

Corollary 5. *In theorem 6, we choose* $s=1$ *and* $\frac{1}{3}<\left(\frac{1}{2p+1}\right)^{\frac{1}{p}}<1$, $p>1$, *we have*

$$\left| f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(t)dt\right| \leq \frac{(b-a)^2}{16}\left[\left[\left|f''\left(\frac{3a+b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]^{\frac{1}{q}}+\left[\left|f''\left(\frac{a+3b}{4}\right)\right|^q+\frac{c}{12}(b-a)^2\right]^{\frac{1}{q}}\right].$$

3 Concluding remarks

In this study, using practical identity for twice differentiable functions proved by Özdemir et al., we present some new upper bounds for Midpoint type inequalities by taking advantageous of mappings whose second derivatives in absolute values are strongly s-convex.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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