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FGP Approach for Solving multi-level multi-objective quadratic fractional programming problem with fuzzy parameters

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Abstract: In this paper, we consider fuzzy goal programming (FGP) approach for solving multi-level multi-objective quadratic fractional programming (ML-MOQFP) problem with fuzzy parameters in the constraints. Firstly, the concept of the α-cut approach is applied to transform the set of fuzzy constraints into a common deterministic one. Then, the quadratic fractional objective functions in each level are transformed into quadratic objective functions based on a proposed transformation. Secondly, the FGP approach is utilized to obtain a compromise solution for the ML-MOQFP problem by minimizing the sum of the negative deviational variables. Finally, an illustrative numerical example is given to demonstrate the applicability and performance of the proposed approach.

Keywords: Multi-level programming, quadratic programming, fractional programming, fuzzy sets, fuzzy goal programming.

1 Introduction

Hierarchical optimization or multi-level mathematical programming (MLMP) techniques are extensions of Stackelberg games for solving decentralized planning problems with multiple decision makers (DMs) in a hierarchical organization where each unit seeks its own interests. The basic concept of multi-level programming technique is that the first-level decision maker (FLDM) sets his goal and/or make decision, set goal/or makes decision that requires each subordinate level in the organization for an independent optimal solution. These solutions are modified by the FLDM in line with the organizational objectives. This process proceeds to a satisfactory solution [1,4,7,20,21].

Over the last few years, rapid improvement in solving MLMP [4,7,12,13] as well as bi-level mathematical programming (BLMP) problems [1-3,5,21] have been witnessed and several methods have been presented. The use of the concept of the membership function of fuzzy set theory to multi-level programming problems for obtaining satisfactory decisions was first presented in [16]. FGP approach has been introduced in [18] for proper distribution of decision powers to the decision maker to arrive at a satisfying decision for the overall benefits of the organization. Sakawa et al. [23] proposed interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters. FGP algorithm for solving a decentralized bi-level multi-objective programming problem was developed in [3]. Arora and Gupta [1] presented interactive FGP approach for linear bi-level programming problem with the characteristics of dynamic programming. Multi-level decision-making problems were recently studied in [7]. Pramanik and Roy [21] adopted fuzzy goals to specify the decision variables of higher level DMs and proposed weighted/ unweighted FGP models for solving MLMP to obtain a satisfactory solution. Also, FGP approach was extended for solving bi-level multi-objective

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programming problems with fuzzy demands [5].

The fractional optimization problem is one of the most difficult problems in the field of optimization. Optimization of the ratio of two functions is called fractional programming (ratio optimization) problem [22,26]. Indeed, in such situations, it is often a question of optimizing a ratio of output/employee, profit/cost, inventory/sales, student/cost, doctor/patient, and so on subject to some constraints [11,22]. A proposal to the solution of multi-objective linear fractional programming has been presented in [10]. Multi-objective quadratic fractional programming models involve optimization of many complex and conflicting objective functions in the mathematical form of quadratic fractional subject to the set of constraints. FGP approach for multi-objective quadratic fractional programming (MOQFP) problem has been presented in [14]. Such type of problems in large hierarchical organizations of complex and conflicting multi-objectives formulate ML-MOQFP problems. Recently Lachhwani [15] proposed FGP approach with some modifications for solving MOQFP model. An interactive FGP algorithm to solve decentralized bi-level multi-objective fractional programming problem was presented in [9]. Baky et al. [25] presented fuzzy goal programming procedures to bi-level multi-objective fractional programming. FGP approach to solve stochastic fuzzy multi-level multi-objective fractional programming problem was extended in [20]. Parametric multi-level multi-objective fractional programming problems with fuzziness in the constraints has been presented in [19].

During the past two decades, the majority of research on the multi-level programming problems have been concentrated on the deterministic version in which the coefficients and decision variables in the objective functions and the constraints are assumed to be crisp values. However, in reality, it is usually difficult to know precisely the values of the coefficients due to the existence of imprecise or uncertain information when establishing multi-level models [24,27]. Thus, lead us to present the current research hoping that the proposed ML-MOQFP problem with fuzzy parameters can contribute to future studies in the field of uncertain multi-level optimization.

This paper presents a FGP approach for solving ML-MOQFP problem with fuzzy parameters. These parameters are expressed as fuzzy numbers based on the fuzzy set theory [24] to account for the uncertainty in decision-making problems. This study also employs α-cut approach to formulate the crisp model at the desired α -level.Then, the quadratic fractional objective functions in each level are transformed into non-linear objective functions based on a proposed transformation, thus the ML-MOQFP problem transformed into multi-level multi-objective non-linear fractional programming (ML-MONFP) problem. Secondly, separate non-linear membership functions for each objective function of the ML-MONFP problem are defined. Then, the FGP approach is utilized to obtain a compromise solution for the ML-MOQFP problem by minimizing the sum of the negative deviational variables. An algorithm for the ML-MOQFP problem is presented in details.

The rest of this paper is organized as follows. The next section describes problem formulation of the ML-MOQFP problem with fuzzy parameters and It's equivalent crisp model is also established. In Section 3, the quadratic model of the ML-MOQFP problem is developed. FGP approach is introduced in Section 4. In Section 5, an algorithm is developed for solving the ML-MOQFP problem. Numerical example was provided in Section 6. Concluding remarks are given at the end.

2 Problem formulation

Multi-level programming problems have more than one decision maker. Consider the hierarchical system be composed of a p-level decision maker. Let the decision maker at the i^{th}-level denoted by DM_i controls over the decision variable

$x_i = (x_{i1}, x_{i2}, \dots, x_{in_i}) \in R^{n_i},\ i = 1,2,\dots,p$. where $x = (x_1, x_2, \dots, x_p) \in R^n$ and $n = \sum_{i=1}^{p} n_i$ and furthermore assumed that

$$F_i(x_1, x_2, \dots, x_p) \equiv\ F_i(x):\ R^{n_1} \times R^{n_2} \times \dots \times R^{n_p} \to R^{k_i}, i = 1,2,\dots,p, \tag{1}$$

are the vector of quadratic fractional objective functions for $DM_i, i = 1,2,\dots,p$. Mathematically, ML-MOQFP problem with fuzzy parameters in the set of constraints [4,7,12,21] follows as.

$[1^{st} Level]$

$$\underbrace{max}_{x_1} F_1(x) = \underbrace{max}_{x_1} \left(f_{11}(x), f_{12}(x), \dots, f_{1k_1}(x)\right), \tag{2}$$

where $x_2, x_3, \dots, x_p$ solves

$[2^{nd} Level]$

$$\underbrace{max}_{x_2} F_2(x) = \underbrace{max}_{x_2} \left(f_{21}(x), f_{22}(x), \dots, f_{2k_2}(x)\right), \tag{3}$$
$$\vdots$$

where x_p solves

$[p^{th} Level]$

$$\underbrace{max}_{x_p} F_p(x) = \underbrace{max}_{x_p} \left(f_{p1}(x), f_{p2}(x), \dots, f_{pk_p}(x)\right), \tag{4}$$

subject to

$$x \in \tilde{G} = \left\{ x \in R^n | \tilde{A}_1 x_1 + \tilde{A}_2 x_2 + \dots + \tilde{A}_p x_p \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \tilde{b}, x \geq 0, \tilde{b} \in R^m \right\}, \tag{5}$$

where the objective function $f_{ij}(x)$ are represented by

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{x^T Q^{ij} x + c^{ij} x + \alpha^{ij}}{x^T R^{ij} x + d^{ij} x + \beta^{ij}}, i = 1,2,\dots,p, j = 1,2,\dots,k_i. \tag{6}$$

where Q^{ij} is an $n \times n$negative definite matrix, $R^{ij}$is an $n \times n$ positive semi-definite matrix c^{ij}, d^{ij} are n-vectors, $\tilde{A}_i$ is an $m \times n_i,\ \ i = 1,2,\dots,p$ fuzzy matrices and $\tilde{b}$ is an m-vector of fuzzy parameters. It is customary to assume that $D_{ij}(x) > 0$ forall $x \in \tilde{G}$, also α^{ij} and β^{ij} are constants and $\tilde{G}$ represents the multi-level convex constraints feasible choice set in the fuzzy environment.

3 Formulation of crisp set of constraints and solution concept

Based on ML-MOQFP model $(2)-(5)$, the coefficients of the set of constraints are represented by fuzzy numbers. Let $\mu_{\tilde{A}_i}$, and $\mu_{\tilde{b}}$ be the membership functions which represents the fuzzy coefficients matrices $\tilde{A}_i$ and the fuzzy numbers in the corresponding vector $\tilde{b}$ respectively. The α-cuts of $\tilde{A}_i$ and $\tilde{b}$ are defined as [5,17,20,23]:

$$(A_i)_\alpha = \left\{ A_i \in \left[(A_i)^L_\alpha,\ (A_i)^U_\alpha \right] \mu_{\tilde{A}_i} \geq \alpha,\ A_i \in S\left(\tilde{A}_i\right) \right\}, \tag{7}$$

$$(b)_\alpha = \left\{ b \in \left[(b)^L_\alpha,\ (b)^U_\alpha \right] \mu_{\tilde{b}} \geq \alpha,\ b \in S\left(\tilde{b}\ \right) \right\}, \tag{8}$$

where $S\left(\tilde{b}\ \right)$, and $S\left(\tilde{A}_i\right)$are the supports of the corresponding vectors and matrix of fuzzy numbers.

For the set of constraints represented by convex constraints feasible choice set in the fuzzy environment denoted by $\tilde{G}$. The numerical interval determined from an α-cut can thus be applied to equation (5) when the value of $\alpha$is specified, for the inequality constraints.

$$\sum_{j=1}^{n} \tilde{A}_{ij}\, x_j \ \geq\ \tilde{b}_i,\ \ \forall\ \ (i = 1,2,\ldots,r_1), \tag{9}$$

$$\sum_{j=1}^{n} \tilde{A}_{ij} x_j \ \leq\ \tilde{b}_i,\ \ \forall\ \ (i = r_1 + 1,2,\ldots,r_2), \tag{10}$$

can be replaced by the following constraints [6,17,23].

$$\sum_{j=1}^{n} (A_{ij})^U_\alpha\, x_j \ \geq (b_i)^L_\alpha, \quad \forall\ (i = 1,2,\ldots,r_1), \tag{11}$$

$$\sum_{j=1}^{n} (A_{ij})^L_\alpha x_j \ \leq (b_i)^U_\alpha, \quad \forall\ (i = r_1 + 1,\ldots,r_2), \tag{12}$$

where the lower bounds $(b_i)^L_\alpha, (A_{ij})^L_\alpha$ and upper bounds $(b_i)^U_\alpha,\ (A_{ij})^U_\alpha$ of the coefficients are set to have the largest feasible region so that the compromise solutions for the objective functions are most likely to be found. For equality constraints.

$$\sum_{j=1}^{n} \tilde{A}_{ij} x_j = \ \tilde{b}_i, \qquad \forall\ (i = r_2 + 1,\ldots,m), \tag{13}$$

can be replaced by two equivalent constraints [6,17,23]:

$$\sum_{j=1}^{n} (A_{ij})^L_\alpha x_j \ \leq\ (b_i)^U_\alpha, \quad \forall\ (i = r_2 + 1,\ldots,m), \tag{14}$$

$$\sum_{j=1}^{n} (A_{ij})^U_\alpha\, x_j \ \geq\ (b_i)^L_\alpha, \quad \forall\, (i = r_2 + 1,\ldots,m), \tag{15}$$

Definition 1. *For any* $x_1 \left(x_1 \in (G_1)_\alpha = \left\{x_1 x = (x_1,x_2,\ldots,x_p) \in (G\ )_\alpha\right\}\right)$ *given by FLDM and* $x_2 \left(x_2 \in (G_2)_\alpha = \left\{x_2 x = (x_1,x_2,\ldots,x_p) \in (G\ )_\alpha\right\}\right)$ *given by SLDM, if the decision variable*$x_p \left(x_p \in (G_p)_\alpha = \left\{x_p x = (x_1,x_2,\ldots,x_p) \in (G\ )_\alpha\right\}\right)$ *is the* α*-Pareto optimal solution of the PLDM, then* $(x_1,x_2,\ldots,x_p)$ *is an* α*-feasible solution of ML-MOQFP problem.*

Definition 2. *If* $x^* = \left(x^*_1,x^*_2,\ldots,x^*_p\right)$ *is an* α *-feasible solution of the ML-MOQFP problem; no other* α*-feasible solution* $x = \left(x_1,x_2,\ldots,x_p\right) \in G_\alpha$*exist, such that* $\left(f_{1j}\left(x^*\right)\right) \ \leq \left(f_{1j}\left(x\ \right)\right)$ *with at least one strict inequality hold for* $j\ \left(j = 1,2,\ldots,k_1\right)$*; so* $\left(x^*_1,x^*_2,\ldots,x^*_p\right)$ *is the* α*-Pareto optimal solution of the ML-MOQFP problem.*

4 Quadratic model development of the ML-MOQFP problem

Since the proposed model (2)-(5) has a vector of quadratic fractional objective functions in each level we can convert them by adopting the variable transformation parallel to those transformation given by Charnes and Cooper [8] for linear fractional programming problem into quadratic programming problem. Thus, in contrast to eq. (6) in this paper we employ the following representation in order to deal with the ML-MOQFP problem.

$$f_{ij}(x)=\frac{Q_1^{ij}x_1^2+Q_2^{ij}x_2^2+\cdots+Q_p^{ij}x_p^2+c_1^{ij}x_1+c_2^{ij}x_2+\cdots+c_p^{ij}x_p+\alpha^{ij}}{R_1^{ij}x_1^2+R_2^{ij}x_2^2+\cdots+R_p^{ij}x_p^2+d_1^{ij}x_1+d_2^{ij}x_2+\cdots+d_p^{ij}x_p+\beta^{ij}}, \quad \forall i,j. \tag{16}$$

Now, we make further extensions on the article of M. Chakraborty and S. Gupta [6], to develop a methodology for obtaining the quadratic model of the ML-MOQFP problem. Since the MOQFP problem for the i^{th}-level decision maker may be written as.

$$\underbrace{max}_{x_i} F_i(x)=\underbrace{max}_{x_i}\left(f_{i1}(x),f_{i2}(x),\ldots,f_{ik_i}(x)\right), \tag{17}$$

subject to

$$x\in G_\alpha=\left\{x\in R^n \,\middle|\, \begin{array}{l}\sum_{j=1}^n (A_{ij})_\alpha^U x_j \ \geq \ (b_i)_\alpha^L,\ i=1,2,\ldots,r_1,r_2+1,\ldots,m,\\ \sum_{j=1}^n (A_{ij})_\alpha^L x_j \ \leq (b_i)_\alpha^U,\ i=r_1+1,\ldots,r_2,\ r_2+1,\ldots,m,\end{array}\right\} \tag{18}$$

where

$$f_{ij}(x)=\frac{Q_1^{ij}x_1^2+Q_2^{ij}x_2^2+\ldots+Q_p^{ij}x_p^2+c_1^{ij}x_1+c_2^{ij}x_2+\ldots+c_p^{ij}x_p+\alpha^{ij}}{R_1^{ij}x_1^2+R_2^{ij}x_2^2+\ldots+R_p^{ij}x_p^2+d_1^{ij}x_1+d_2^{ij}x_2+\ldots+d_p^{ij}x_p+\beta^{ij}}, \ \forall\ i,j. \tag{19}$$

Let us take the least value of $\left(\frac{1}{R_1^{ij}x_1^2+R_2^{ij}x_2^2+\ldots+R_p^{ij}x_p^2+d_1^{ij}x_1+d_2^{ij}x_2+\ldots+d_p^{ij}x_p+\beta^{ij}}\right)$ is t^2 i.e.

$$\bigcap_{\forall i,j}\frac{1}{R_1^{ij}x_1^2+R_2^{ij}x_2^2+\ldots+R_p^{ij}x_p^2+d_1^{ij}x_1+d_2^{ij}x_2+\ldots+d_p^{ij}x_p+\beta^{ij}}=t^2, \tag{20}$$

which is equivalent to

$$\frac{1}{R_1^{ij}x_1^2+R_2^{ij}x_2^2+\ldots+R_p^{ij}x_p^2+d_1^{ij}x_1+d_2^{ij}x_2+\ldots+d_p^{ij}x_p+\beta^{ij}}\geq t^2, \ \text{ and } y=tx. \tag{21}$$

So, each quadratic fractional objective function is transformed into the following quadratic function

$$f_{ij}(y,t)=\left[Q_1^{ij}\frac{y_1^2}{t^2}+Q_2^{ij}\frac{y_2^2}{t^2}+\ldots+Q_p^{ij}\frac{y_p^2}{t^2}+c_1^{ij}\frac{y_1}{t}+c_2^{ij}\frac{y_2}{t}+\ldots+c_p^{ij}\frac{y_p}{t}+\alpha^{ij}\right]\times t^2 \tag{22}$$

or,

$$f_{ij}(y,t)=Q_1^{ij}y_1^2+Q_2^{ij}y_2^2+\ldots+Q_p^{ij}y_p^2+c_1^{ij}ty_1+c_2^{ij}ty_2+\ldots+c_p^{ij}ty_p+\alpha^{ij}t^2. \tag{23}$$

Based on the transformation $y=tx\ (t>0)$, $y\in R^n$, $t\in R$, and the above eq. (21) therefore, the quadratic model of the MOQFP problem for i^{th} level decision maker is formulated as follows.

$$\underbrace{\max}_{y_i} f_{ij}(y,t)=Q_1^{ij}y_1^2+Q_2^{ij}y_2^2+\ldots+Q_p^{ij}y_p^2+c_1^{ij}ty_1+c_2^{ij}ty_2+\ldots+c_p^{ij}ty_p+\alpha^{ij}t^2, \tag{24}$$

subject to

$$R_1^{ij}y_1^2+R_2^{ij}y_2^2+\ldots+R_p^{ij}y_p^2+d_1^{ij}ty_1+d_2^{ij}ty_2+\ldots+d_p^{ij}ty_p+\beta^{ij}t^2\le 1,\ j=1,\ldots,k_i, \tag{25}$$

$$\sum_{j=1}^{n}(A_{ij})_\alpha^U\left(\frac{y_j}{t}\right)\ \ge\ (b_i)_\alpha^L,\ i=1,2,\ldots,r_1,r_2+1,\ldots,m, \tag{26}$$

$$\sum_{j=1}^{n}(A_{ij})_\alpha^L\left(\frac{y_j}{t}\right)\le (b_i)_\alpha^U,\ i=r_1+1,\ldots,r_2,r_2+1,\ldots,m,t>0. \tag{27}$$

Following the above discussion thus, the ML-MOQFP model is transformed into the following ML-MOQP model.

$[1^{st}\ Level]$

$$\underbrace{\max}_{y_1}F_1(y,t)=Q_1^{1j}y_1^2+Q_2^{1j}y_2^2+\ldots+Q_p^{1j}y_p^2+c_1^{1j}ty_1+c_2^{1j}ty_2+\ldots+c_p^{1j}ty_p+\alpha^{1j}t^2,j=1,\ldots,k_1 \tag{28}$$

where $y_2,y_3,\ldots,y_p$ solves

$[2^{nd}\ Level]$

$$\underbrace{\max}_{y_2}F_2(y,t)=Q_1^{2j}y_1^2+Q_2^{2j}y_2^2+\ldots+Q_p^{2j}y_p^2+c_1^{2j}ty_1+c_2^{2j}ty_2+\ldots+c_p^{2j}ty_p+\alpha^{2j}t^2,j=1,\ldots,k_2 \tag{29}$$

$$\vdots$$

where y_p solves

$[p^{th}\ Level]$

$$\underbrace{\max}_{y_p}F_p(y,t)=Q_1^{pj}y_1^2+Q_2^{pj}y_2^2+\ldots+Q_p^{pj}y_p^2+c_1^{pj}ty_1+c_2^{pj}ty_2+\ldots+c_p^{pj}ty_p+\alpha^{pj}t^2,j=1,\ldots,k_p \tag{30}$$

subject to

$$R_1^{1j}y_1^2+R_2^{1j}y_2^2+\ldots+R_p^{1j}y_p^2+d_1^{1j}ty_1+d_2^{1j}ty_2+\ldots+d_p^{1j}ty_p+\beta^{1j}t^2\le 1,\ j=1,\ldots,k_1 \tag{31}$$

$$R_1^{2j}y_1^2+R_2^{2j}y_2^2+\ldots+R_p^{2j}y_p^2+d_1^{2j}ty_1+d_2^{2j}ty_2+\ldots+d_p^{2j}ty_p+\beta^{2j}t^2\le 1,\ j=1,\ldots,k_2 \tag{32}$$

$$R_1^{pj}y_1^2+R_2^{pj}y_2^2+\ldots+R_p^{pj}y_p^2+d_1^{pj}ty_1+d_2^{pj}ty_2+\ldots+d_p^{pj}ty_p+\beta^{pj}t^2\le 1,\ j=1,\ldots,k_p \tag{33}$$

$$\sum_{j=1}^{n}(A_{ij})_\alpha^U\left(\frac{y_j}{t}\right)\ \ge\ (b_i)_\alpha^L,\ i=1,2,\ldots,r_1,r_2+1,\ldots,m, \tag{34}$$

$$\sum_{j=1}^{n}(A_{ij})_\alpha^L\left(\frac{y_j}{t}\right)\le (b_i)_\alpha^U,\ i=r_1+1,\ldots,r_2,r_2+1,\ldots,m,\ t>0, \tag{35}$$

where the system of constraints, in equations (31)-(35), at an α-level denoted by S_α, which form a nonempty convex set.

5 Fuzzy goal programming approach for ML-MOQFP problem

In the proposed FGP approach, in order to obtain the compromise solution which is a Pareto optimal solution. The vector of quadratic objective functions of the model (28)-(35) for each DM is formulated as a fuzzy goal characterized by its membership function.

5.1 Characterization of membership functions

To define the membership functions of the fuzzy goals [5,25], each objective function's individual maximum is taken as the corresponding aspiration level, as follows

$$u_{ij} = \underbrace{\max}_{y \in S_\alpha} f_{ij}(y,t), (i = 1,2,\ldots,p), (j = 1,2,\ldots,k_i). \tag{36}$$

where u_{ij}, $(i = 1,2,\ldots,p)$, $(j = 1,2,\ldots,k_i)$, give the upper tolerance limit or aspired level of achievement for the membership function of ij^{th} objective function. Similarly, each objective function's individual minimum is taken as the corresponding aspiration level, as follows

$$g_{ij} = \underbrace{\max}_{y \in s_\alpha} f_{ij}(y,t), (i = 1,2,\ldots,p), (j = 1,2,\ldots,k_i). \tag{37}$$

where g_{ij}, $(i = 1,2,\ldots,p)$, $(j = 1,2,\ldots,k_i)$, give the lower tolerance limit or lowest acceptable level of achievement for the membership function of ij^{th} objective function. It can be assumed reasonably the values of $f_{ij}(y,t) \geq u_{ij}$, $(i = 1,2,\ldots,p)$, $(j = 1,2,\ldots,k_i)$, are acceptable and all values less than $g_{ij} = \underbrace{\max}_{y \in S_\alpha} f_{ij}(y,t)$, are absolutely unacceptable. Then, the membership function $\mu_{ij}(f_{ij}(y,t))$, for the ij^{th} fuzzy goal can be formulated as.

$$\mu_{ij}(f_{ij}(y,t)) = \begin{cases} 1, \text{ if } & f_{ij}(y,t) \geq u_{ij}, \\ \frac{f_{ij}(y,t) - g_{ij}}{u_{ij} \ldots g_{ij}}, \text{ if } & g_{ij} \leq f_{ij}(y,t) \leq u_{ij}, \ (i = 1,\ldots,p), \ (j = 1,\ldots,k_i), \\ 0, \text{ if } & f_{ij}(y,t) \leq g_{ij}, \end{cases} \tag{38}$$

5.2 Fuzzy Goal programming model

In the decision-making context, each decision maker is interested in maximizing his or her own objective function; the optimal solution of each DM, when calculated in isolation, would be considered as the best solution and the associated value of the objective function can be considered as the aspiration level of the corresponding fuzzy goal. In fuzzy programming approach, the highest degree of membership is one [18]. So, for the defined membership functions in equations (38), the flexible membership goals having the aspired level unity can be represented as follows.

$$\mu_{f_{ij}}(f_{ij}(y,t)) + d_{ij}^- - d_{ij}^+ = 1, \ (i = 1,2,\ldots,p), \ (j = 1,2,\ldots,k_i), \tag{39}$$

or equivalently as

$$\frac{f_{ij}(y,t) - g_{ij}}{u_{ij} \ \ldots g_{ij}} + d_{ij}^- - d_{ij}^+ = 1, \ (i = 1,2,\ldots,p), \ (j = 1,2,\ldots,k_i), \tag{40}$$

where $d_{ij}^-, d_{ik}^- \geq 0$, with $d_{ij}^- \times d_{ij}^+ = 0$, represent the under- and over- deviations, respectively, from the aspired levels [21].

Following the basic concept of MLMP, the FLDM decides his/her objectives and/or choices, hence asks each inferior level of the association for their solutions, which obtained individually. The lower level decision makers' choices are then presented and altered by the FLDM in light of the general advantage for the organization. Thus, the vector of decision variables y_{il}, $(i=1,2,\ldots,p-1)$, $(l=1,2,\ldots,n_i)$, for the top levels are taken as a binding constraints for the p^{th}-level problem as follows.

$$y_{il}=y_{il}^{*},\ (i=1,2,\ldots,p-1),\ (l=1,2,\ldots,n_i). \tag{41}$$

In the classical methodology of goal programming, the under- and over- deviational variables are included in the achievement function for minimizing them depends upon the type of the objective functions to be optimized. In the proposed FGP approach, the sum of under deviational variables is required to be minimized to achieve the aspired level. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value [5,21]. Thus considering the goal achievement problem at the same priority level, the proposed final FGP model for the ML-MOQFP problem follows as

$$minZ=\sum_{j=1}^{k_1}w_{1j}^{-}d_{1j}^{-}+\sum_{j=1}^{k_2}w_{2j}^{-}d_{2j}^{-}+\ldots+\sum_{j=1}^{k_p}w_{pj}^{-}d_{pj}^{-}, \tag{42}$$

subject to

$$\frac{f_{ij}(y,t)-g_{ij}}{(u_{ij}\ \ldots g_{ij})}+d_{ij}^{-}-d_{ij}^{+}=1,\ i=1,2,\ldots,p,\ j=1,2,\ldots,k_i, \tag{43}$$

$$R_1^{1j}y_1^2+R_2^{1j}y_2^2+\ldots+R_p^{1j}y_p^2+d_1^{1j}ty_1+d_2^{1j}ty_2+\ldots+d_p^{1j}ty_p+\beta^{1j}t^2\leq 1,\ j=1,2,\ldots,k_1, \tag{44}$$

$$R_1^{2j}y_1^2+R_2^{2j}y_2^2+\ldots+R_p^{2j}y_p^2+d_1^{2j}ty_1+d_2^{2j}ty_2+\ldots+d_p^{2j}ty_p+\beta^{2j}t^2\leq 1,\ j=1,2,\ldots,k_2, \tag{45}$$

$$R_1^{pj}y_1^2+R_2^{pj}y_2^2+\ldots+R_p^{pj}y_p^2+d_1^{pj}ty_1+d_2^{pj}ty_2+\ldots+d_p^{pj}ty_p+\beta^{pj}t^2\leq 1,\ j=1,2,\ldots,k_p, \tag{46}$$

$$y_{il}=y_{il}^{*},\ (i=1,2,\ldots,p-1),\ (l=1,2,\ldots,n_i). \tag{47}$$

$$\sum_{j=1}^{n}(A_{ij})_{\alpha}^{U}\left(\frac{y_j}{t}\right)\ \geq\ (b_i)_{\alpha}^{L},\ i=1,2,\ldots,r_1,r_2+1,\ldots,m, \tag{48}$$

$$\sum_{j=1}^{n}(A_{ij})_{\alpha}^{L}\left(\frac{y_j}{t}\right)\leq (b_i)_{\alpha}^{U},\ i=r_1+1,\ldots,r_2,r_2+1,\ldots,m,\ t>0, \tag{49}$$

$$d_{ij}^{-}\times d_{ij}^{+}=0,\quad \text{and}\quad d_{ij}^{-},d_{ij}^{+}\geq 0,\ (i=1,2,\ldots,t),\ (j=1,2,\ldots,k_i), \tag{50}$$

where Z represents the achievement function consisting of the weighted under-deviational variables of the fuzzy goals. The numerical weights w_{ij}^{-} represent the relative importance of achieving the aspired levels of the respective fuzzy goals. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested in [18] is used to assign the values to, w_{ij}^{-}. These values are determined as

$$w_{ij}^{-}=\frac{1}{u_{ij}-g_{ij}},\ (i=1,2,\ldots,p),\ (j=1,2,\ldots,k_i), \tag{51}$$

6 The FGP algorithm for ML-MOQFP problem with fuzzy parameters

Following the above discussion, the proposed FGP algorithm will be constructed for solving the ML-MOQFP problems with fuzzy parameters as follows

Step 1.	Set the value of α,acceptable for all decision makers, for the degree of all membership functions of the fuzzy parameters.
Step 2.	Formulate the crisp set of constraints for the ML-MOQFP problem at the given α-level.
Step 3.	Formulate the ML-MOQP model, equation (28)-(35), of the ML-MOQFP problem.
Step 4.	Calculate the individual maximum and minimum values for each objective function $f_{ij}(x,y)$ in all levels subject to the set of constraints.
Step 5.	Set the goals and the upper tolerance limits for each objective function in all levels.
Step 6.	Evaluate the weights w_{ij}^{-} as defined in equation (51).
Step 7.	Set $l=1$,for the i^{th} level decision-making problem.
Step 8.	Build the membership functions $\mu_{f_{lj}}\left(f_{lj}(x,y)\right)$ $j=1,2,\ldots,m_l$, as in equation (38).
Step 9.	Solve l_{th}-level FGP model sequentially to get $x_{lk}=x_{lk}^{*}$.
Step 10.	If $l>t-1$,then go to the Step 11; otherwise set $l=l+1$,and go to Step 8.
Step 11.	Solve the final FGP model for the ML-MOQFP problem with fuzzy parameters.
Step 12.	If all decision makers are satisfied with the compromise solution in Step11; then go to Step 14; otherwise go to Step 13.
Step 13.	Improve the upper and lower tolerance limits g_{ij}, u_{ij},for all objective goals in all levels, go to Step 6.
Step 14.	Stop with the satisfactory solution for all decision makers in the problem.

7 Illustrative example

To demonstrate the proposed FGP approach, consider the following ML-MOQFP problem with fuzzy parameters in the constraints.

$[1^{st}\ Level]$

$$\underbrace{\max}_{x_1}\left(f_{11}=\frac{-x_1^2-2x_2^2-x_3^2+5x_2+10}{x_1^2+3x_2+5},f_{12}=\frac{-x_1^2-x_2^2-4x_3^2+5x_1+12}{x_1^2+3x_2+5}\right),$$

where x_2,x_3 solves

$[2^{nd}\ Level]$

$$\underbrace{\max}_{x_2}\left(f_{21}=\frac{-x_1^2-2x_2^2-2x_3^2+5x_3+6}{x_2^2+3x_1+1},f_{22}=\frac{-3x_1^2-x_2^2-x_3^2+7x_3+8}{x_2^2+3x_1+1}\right),$$

where x_3 solves

$[3^{rd}\ Level]$

$$\underbrace{\max}_{x_3}\left(f_{31}=\frac{-x_1^2-4x_2^2-x_3^2+6x_2+7}{x_3^2+5x_2+2},f_{32}=\frac{-x_1^2-2x_2^2-x_3^2+9}{x_3^2+5x_2+2}\right),$$

subject to

$$\tilde{4}x_1+\tilde{7}x_2+\tilde{2}x_3\leq \tilde{30},$$
$$\tilde{3}x_1-\tilde{0}x_2+\tilde{14}x_3\leq \tilde{18},$$
$$\tilde{7}x_2+\tilde{8}x_3\leq \tilde{12}.$$

Here, the fuzzy numbers are assumed to be LR-fuzzy numbers and are given as follows. $\tilde{4}=(4,2,1)_{LR}$, $\tilde{7}=(7,4,2)_{LR}$, $\tilde{2}=(2,2,3)_{LR}$, $\tilde{3}=(3,2,2)_{LR}$, $\tilde{0}=(0,1,2)_{LR}$, $\tilde{14}=(14,4,2)_{LR}$, $\tilde{8}=(8,4,2)_{LR}$, $\tilde{30}=(30,5,10)_{LR}$, $\tilde{18}=(18,3,4)_{LR}$, $\tilde{12}=(12,2,8)_{LR}$. Following the proposed algorithm, the solution of the ML-MOQFP problem for a desired value of α, assume that an α-level of 0.5 is accepted by the three level DMs. Thus the deterministic model of the ML-MOQFP problem is obtained as follows.

$[1^{st}$ *Level*$]$

$$\underbrace{\max}_{x_1}\left(f_{11}=\frac{-x_1^2-2x_2^2-x_3^2+5x_2+10}{x_1^2+3x_2+5},f_{12}=\frac{-x_1^2-x_2^2-4x_3^2+5x_1+12}{x_1^2+3x_2+5}\right),$$

where x_2,x_3 solves

$[2^{nd}$ *Level*$]$

$$\underbrace{\max}_{x_2}\left(f_{21}=\frac{-x_1^2-2x_2^2-2x_3^2+5x_3+6}{x_2^2+3x_1+1},\quad f_{22}=\frac{-3x_1^2-x_2^2-x_3^2+7x_3+8}{x_2^2+3x_1+1}\right)$$

where x_3 solves

$[3^{rd}$ *Level*$]$

$$\underbrace{\max}_{x_3}\left(f_{31}=\frac{-x_1^2-4x_2^2-x_3^2+6x_2+7}{x_3^2-5x_2+2},\quad f_{32}=\frac{-x_1^2-2x_2^2-x_3^2+9}{x_3^2-5x_2+2}\right),$$

subject to

$$3x_1+5x_2+x_3\leq 35,$$
$$2x_1-1x_2+12x_3\leq 20,$$
$$5x_2+6x_3\leq 16.$$

Then the ML-MOQFP problem is transformed into the ML-MOQP model based on the proposed transformation as follows.

$[1^{st}$ *Level*$]$

$$\underbrace{\max}_{y_1}\begin{pmatrix} f_{11}=-y_1^2-2y_2^2-y_3^2+5ty_2+10t^2, \\ f_{12}=-y_1^2-y_2^2-4y_3^2+5ty_1+12t^2, \end{pmatrix},$$

where y_2,y_3 solves

[2^{nd} *Level*]

$$\underbrace{\max}_{y_2}\begin{pmatrix} f_{21} = -y_1^2 - 2y_2^2 - 2y_3^2 + 5ty_3 + 6t^2, \\ f_{22} = -3y_1^2 - y_2^2 - y_3^2 + 7ty_2 + 8t^2, \end{pmatrix}$$

where y_3 solves

[3^{rd} *Level*]

$$\underbrace{\max}_{y_3}\begin{pmatrix} f_{31} = -y_1^2 - 4y_2^2 - y_3^2 + 6ty_2 + 7t^2, \\ f_{32} = -y_1^2 - 2y_2^2 - y_3^2 + 9t^2, \end{pmatrix},$$

subject to

$$\begin{aligned} &y_1^2 + 3ty_2 + 5t^2 \leq 1, \\ &y_2^2 + 3y_1 + t^2 \leq 1, \\ &y_3^2 - 5y_2 + 2t^2 \leq 1, \\ &3y_1 + 5y_2 + y_3 - 35t \leq 0, \\ &2y_1 - 1y_2 + 12y_3 - 20t \leq 0, \\ &5y_2 + 6y_3 - 16t \leq 0. \end{aligned}$$

Table 1: Individual maximum, minimum values, u_{ij}, $g_{ij}$and weights w_{ij}.

	$f_{11}(y)$	$f_{12}(y)$	$f_{21}(y)$	$f_{22}(y)$	$f_{31}(y)$	$f_{32}(y)$
$max\ f_{ij}(y)$	2	2.73	1.825	1.752	1.436	1.8
$min\ f_{ij}(y)$	-0.86	-0.387	-0.998	-2.78	-1	-0.94
u_{ij}	2	2.73	1.825	1.752	1.436	1.8
g_{ij}	-0.86	-0.387	-0.998	-2.78	-1	-0.94
w_{ij}	0.35	0.32	0.355	0.22	0.41	0.36

Therefore, Solve the FGP models sequentially to get $y_1 = y_1^*$ and $y_2 = y_2^*$. Thus the first level FGP model follows as.

$$min\ Z = 0.35d_{11}^- + 0.32d_{12}^-,$$

subject to

$$
\begin{aligned}
&-y_1^2-2y_2^2-y_3^2+5ty_2+10t^2+2.86d_{11}^- -2.86d_{11}^+ =2,\\
&-y_1^2-y_2^2-4y_3^2+5ty_1+12t^2+3.126d_{12}^- -3.126d_{12}^+ =2.73,\\
&y_1^2+3ty_2+5t^2\le 1,\\
&y_2^2+3y_1+t^2\le 1,\\
&y_3^2-5y_2+2t^2\le 1,\\
&3y_1+5y_2+y_3-35t\le 0,\\
&2y_1-1y_2+12y_3-20t\le 0,\\
&5y_2+6y_3-16t\le 0,\\
&d_{11}^-,\ d_{11}^+,\ d_{12}^-,\ d_{12}^+\ge 0,\ \ t>0.
\end{aligned}
$$

Using Lingo programming, the compromise solution of the first level decision making problem is obtained as; $(y_1,y_2,\ y_3,t)=(0.1544,0,0,\ 0.4419)$. Then assuming that the FLDM set $y_1^*=0.1544$.

The second level decision maker FGP model follows as.

$$min\ Z=0.35d_{11}^- +0.32d_{12}^- +0.355d_{21}^- +0.22d_{22}^-,$$

subject to

$$
\begin{aligned}
&-y_1^2-2y_2^2-y_3^2+5ty_2+10t^2+2.86d_{11}^- -2.86d_{11}^+ =2,\\
&-y_1^2-y_2^2-4y_3^2+5ty_1+12t^2+3.126d_{12}^- -3.126d_{12}^+ =2.73,\\
&-y_1^2-2y_2^2-2y_3^2+5ty_3+6t^2+2.823d_{21}^- -2.823d_{22}^+ =1.825,\\
&-3y_1^2-y_2^2-y_3^2+7ty_2+8t^2+4.532d_{22}^- -4.532d_{22}^+ =1.752,\\
&y_1^2+3ty_2+5t^2\le 1,\\
&y_2^2+3y_1+t^2\le 1,\\
&y_3^2-5y_2+2t^2\le 1,\\
&3y_1+5y_2+y_3-35t\le 0,\\
&2y_1-1y_2+12y_3-20t\le 0,\\
&5y_2+6y_3-16t\le 0,\\
&y_1^*=0.1544,\\
&d_{11}^-,\ d_{11}^+,\ d_{12}^-,\ d_{12}^+,\ d_{21}^-,\ d_{21}^+,\ d_{22}^-,\ d_{22}^+\ge 0,\quad t>0.
\end{aligned}
$$

Using Lingo programming, the compromise solution of the second level decision making problem is obtained as; $(y_1,y_2,\ y_3,t)=(0.1544,0,0.1652,\ 0.4419)$. Also, the SLDM sets $y_2^*=0$.

Hence, the final FGP model for the ML-MOQFP problem with fuzziness in the constraints is obtained as follows

$$min\ Z=0.35d_{11}^- +0.32d_{12}^- +0.355d_{21}^- +0.22d_{22}^- +0.41d_{31}^- +0.36d_{32}^-,$$

subject to

$$
\begin{aligned}
&-y_1^2-2y_2^2-y_3^2+5ty_2+10t^2+2.86d_{11}^- -2.86d_{11}^+=2,\\
&-y_1^2-y_2^2-4y_3^2+5ty_1+12t^2+3.126d_{12}^- -3.126d_{12}^+=2.73,\\
&-y_1^2-2y_2^2-2y_3^2+5ty_3+6t^2+2.823d_{21}^- -2.823d_{22}^+=1.825,\\
&-3y_1^2-y_2^2-y_3^2+7ty_2+8t^2+4.532d_{22}^- -4.532d_{22}^+=1.752,\\
&-y_1^2-4y_2^2-y_3^2+6ty_2+7t^2+2.436d_{31}^- -2.436d_{31}^+=1.436,\\
&-y_1^2-2y_2^2-y_3^2+9t^2+2.74d_{32}^- -2.74d_{32}^+=1.8,\\
&y_1^2+3ty_2+5t^2\le 1,\\
&y_2^2+3y_1+t^2\le 1,\\
&y_3^2-5y_2+2t^2\le 1,\\
&3y_1+5y_2+y_3-35t\le 0,\\
&2y_1-1y_2+12y_3-20t\le 0,\\
&5y_2+6y_3-16t\le 0,\\
&y_1^*=0.1544,\\
&y_2^*=0,\\
&d_{11}^-,\ d_{11}^+,\ d_{12}^-,\ d_{12}^+,\ d_{21}^-,\ d_{21}^+,\ d_{22}^-,\ d_{22}^+,d_{31}^-,\ d_{31}^+,\ d_{32}^-,\ d_{32}^+\ge 0,\quad t>0.
\end{aligned}
$$

Using Lingo programming, the compromise solution of the ML-MOQFP problem is obtained as $(y_1,y_2,\ y_3,t)=(0.1544,0,\ 0.1213,0.4419)$ with the corresponding values $x_i=\frac{y_i}{t}$, $(x_1,x_2,\ x_3)=(0.349,0,\ 0.274)$, with the corresponding objective function values $f_{11}(x)=1.914,\ f_{12}(x)=2.6,,\ f_{21}(x)=3.47,\ f_{22}(x)=3.69,,$ $f_{31}(x)=3.28,\ f_{32}(x)=4.24.$

8 Summary and conclusion

This paper reveals how the concept of FGP approach can be efficiently used for solving ML-MOQFP problems with fuzzy parameters. Based on the α-level properties a numerical general model is constructed. An effort has been made to solve the ML-MOQFP problem with fuzzy parameters based on the fuzzy set theory and goal programming approach. Thus, the numerical results for the given example obtained to validity of the proposed method. The FGP approach appears to be promising and computationally easy to implement.

However there are many open points for discussion in future, which should be explored in the area of multi-level quadratic fractional optimization such as:

(1) Interactive algorithm is needed for dealing with multi-level multi-objective quadratic fractional programming with fuzzy parameters.
(2) Fuzzy goal programming algorithm is required for treating multi-level integer multi-objective quadratic fractional with fuzzy parameters.
(3) Fuzzy goal programming algorithm is required for treating multi-level integer multi-objective quadratic fractional in rough environment

Competing interests

The authors declare that they have no competing interests.

Authors’ contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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