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Heat transfer study in porous fin with temperature-dependent thermal conductivity and internal heat generation using Legendre wavelet collocation method

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Abstract: In this paper, heat transfer study of porous fin with temperature-dependent thermal conductivity and internal heat generation is analyzed numerically using Legendre wavelet collocation method. The numerical solutions are used effects of nonlinear thermal conductivity, convective and porosity parameters on the thermal conductivity of the fin. The Legendre wavelet collocation method is verified with the results of numerical method using Runge-Kutta method and good agreements are established.

Keywords: Legendre wavelet collocation method, porous fin, thermal performance, temperature-dependent thermal conductivity and internal heat generation.

1 Introduction

The thermal performance of heat transfer equipment has been significantly enhanced in recent times through the use of porous fin. The importance and applications of such fins has aroused many research interests to analyze the thermal performance of porous fins under various conditions. In such quest, a pioneer work on the heat transfer enhancement through the use of porous was carried out by Kiwan and Al-Nimr [1] who applied numerical method to investigate the thermal analysis of porous fin while Kiwan [2-4] developed a simple method to study the performance of porous fins in natural convection environment. Also, the same author investigated the effects of radiative losses on the heat transfer from porous fins. Gorla and Bakier [5] numerically carried out the thermal analysis of natural convection and radiation in a rectangular porous fin. Kundu and Bhanja [6] presented analytical model for the analysis of performance and optimization of porous fins. Kundu *et al.* [7] proposed a model for computing maximum heat transfer in porous fins. Taklifi *et al.* [8] investigated the effects of magnetohydrodynamics (MHD) on the performance of a rectangular porous fin. In the work, that by imposing MHD in system except near the fin tip, heat transfer rate from the porous fin decreases. Bhanja and Kundu [9] analytically investigated thermal analysis of a constructal T-shape porous fin with radiation effects. An increase in heat transfer is found by choosing porous medium condition in the fin. Recently, Kundu *et al.* [10] applied Adomian decomposition method on the performance and optimum design analysis of porous fin of various profiles operating in convection environment transient heat transfer analysis of variable section pin fins. Saedodin and Sadeghi [11] analyzed the heat transfer in a cylindrical porous fin while Saedodin and Olank [12]. Darvishi *et al.* [13] studied the thermal performance of a porous radial fin with natural convection and radiative heat losses while Hatami and Ganji [14] investigated the thermal performance of circular convective-radiative porous fins with different section shapes

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and materials. Hatami *et al.* [15 -18] presented various heat transfer studies in both dry and wet porous fins. All the studies on porous fin cited above were based on constant thermal conductivity. Such assumption might be correct because, for ordinary fins problem, the thermal conductivity of the fin might be taken to be constant. However, if large temperature difference exists within the fin, typically, between tip and the base of the fin, the thermal conductivity is not constant but temperature-dependent. Also, in their work on porous fins, Gorla *et al.* [19] and Moradi *et al.* [20] pointed out that for most materials, the effective thermal conductivity increases with temperature. Therefore, while analyzing the fin, effects of the temperature-dependent thermal conductivity must be taken into consideration. In carrying out such analysis, the thermal conductivity may be modelled for such and other many engineering applications by linear dependency on temperature. Such dependency of thermal conductivity on temperature renders the problem highly non-linear and difficult to solve exactly. It is also very realistic to consider the temperature-dependent internal heat generation in the fin (electric-current carrying conductor, nuclear rods or any other heat generating components of thermal systems). In solving the heat transfer problem in porous fin, Kundu [6-7, 10] applied Adomian decomposition method (ADM) on the performance and optimum design analysis of the fins while Saedodin and Sadeghi [11], Kiwan [1-5] applied Runge-Kutta for the thermal analysis in porous fin. Golar and Baker [5] and Gorla *et al.* [19] applied Spectral collocation method (SCM) to study the effects of variable thermal conductivity on the natural convection and radiation in porous fin. Saedodin and Shahababaei [21] adopted Homotopoy perturbation method (HPM) to analyse heat transfer in longitudinal porous fins while Darvishi *et al.* [13] and Moradi *et al.* [20] and Ha *et al.* [22] adopted Homotopy analysis method (HAM) to provide solution to the natural convection and radiation in a porous and porous moving fins while Hoshyar *et al.* [23] used Homotopy perturbation method and collocation method for Thermal performance analysis of porous fins with temperature-dependent heat generation. Hatami and Ganji [14] applied least square method (LSM) to study the thermal behaviour of convective-radiative in porous fin with different sections and ceramic materials. Also, Rostamiyaan *et al*. [24] applied variational iterative method (VIM) to provide analytical solution for heat transfer in porous fin. Ghasemi *et al.* [25] used differential transformation method (DTM) for heat transfer analysis in porous and solid fin. The approximate analytical methods as applied by past researchers solve the differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. However, the search for a particular value that will satisfy second the boundary condition or the determination of auxiliary parameters necessitated the use of software and such could result in additional computational cost in the generation of solution to the problem. Also, most of the approximate methods give accurate predictions only when the nonlinearities are weak or for small values of the fin thermo-geometric parameter, they fail to predict accurate solutions for strong nonlinear models. Also, the methods often involved complex mathematical analysis leading to analytic expression involving a large number terms and when they are routinely implemented, they can sometimes lead to erroneous results [26, 27]. Moreover, in practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers. Inevitably, simple yet accurate expressions are required to determine the fin temperature distribution, efficiency, effectiveness and the optimum parameter. In other to reduce the computation cost and time in the analysis of nonlinear problems, different wavelet collocation methods such as Legendre, Haar, Chebyshev, Leibnitz-Haar, cubic B-spline, sympletic, multi-sympletic, adaptive, multi-level, interpolating, rational, spectral, ultraspherical, first split-step, sine-cosine and semiorthogonal B-spline wavelet collocation methods have adopted to solve different nonlinear equations. The ease of use, simplicity and fast rate of convergence have in recent times made these methods gain popularity in nonlinear analysis of systems and they have been applied to nonlinear problems in heat transfer analysis of fins [28-32]. Also, the ability of these wavelet collocation methods to solve the nonlinear differential equations directly without simplification, linearization, perturbation, Taylor's series expansion, mesh independent study, determination of auxiliary parameters, functions, Lagrange multiplier, Adomian polynomials and recursive relations as carried out HAM, VIM, ADM, VIM, DTM etc. Therefore, in this paper, Legendre wavelet collocation method is applied to analyze heat transfer in porous fin with temperature-dependent thermal conductivity and internal heat generation. The numerical solutions are used effects of nonlinear thermal conductivity, convective and porosity parameters on the thermal conductivity of the fin. The results of obtained by LWCM are in excellent agreements with exact analytical solutions (for the linear model) and the direct numerical solutions (for the nonlinear model).

2 Problem formula

Consider a straight porous fin of length L and thickness t exposed on both faces to a convective environment at temperature T_∞ as shown in Fig.1.The dimension x pertains to the height coordinate which has its origin at the fin tip and has a positive orientation from fin tip to fin base. Assuming that the porous medium is homogeneous, isotropic and saturated with a single phase fluid, the physical properties of solid as well as fluid are constant except density variation of liquid, the fluid and porous mediums are locally thermodynamic equilibrium in the domain, surface radiative transfers and non-Darcian effects are negligible and there is no thermal contact resistance at the fin base and the fin tip is adiabatic type.

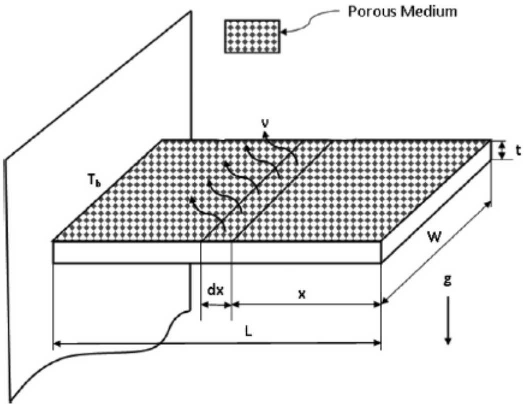

Fig. 1: Schematic of the longitudinal porous fin geometry with the internal heat generation.

Based on Darcy's model and the above assumptions, the governing equation for one-dimensional steady state heat transfer in the fin is given as

$$\frac{d}{dx}\left[k_{eff}(T)\frac{dT}{dx}\right]-\frac{hP(T-T_\infty)}{A}-\frac{\rho c_p g\beta' KP(T-T_\infty)^2}{Av_f}+q_a(T)=0 \tag{1}$$

The boundary conditions are

$$\begin{aligned} x=0, &\quad \tfrac{dT}{dx}=0 \\ x=L, &\quad T=T_b \end{aligned} \tag{2}$$

For many engineering applications, the thermal conductivity and the coefficient of heat transfer are temperature-dependent. Therefore, the temperature-dependent thermal properties and internal heat generation are given by

$$k_{eff}(T) = \phi k_f + (1-\phi)k_s = k_{eff,a}[1+\lambda(T-T_\infty)] \tag{3}$$

$$q_{\text{int}}(T) = q_a[1+\psi(T-T_\infty)] \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (1), we have

$$\frac{d}{dx}\left[[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right] - \frac{h(T-T_\infty)}{k_{eff,a}t} - \frac{\rho c_p g K\beta'(T-T_\infty)^2}{k_{eff,a}t\nu_f} + \frac{q_a}{k_{eff,a}}[1+\psi(T-T_\infty)] = 0 \tag{5}$$

On introducing the following dimensionless parameters in Eq. (6) into Eq. (5);

$$X = \frac{x}{L},\quad \theta = \frac{T-T_\infty}{T_b-T_\infty},\ Ra = Gr.\Pr = \left(\frac{\beta' g T_b t^3}{\nu_f^2}\right)\left(\frac{\rho c_p \nu_f}{k_{eff,a}}\right),\quad Da = \frac{K}{t^2},\quad Q = \frac{q\nu_f t}{\rho c_p \beta' g K (T_b-T_\infty)^2},$$
$$M^2 = \frac{hL^2}{k_{eff,a}t}, S_h = \left(\frac{\beta' g (T_b-T_\infty)t^3}{\nu_f^2}\right)\left(\frac{\rho c_p \nu_f K}{k_{eff,a}t^2}\right)\frac{(L/t)^2}{k_{eff,a}} = \frac{RaDa(L/t)^2}{k_{eff,a}},\ \gamma = \psi(T_b-T_\infty),\quad \beta = \lambda(T_b-T_\infty) \tag{6}$$

We arrived at the dimensionless governing differential Eq. (7) and the boundary conditions

$$\frac{d}{dX}\left[(1+\beta\theta]\frac{d\theta}{dX}\right] - M^2\theta - S_H\theta^2 + S_H Q\gamma\theta + S_H Q = 0 \tag{7}$$

If we expand Eq. (7), we have;

$$\frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)^2 - M^2\theta - S_H\theta^2 + S_H Q\gamma\theta + S_H Q = 0 \tag{8}$$

The boundary conditions are

$$\begin{array}{ll} X = 0, & \frac{d\theta}{dX} = 0 \\ X = 1, & \theta = 1 \end{array} \tag{9}$$

3 Method of solution: Legendre wavelet collocation method

There is a difficulty in developing an explicit exact analytical/closed-form solution for the above non-linear Eq. (8). Therefore, in this work, we apply Legendre wavelet collocation method. The wavelet algorithm is based on collocation method and the procedures for applications are described as follows.

Wavelets: Continuous wavelet are defined by the following formula

$$\psi_{a,b}(X) = |a|^{\frac{-1}{2}}\psi\left(\frac{X-b}{a}\right), a,b \in R, a \neq 0 \tag{10}$$

where a and b are dilation and translation parameters, respectively. The Legendre wavelets defined on the interval $[0,1]$ is given by

$$\psi_{n,m}(X) = \begin{cases} \sqrt{(m+1/2)}2^{k/2}p_m\left(2^kX-\hat{n}\right), & \frac{\hat{n}-1}{2^k} \leq \frac{\hat{n}-1}{2^k} \\ 0 & otherwise \end{cases} \tag{11}$$

where m=0,1,...,M-1 and n=1,2,...2^{k-1}. P_m(x) is the Legendre polynomial of order m

$$\begin{aligned}&P_0(X)=1, P_1(X)=X,\\&P_{m+1}(X)=\tfrac{2m+1}{m+1}XP_m(X)-\tfrac{m}{m+1}P_{m-1}(X)\\&m=1,2,3......,M-1.\end{aligned}\tag{12}$$

A function $f(x)$ defined in domain $[0,1]$ can be expressed as

$$f(X)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}c_{n,m}\psi_{n,m}(X)\tag{13}$$

where $c_{n,m}=<f(X),\psi_{n,m}(X)>$ in which $<\ldots>$ denotes the inner product. Taking some terms in infinite series, we can write Eq. (10) as

$$f(X)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n,m}(X)=C^T\psi(X)\tag{14}$$

where C and $\psi(X)$ are M x1 matrices given by

$$\begin{aligned}&C=[c_1,0,c_1,1,.....,c_1,M-1,c_2,1,......,c_2,M-1,c_{2^{k-1}},1,...,c_{2^{k-1}},M-1]^T\\&\psi(X)=\begin{bmatrix}\psi_{1,0}(X),\psi_{1,1}(X),...,\psi_{1,M-1}(X),\psi_{2,0}(X),......,\\ \psi_{2,M-1}(X),........,\psi_{2^{k-1},0}(X)\,\psi_{2^{k-1},1}(X),....,\psi_{2^{k-1},M-1}(X)\end{bmatrix}^T\end{aligned}\tag{15}$$

(i) Property of the product of two Legendre wavelets. If E is a given wavelets vector, then we have the property

$$E^T\psi\psi^T=\psi^T\hat{E}\tag{16}$$

(ii) Operational matrix of integration: The integration of wavelets $\psi(X)$ which is defined in Eq. (8) can be obtained as

$$\int_0^X\psi(s)ds=P\psi(X), X\in[0,1]$$

where P is $2^{k-1}Mx2^{k-1}M$, the operational matrix of integration is given by

$$P=\frac{1}{2}\begin{pmatrix}1&\frac{1}{3}&0&\cdots\cdots\cdots\cdots&0\\ \frac{-1}{\sqrt{3}}&0&\frac{1}{\sqrt{15}}&\cdots\cdots\cdots\cdots&0\\ 0&\frac{-1}{\sqrt{15}}&0&\cdots\cdots\cdots\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&0&\cdots\cdots\cdots\ddots&\frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}}\\ 0&0&0&\cdots\cdots\cdots\cdots&\frac{-\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}}\end{pmatrix}.\tag{17}$$

3.1 Legendre wavelet collation method

Let

$$\theta''(X) = C^T \psi(X) \tag{18}$$

Integrating Eq. (15) with respect to x from 0 to x, we have

$$\theta'(X) = \theta'(0) + C^T P \psi(x) \qquad \Rightarrow \theta'(X) = C^T P \psi(x) \quad since \quad \theta'(0) = 0. \tag{19}$$

If we integrate Eq. (16) and use the boundary conditions, we arrived at

$$\theta(X) = \theta(0) + C^T P^2 \psi(X). \tag{20}$$

Put $X=1$ in (16), we have

$$\theta(0) = 1 - C^T P^2 \psi(1), \quad \text{since} \quad \theta(1) = 1, \tag{21}$$

on substituting Eq. (18) into E.(17)

$$\theta'(X) = 1 - C^T P^2 \psi(1) + C^T P^2 \psi(X). \tag{22}$$

Again, integrating above equation, with respect to X from 0 to X, we obtain

$$\theta(X) = 1 - C^T P^2 \psi(1) + C^T P^2 \psi(X) \tag{23}$$

Substituting, $\theta''(X)$, $\theta''(X)$and $\theta''(X)$ in Eq. (6), we arrived at

$$\begin{aligned}&(1+\beta\theta) C^T \psi(X) + \beta\left(C^T P \psi(X)\right)^2 - \left(M^2 - S_H Q \gamma\right)\left\{1 - C^T P \psi(1) + C^T P \psi(X)\right\}\\ &-S_H\left\{1 - C^T P \psi(1) + C^T P \psi(X)\right\}^2 + S_H Q = R(X, c_1, c_2, \ldots.., c_n)\end{aligned} \tag{24}$$

On expanding Eq. (21), we have

$$\begin{aligned}&\left[1 + \beta\left\{1 - C^T P^2 \psi(1) + C^T P^2 \psi(X)\right\}\right] C^T \psi(X) + \beta\left\{C^T P \psi(X)\right\}^2\\ &-\left(M^2 - S_H Q \gamma\right)\left\{1 - C^T P^2 \psi(1) + C^T P^2 \psi(X)\right\} - S_H\left\{1 - C^T P \psi(1) + C^T P \psi(X)\right\}^2\\ &+S_H Q = R(X, c_1, c_2, \ldots.., c_n)\end{aligned} \tag{25}$$

Choosing n collocation points i.e. x_i, $i=1,2,3,\ldots,n$ in the interval (0,1), at which residual $R(x, c_i)$ equal to zero. The number of such points gives the number of coefficient c_i, $i=1,2,3,..,n$.

$$C = [c_1, 0, c_1, 1, \ldots.., c_1, M-1, c_2, 1, \ldots..., c_2, M-1, c_{2^{k-1}}, 1, \ldots, c_{2^{k-1}}, M-1]^T$$

Thus, we get $R(X, c_1, c_2, c_3, \ldots, c_n) = 0$, $i=1,2,3,\ldots,n$. The above Eq. (22) gives system of nonlinear equations which are solved simultaneously using Newton-Raphson method and the values of C are obtained. Substituting the values of C in Eq. (20), the approximate solution of $\theta(X)$ is found.

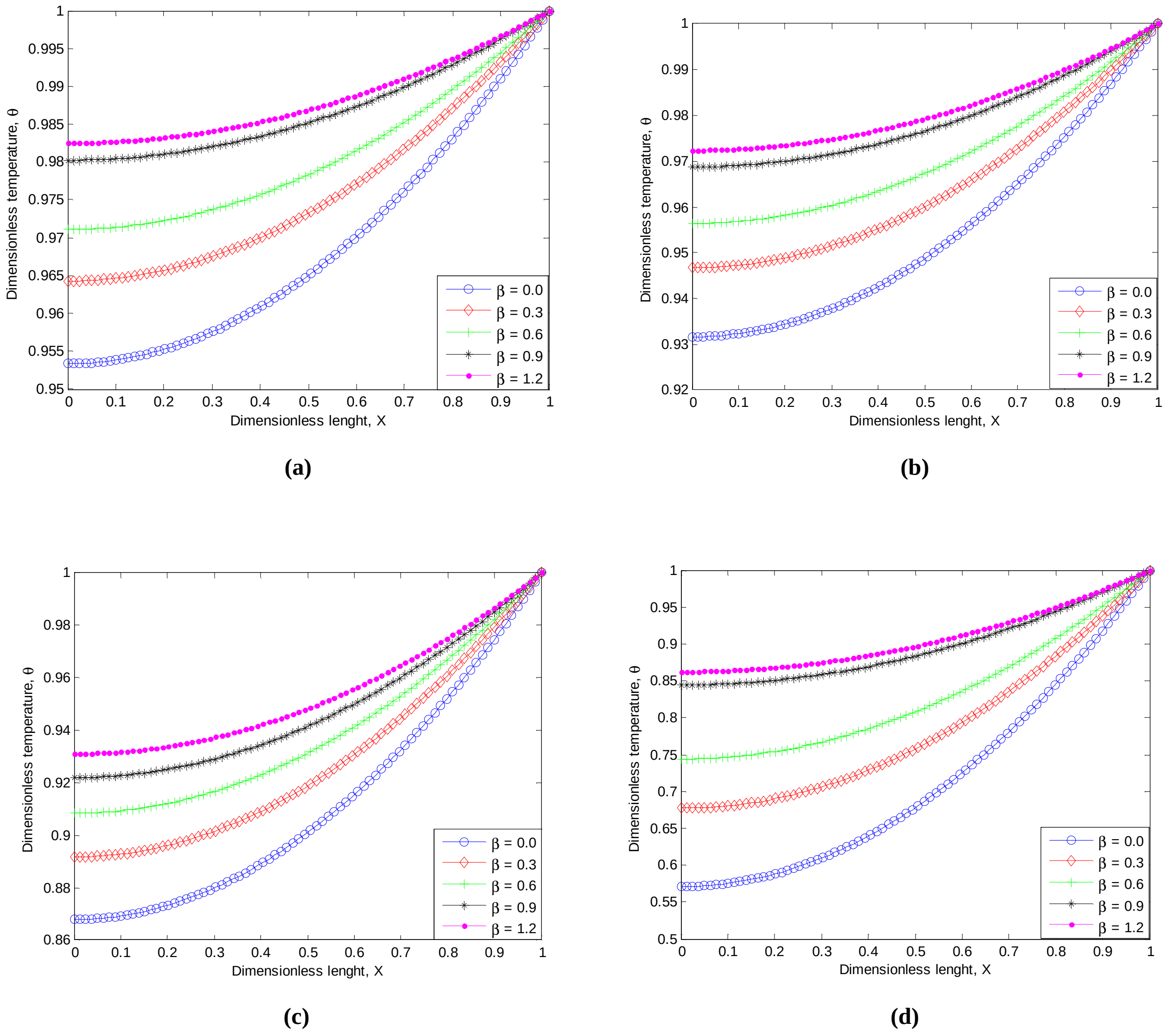

Fig. 2: Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) $M = 0.3, S_h, = 0, Q = 0.4, \gamma = 0.2,$ (b) $M = 0.3, S_h, = 0.1, Q = 0.4, \gamma = 0.2,$ (c) $S_h, = 0.5, M = 0.3, Q = 0.4, \gamma = 0.2,$ (d) $M = 0.8, S_h, = 0.1, Q = 0.4, \gamma = 0..$

Figs. 2a-2d show effects of nonlinear thermal conductivity parameters on the dimensionless temperature distribution and by extension on the rate of heat transfer. From the figures, it is shown that as the non-linear thermal conductivity parameter increases, the dimensionless temperature distribution in the fin decreases. The effects of porous parameter or porosity on the temperature distribution in the porous fin are shown in Fig 3a-d. From the figures, as the porosity parameter increases, the temperature decreases rapidly and the rate of heat transfer (the convective heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates) as depicted in the figures. The rapid decrease in fin temperature due to increase in the porosity parameter is because as porosity parameter, $S_h$increases and in consequent, the Darcy and Raleigh number increase, the permeability of the porous fin increases and therefore the ability of the working fluid to penetrate through the fin pores increases, the effect of buoyancy force increases and thus the fin convects more heat, the rate of heat transfer from the fin is enhanced and the thermal performance of the fin is increased. Therefore, increase in the porosity of the fin improves fin efficiency due to increasing in convection heat transfer.

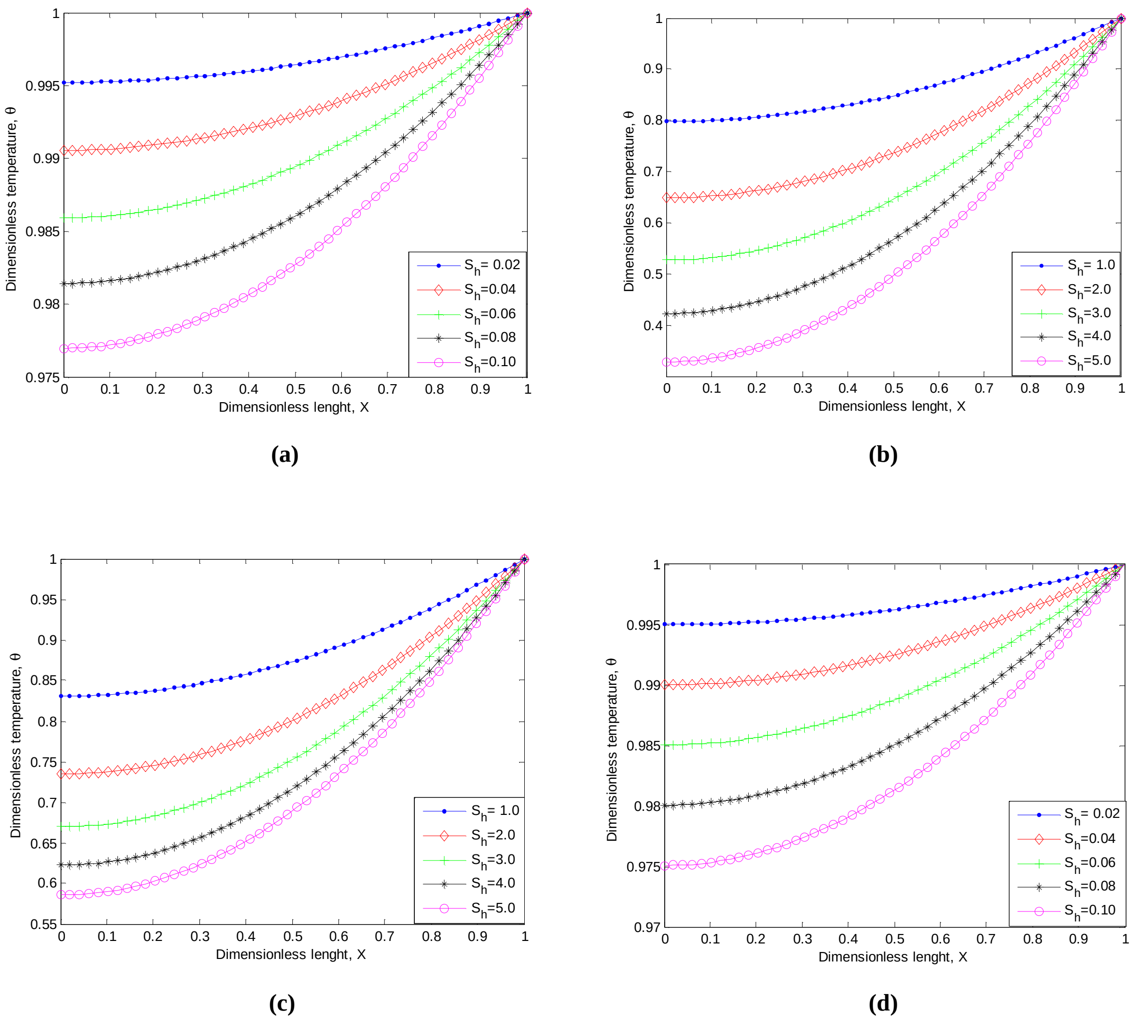

Fig. 3: Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when
(a) $\beta = 0.5, M = 0.5, Q = 0.2, \gamma = 0.4,$ (b) $\beta, = 0.5, M = 1.0, Q = 0.2, \gamma = 0.4,$
(c) $\beta = 0.5, M = 2.0; Q = 0.2, \gamma = 0.4,$ (d) $\beta = 0.5, M = 10, Q = 0.2, \gamma = 0.4.$

The effects of the internal heat generation on the thermal stability of the fin is shown in Fig. 4a-d and Fig. 5a-b. It is obvious that as porous parameter, S_h increases to a certain value, the dimensionless temperature distribution decreases. The effects of the internal heat generation on the thermal stability of the fin is shown in Fig. 4a-b, it is obvious that as porous parameter, S_h increases to a certain value, the dimensionless temperature distribution at the fin tip results in negative value (which shows thermal instability) at *x=0*, contradicting the assumption made in the analysis. However, value of porosity parameter for the thermal stability increases with increase in internal heat generation parameter, Q (Fig. 4c) and thermal conductivity parameters, β. This fact was not established in the Kiwan [3] numerical analysis of the same problem for the large values of S_h.

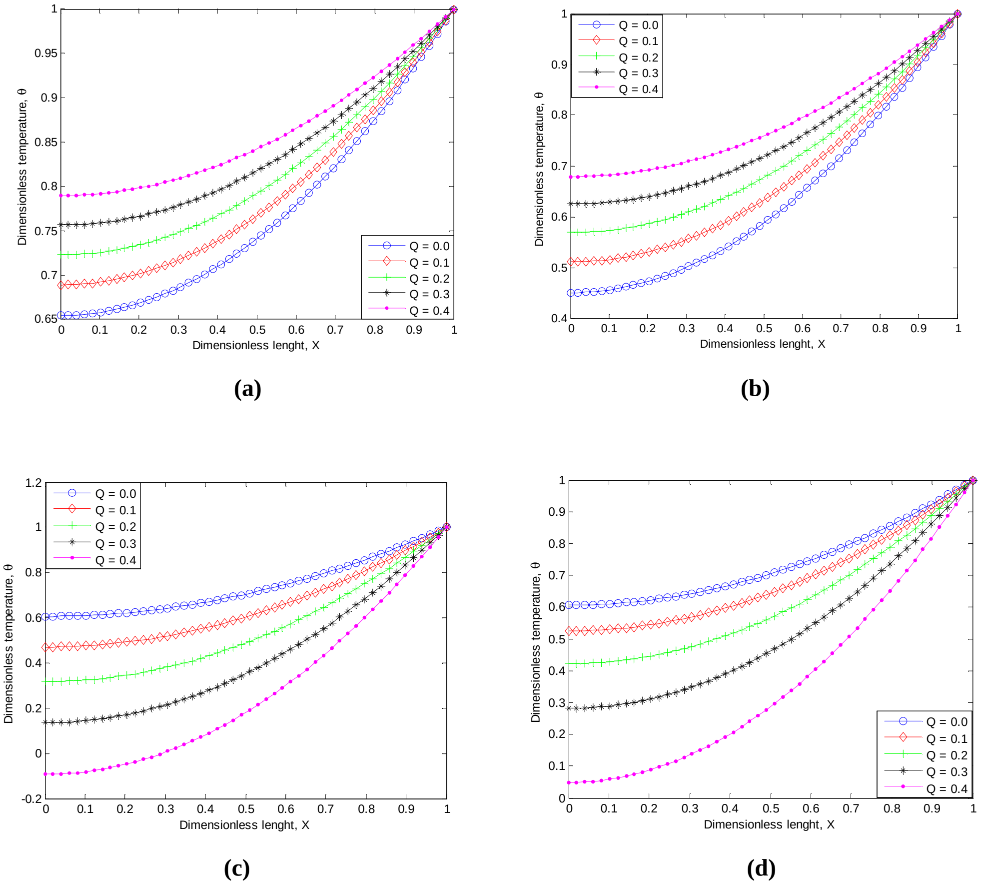

Fig. 4: Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when
(a) $M = 0.5, S_h = 2.0, \beta = 0.5, \gamma = 0.2,$ (b) $M = 0.5, S_h = 5.0, \beta, = 0.5, \gamma = 0.2,$
(c) $M = 2.0, S_h = 5.0, \beta = 0.5, \gamma = 0.2,$ (d) $M = 2.0, S_h = 5.0, \beta = 0.5, \gamma = 2.0.$

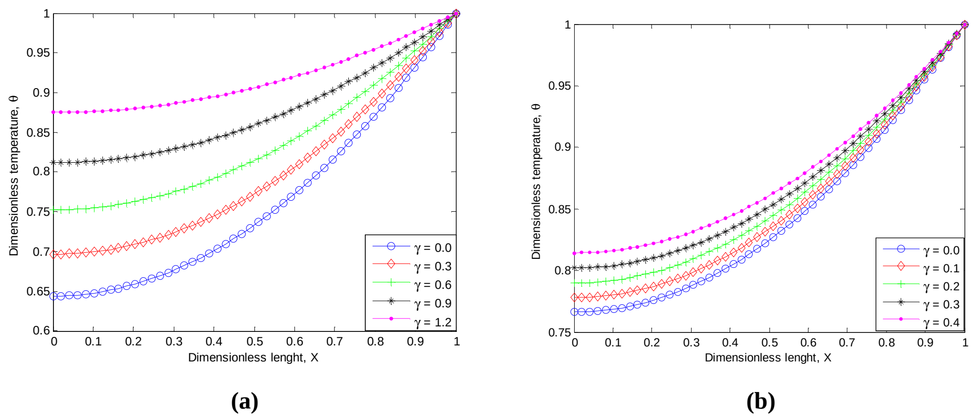

Fig. 5: Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when
(a) $S_h = 5.0, M = 0.5, \beta = 0.5, Q = 0.4, \gamma = 0.2$ (b) $S_h = 5.0, \beta = 0.5, M = 0.5, Q = 0.4, \gamma = 0.2.$

X	Exact	LWCM (The Present study)	X	NM	LWCM (The Present study)
0.0	0.6481	0.6481	0.0	0.9581	0.9581
0.1	0.6513	0.6513	0.1	0.9585	0.9585
0.2	0.6611	0.6611	0.2	0.9597	0.9597
0.3	0.6774	0.6774	0.3	0.9618	0.9618
0.4	0.7006	0.7006	0.4	0.9647	0.9647
0.5	0.7308	0.7308	0.5	0.9685	0.9685
0.6	0.7682	0.7682	0.6	0.9730	0.9730
0.7	0.8134	0.8134	0.7	0.9785	0.9785
0.8	0.8667	0.8667	0.8	0.9846	0.9846
0.9	0.9287	0.9287	0.9	0.9919	0.9919
1.0	1.0000	1.0000	1.0	1.0000	1.0000

Fig. 6: Table of comparison of results

4 Conclusion

In this work, heat transfer study of porous fin with temperature-dependent thermal conductivity and internal heat generation has been analyzed numerically using Legendre wavelet collocation method. The numerical solutions are used effects of nonlinear thermal conductivity, convective and porosity parameters on the thermal conductivity of the fin. The Legendre wavelet collocation method is verified with the results of numerical method using Runge-Kutta method and good agreements are established.

Nomenclature

A cross sectional area of the fins, m^2
A_b porous fin base area
A_s porous fin surface area
h heat transfer coefficient, Wm $^{-2}k^{-1}$
h_b heat transfer coefficient at the base of the fin, Wm $^{-2}k^{-1}$
c_p specific heat of the fluid passing through porous fin $(J/kg-K)$
Da Darcy number

g gravity constant (m/s^2)
h heat transfer coefficient over the fin surface (W/m^2K)
H dimensionless heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
k thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
k_b thermal conductivity of the fin material at the base of the fin, $Wm^{-1}k^{-1}$
k_{eff} effective thermal conductivity ratio
K permeability of the porous fin (m^2)
L Length of the fin, m
M dimensionless thermo-geometric parameter
m mass flow rate of fluid passing through porous fin(kg/s)
P perimeter of the fin(m)
Q dimensionless heat transfer rate per unit area
q_b heat transfer rate per unit area at the base (W/m^2)
Q_b dimensionless heat transfer rate the base in porous fin
Q_s dimensionless heat transfer rate the base in solid fin
Ra Rayleigh number
S_h Porosity parameter
t thickness of the fin
T_b base temperature(K)
T fin temperature (K)
T_a ambient temperature, K
T_b Temperature at the base of the fin, K
v average velocity of fluid passing through porous fin(m/s)
x axial length measured from fin tip (m)
X dimensionless length of the fin
w width of the fin
q internal heat generation in W/m^3

Greek Symbols

β thermal conductivity parameter or non-linear parameter
δ thickness of the fin, m
δ_b fin thickness at its base.
γ dimensionless internal heat generation parameter
θ dimensionless temperature
θ_b dimensionless temperature at the base of the fin
η efficiency of the fin
ε effectiveness of the fin
β' coefficient of thermal expansion (K^{-1})
ε porosity or void ratio
υ kinematic viscosity (m^2/s)
ρ density of the fluid (kg/m^3)

Subscripts s solid properties
f fluid properties
eff effective porous properties

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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