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# Application of the method of lines for solving the KdV-Burger equation

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**Abstract:** This paper presents two methods for obtaining the solutions to the nonlinear Korteweg-de Vries–Burgers (KdVB) equation. The first is the method of lines (MOL). The second method is Adomian decomposition method (ADM). The numerical results of the MOL are compared with the analytical results of the ADM. In order to show the reliability of the considered methods we have compared the obtained solutions with the exact ones. The results reveal that the both methods are effective and convenient for solving such types of partial differential equations but the method of lines gives accurate results over the analytical method.

**Keywords:** KdV-Burger equation, the method of lines, Adomian decomposition method, finite difference scheme, Runge–Kutta method.

## 1 Introduction

This paper is concerned with the initial-boundary value problem associated with the nonlinear dispersive and dissipative wave which was formulated by Korteweg, de Vries and Burgers in the form

$$\frac{\partial u}{\partial t} + \mu\, u\, \frac{\partial u}{\partial x} - \theta\, \frac{\partial^2 u}{\partial x^2} + \delta\, \frac{\partial^3 u}{\partial x^3} = 0 \quad (1)$$

where $\mu$ , $\theta$, $\delta$ are constant coefficients.

It is well known that many physical phenomena can be described by the Korteweg-de Vries–Burgers equation. Eq. (1) can serve as a nonlinear wave model of a fluid in an elastic tube [1],of a liquid with small bubbles [2,] and turbulence [3,4].The coefficients $\theta$and $\delta$in Eq. (1) represent the damping and the dispersion coefficients, respectively. We note that Eq.(1) is non integrable.

Soliton solutions of the KdV equation are known since long time [5,6]. Many problems, however, involve not only dispersion but also dissipation, and these are not governed by the KdV equation. More complicated problems are the flow of liquids containing gas bubbles [7,8],and the propagation of waves in an elastic tube filled with a viscous fluid [9,10]. Other cases regarded the governing evolution equation can be shown to be the so-called Korteweg-de Vries–Burgers equation.

In particular, the travelling wave solution to the KdVB equation has been studied extensively. Johnson [11], Demiray [12] and Antar and Demiray [13] derived KdVB equation as the governing evolution equation for waves propagating in fluid-filled elastic or viscoelastic tubes in which the effects of dispersion, dissipation and nonlinearity are present.

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The KdVB equation is a one-dimension generalization of the model description of the density and velocity fields that takes into account pressure forces as well as the viscosity and the dispersion. It may be a more flexible tool for physicists than the Burgers equation. Several studies in the literature, employing a large variety of methods to derive explicit solutions for KdVB equation (1).

## 2 The method of lines

The method of lines [14] is a well established numerical technique (or rather a semi analytical method) for the analysis of transmission lines, waveguide[15-18]. The method of lines is regarded as a special finite difference method but more effective with respect to accuracy and computational time than the regular finite difference method. It basically involves discretising a given differential equation in one or two dimensions while using analytical solution in the remaining direction. The MOL has the merits of both the finite difference method and analytical method, it does not yield spurious modes nor have the problem of relative convergence. The MOL is generally recognized as a comprehensive and powerful approach to the numerical solution of time-dependent partial differential equations (PDEs). This method usually proceeds in two separate steps: first, approximating the spatial derivatives. Second, the resulting system of semi discrete (discrete in space–continuous in time) ordinary differential equations (ODEs) is integrated in time. The essence of the method of lines is a way of approximating PDEs by ODEs. Obviously, an advantage of the MOL is that one can use all kinds of ODE solvers and techniques to solve the semi-discrete ODEs directly.

## 3 Solving the KdV-Burger equation using the MOL

Consider KdV-Burger equation (1) with the initial condition

$$u(x,0) = (\frac{1}{25})(\frac{25\,c}{v} - 100v^2\delta + \frac{\theta^2}{\delta} + 12v^2\delta sech^2(vx) - \frac{12}{5}v\theta\tanh(x)) \tag{2}$$

and the boundary conditions

$$u(a,t) = 0.98, u(b,t) = 0.02. \tag{3}$$

The exact solution of this problem is given by

$$u(x,t) = ((\frac{1}{25})(\frac{25\,c}{v} - 100v^2\delta + \frac{\theta^2}{\delta}) + 12v^2\delta sech^2(vx-ct) - \frac{12}{5}v\theta\tanh(vx-ct)) \tag{4}$$

The solution domain of the KdV- Burger equation (1) is the rectangle a $\leq$ x $\leq$ b , $0 \leq t \leq T$.

Let us subdivide it into uniform rectangular meshes by the lines $x_i = ih\,(i = 0,1,2,3,\ldots..N)$ and the lines $t_j = jk\,(j = 1,2,3,\ldots..)$, We replace the partial derivatives depend on spatial variables $u_x$, dissipation term $u_{xx}$ and dispersion term $u_{xxx}$ in KdV-Burger equation (1) with known finite difference approximations at point $x_i$.

The solution of the method of lines using fourth order finite difference scheme for $u_x$ ,$u_{xx}$ , and $u_{xxx}$ is denoted by MOLI, however the solution using a second order finite difference scheme for $u_x$ ,$u_{xx}$ and $u_{xxx}$ is denoted by MOLII.

The derivative $u_x$ in KdV-Burger equation (1) is computed by finite differences scheme in two way

(1) second order approximations $u_x = \frac{u_{i+1}-u_{i-1}}{2h} + O(h^2)$.
(2) fourth order approximations $u_x = \frac{u_{i-2}-8u_{i-1}+8u_{i+1}-u_{i+2}}{12h} + O(h^4)$.

The derivative $u_{xx}$ in KdV-Burgers equation (1) is computed by finite differences in two ways

(1) second order approximations $u_{xx} = \frac{u_{i-1}-2u_i+u_{i+1}}{h^2} + O(h^2)$.
(2) fourth order approximations $u_{xx} = \frac{-u_{i-2}+16u_{i-1}-30\,u_i+16u_{i+1}-u_{i+2}}{12h^2} + O(h^4)$.

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(2) fourth order approximations $u_{xxx} = \frac{u_{i-3}-8u_{i-2}+13u_{i-1}-13u_{i+1}+8u_{i+2}-u_{i+3}}{8h^3} + O(h^4)$.

Applying the above finite difference schemes to Eq. (1) yields a system of ordinary differential equations for the unknown $u_i$ as functions in $t$ as follows:

$$\frac{du_i(t)}{dt} = f(u_i), i = 1(1)N-1. \tag{5}$$

Using the fourth order finite difference scheme for $u_x$ ,$u_{xx}$ *and* $u_{xxx}$ (MOLI), we have

$$\begin{aligned}\frac{du_i(t)}{dt} = {} & \delta\frac{(u_{i-3}(t)-8u_{i-2}(t)+13u_{i-1}(t)-13u_{i+1}(t)+8u_{i+2}(t)-u_{i+3}(t))}{8h^3}\\ & -u_i(t)\frac{u_{i-2}(t)-8u_{i-1}(t)+8u_{i+1}(t)-u_{i+2}(t)}{12h}\\ & +\theta\frac{-u_{i-2}+16u_{i-1}-30\,u_i+16u_{i+1}-u_{i+2}}{12h^2},\quad i = 1,2,3,...,N-1.\end{aligned}$$

And for the second order finite difference scheme both $u_x$ *and* $u_{xxx}$ (MOLII), we have

$$\begin{aligned}\frac{du_i(t)}{dt} = {} & -\delta\frac{(-u_{i-2}(t)+2u_{i-1}(t)+2u_{i+1}(t)+8u_{i+2}(t))}{2h^3}\\ & +\theta\frac{u_{i-1}-2u_i+u_{i+1}}{h^2} - u_i(t)\frac{u_{i+1}(t)-u_{i-1}(t)}{2h},\ i = 1,2,3,...,N-1.\end{aligned}$$

Thus, we have the system of differential equations of one independent variable t. This system can be easily solved by using fourth order Runge–Kutta scheme

$$\begin{aligned} U^{n+1} &= U^n + \frac{\Delta t(K_1+2K_2+2K_3+K_4)}{6},\ K_1 = F(U^n),\\ K_2 &= F\left(U^n+\frac{\Delta t}{2}K_1\right),\ K_3 = F\left(U^n+\frac{\Delta t}{2}K_2\right),\ K_4 = F(U^n+\Delta t\,K_3).\end{aligned}$$

The computational domain is $[-20,20]*[0,30]$. The computational results are listed in Tables $1\ldots5$.

The results obtained using the method of lines have been compared with the exact solution as a plots of the solution and the absolute error (AE) profiles of the KdV-Burgers equation where $\theta$ and $\delta$ are constants at $c = 0.5, \Delta t = 10^{-3}$, $v = \frac{\theta}{10\delta}$, $t \in [0,30]$.
We obtain the MOLI solutions of KdV–Burgers equation with higher accuracy than MOLII. The obtained results demonstrate the reliability of the MOL and its wider applicability to nonlinear evolution equations.

## 4 Adomian decomposition method

Following the analysis of Adomian [Adomian, 1994] equation (1) can be rewritten in an operator form as the following:

$$L(u)+R(u)+N(u) = g(t) \tag{6}$$

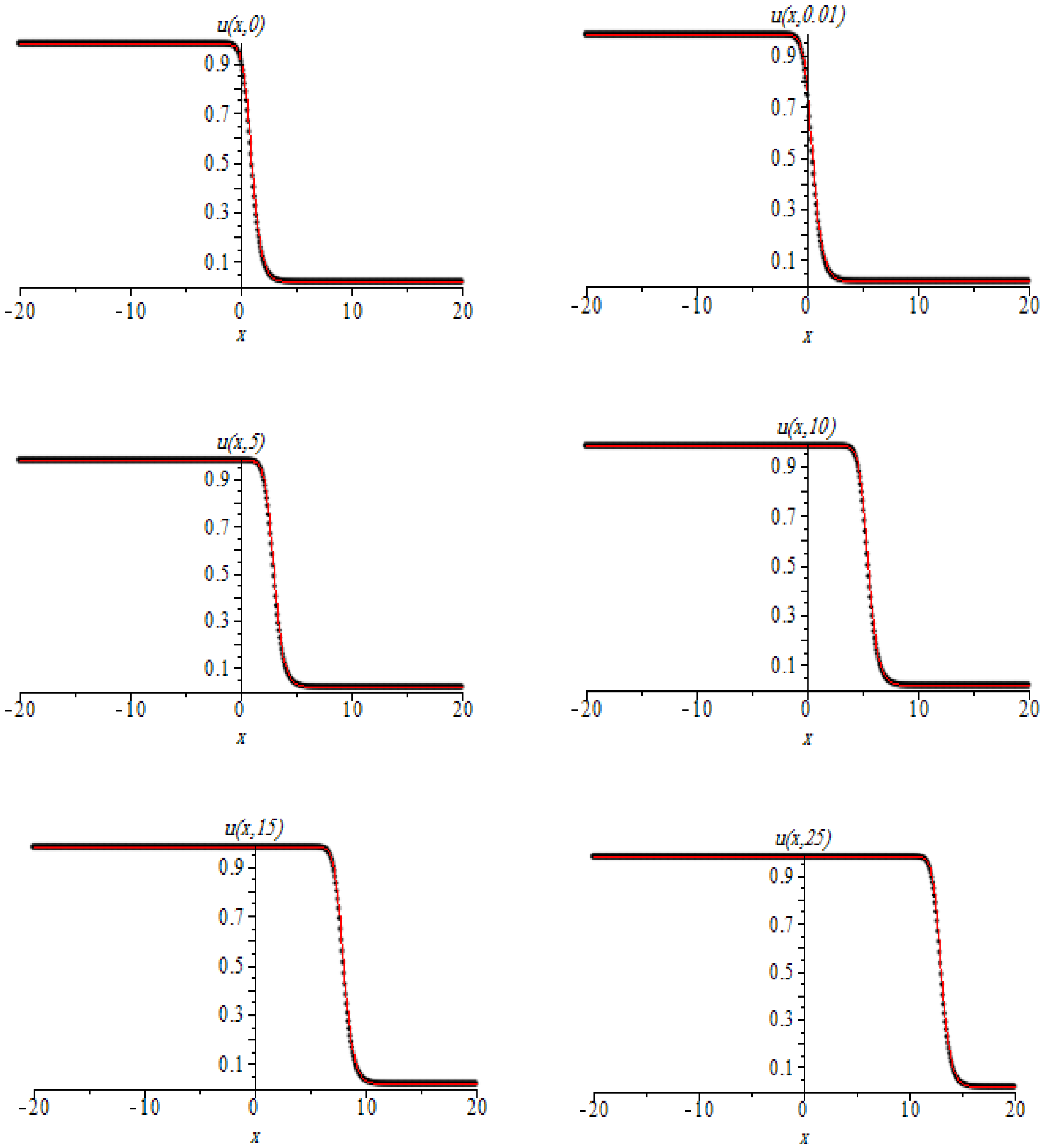


**Fig. 1:** Comparison of MOL I (dotted line) and exact (solid line) solutions at $N = 500, \delta = 0.02, \theta = 0.2, c = 0.5, \nu = \frac{\theta}{10\delta}$ and $t \in [0,30]$.

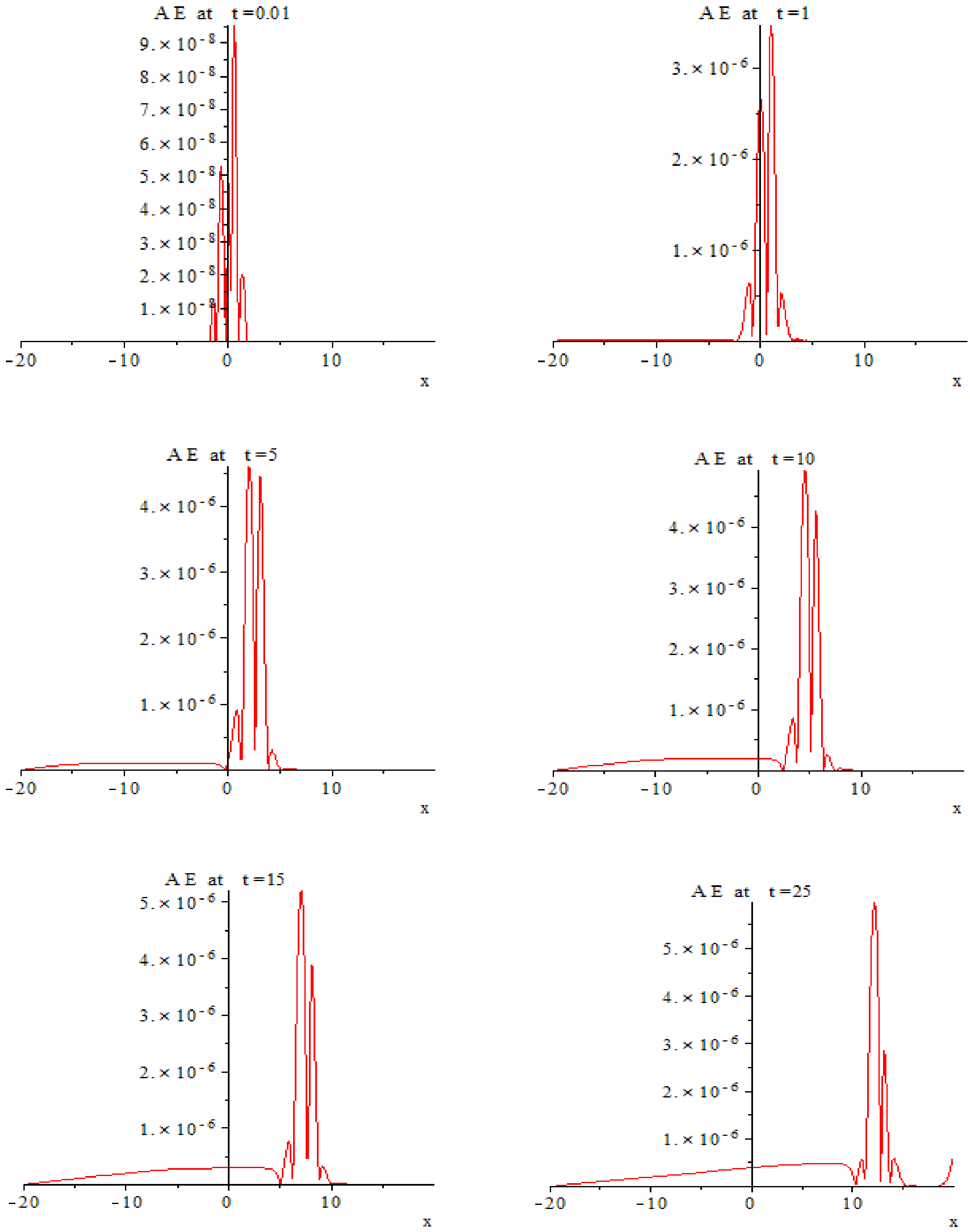


**Fig. 2:** The absolute error between the exact solution $tu(x,t)$ and the (MOL I) solution for KdV-Burger equation for $t \in [0,30]$.

where $L = \frac{\partial}{\partial t}$ is the operator of the highest-ordered derivatives with respect to $t$ and $R$ is the remainder of the linear operator. The nonlinear term is represented by $N(u)$. Thus we get

$$L(u) = g(t) - R(u) - N(u) \tag{7}$$

The inverse $L^{-1}$ is assumed an integral operator given by

$$L^{-1} = \int_0^t (.)\, dt. \tag{8}$$

The operating with the operator $L^{-1}$ on both sides of Eq. (9) we have

$$u = f_0 + L^{-1}(g(t) - R(u) - N(u)) \tag{9}$$

where $f_0$ is the solution of homogeneous equation

$$L(u) = 0. \tag{10}$$

The integration constants involved in the solution of homogeneous equation (10) are to be determined by the initial or boundary condition according as the problem is initial-value problem or boundary - value problem. The ADM assumes that the unknown function $u(x,t)$ can be expressed by an infinite series of the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{11}$$

and the nonlinear operator $F(u)$ can be decomposed by an infinite series of polynomials given by

$$F(u) = \sum_{n=0}^{\infty} A_n. \tag{12}$$

where $u_n(x,t)$ will be determined recurrently, and $A_n$ are the so-called polynomials of $u_0$ , $u_1$ , $u_2$, ..., $u_n$ defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[F(\sum_{i=0}^{\infty}\lambda^i]_{\lambda=0, \quad n=0,1,2,3,...}. \tag{13}$$

It is now well known in the literature that these polynomials can be constructed for all classes of nonlinearity according to algorithms set by Adomian [19,22].

## 5 Solution of KdV-Burger equation using ADM

In the following section, we discuss the solution of the KdV-Burger equation using ADM. Eq. (1) can be written in an operator form:

$$Lu = -\delta\, u_{xxx} - u\, u_x + \theta\, u_{xx} \tag{14}$$

where the differential operator L is L = $\frac{\partial}{\partial t}$.

Applying the inverse operator $L^{-1}$ on both sides of (14) and using the decomposition series (12) and (13) yield

$$sum_{n=0}^{\infty} u_n(x,t) = \left(\frac{1}{25}\right)\left(\frac{25\,c}{v} - 100v^2\delta + \frac{\theta^2}{\delta} + v^2\delta sech^2(vx) - \frac{12}{5}v\,\theta\,tanh(x)\right) + L^{-1}\left(-\left(\sum_{n=0}^{\infty} A_n\right) - \delta\left(\sum_{n=0}^{\infty} u_n\right)_{xxx} + \theta\left(\sum_{n=0}^{\infty} u_n\right)_{xx}\right)$$

where $A_n$ are Adomian polynomials that represent the nonlinear term $uu_x$ and given by

$$\begin{aligned} A_0 &= u_{0x}u_0 \\ A_1 &= u_{0x}u_1 + u_{1x}u_0 \\ A_2 &= u_{0x}u_2 + u_{1x}u_1 + u_{2x}u_0 \\ A_3 &= u_{0x}u_3 + u_{1x}u_2 + u_{2x} \\ A_4 &= u_{0x}u_4 + u_{1x}u_3 + u_{2x}u_2 + u_{3x}u_1 + u_{4x}u_0. \end{aligned} \tag{15}$$

Other polynomials can be generated in a like manner. The first few components of $u_n(x,t)$ follows as

$$\begin{aligned} u_0(x) &= f(x) \\ u_1(x,t) &= L^{-1}(-A_0 + \theta u_{0xx} - \delta u_{0xxx}) \\ u_2(x,t) &= L^{-1}(-A_1 + \theta u_{1xx} - \delta u_{1xxx}) \\ u_3(x,t) &= L^{-1}(-A_2 + \theta u_{2xx} - \delta u_{2xxx}) \end{aligned} \tag{16}$$

The scheme in (16) can easily determine the components $u_n(x,t), n \geq 0$. So it is possible to calculate more components in the decomposition series to enhance the approximation. The resulting components using initial condition (2) and $\delta = 0.02, \theta = 0.2, c = 0.5, v = \frac{\theta}{10\delta}$.

$$\begin{aligned} u_0(x,t) &= u_{(x,0)} = \left(\frac{1}{25}\right)\left(\frac{25\,c}{v} - 100v^2\delta + \frac{\theta^2}{\delta} + 12v^2\delta\;sech^2(vx) - \frac{12}{5}v\theta\;tanh(x)\right) \\ u_1(x,t) &= L^{-1}(-A_0 + \theta u_{0xx} - \delta u_{0xxx}) = \frac{0.24t(\sinh(x) + cosh(x))}{cosh^3(x)} \\ u_2(x,t) &= L^{-1}(-A_1 + \theta u_{1xx} - \delta u_{1xxx}) = \frac{0.06t^2(2\;cosh(x)\;sinh(x) + 2\;cosh^2(x) - 3)}{cosh^4(x)} \\ u_3(x,t) &= L^{-1}(-A_2 + \theta u_{2xx} - \delta u_{2xxx}) = \frac{0.02t^3(-6\;sinh(x) + 2\;sinh(x)\;\;cosh^2(x) - 3\;cosh(x) + 2\;cosh^3(x))}{cosh^5(x)} \end{aligned} \tag{17}$$

So, the solution in a series form is given by

$$\begin{aligned} u(x,t) = 0.5 + 0.24\,sech^2(x) - 0.48\tanh(x) &+ \frac{0.24t(\sinh(x) + \cosh(x))}{cosh^3(x)} \\ &+ \frac{0.06t^2(2\cosh(x)\sinh(x) + 2\,cosh^2(x) - 3)}{cosh^4(x)} \\ &+ \frac{0.02t^3(-6\sinh(x) + 2\sinh(x)\;\;cosh^2(x) - 3\cosh(x) + 2\,cosh^3(x))}{cosh^5(x)}. \end{aligned}$$

We plot the solution and AE profiles of KdV-Burger equation at t= 0.01, 1, 2, 2.5 using ADM.

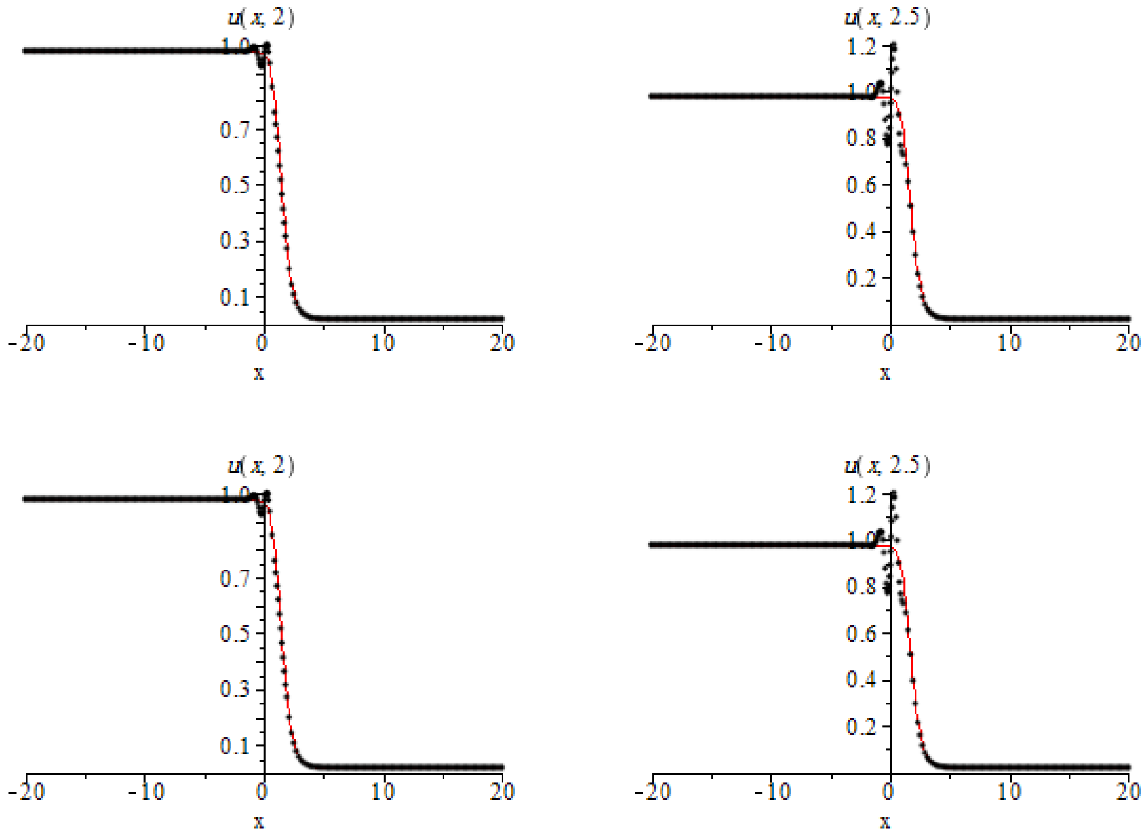


**Fig. 3:** Comparison of ADM (dotted line) and exact (solid line) solutions corresponding to KdV-Burger equation at $t = 0.01, 1, 2$ and 2.5 where $-20 \le x \le 20$.

## 6 Numerical results and some illustrations

In this section, we present the following tables to describe the absolute errors between the exact and numerical solutions. The tables illustrate the errors for both methods, the Adomian decomposition method and the method of lines compared with the exact solution, at different values of $t$.

It is observed that if we increase the number of terms in algorithm (17), the size of calculation is maximized with no increase in accuracy so the reduction of terms facilities the construction of Adomian polynomials for nonlinear operators and gives the same accuracy. ADM can provide the solution with minimal number of iterations.

A comparison between the numerical MOL and the decomposition methods with those obtained by exact solution are given for $\Delta t = 10^{-3}$. From the tables, we can observe that the decomposition method is accurate as compared with MOL at small period of time but with increasing the time, the MOL is more accurate when compared with ADM.

It is noted that when the time increase by using ADM gradually less accuracy and leads to increased errors. From the comparative study between ADM and the MOL we may conclude that the MOL is more accurate than ADM. To demonstrate the efficiency of our methods we report the absolute errors in some arbitrary points in Tables 1-5.
From the above tables we can infer that ADM have better convergence at small $t$. However, a closer look at the errors of ADM reveals that the error considerably increments with increasing the time. This is an indication of little stability on the part of ADM, in contrast to the MOL. By increasing the number of terms not affect on the accuracy of solution.

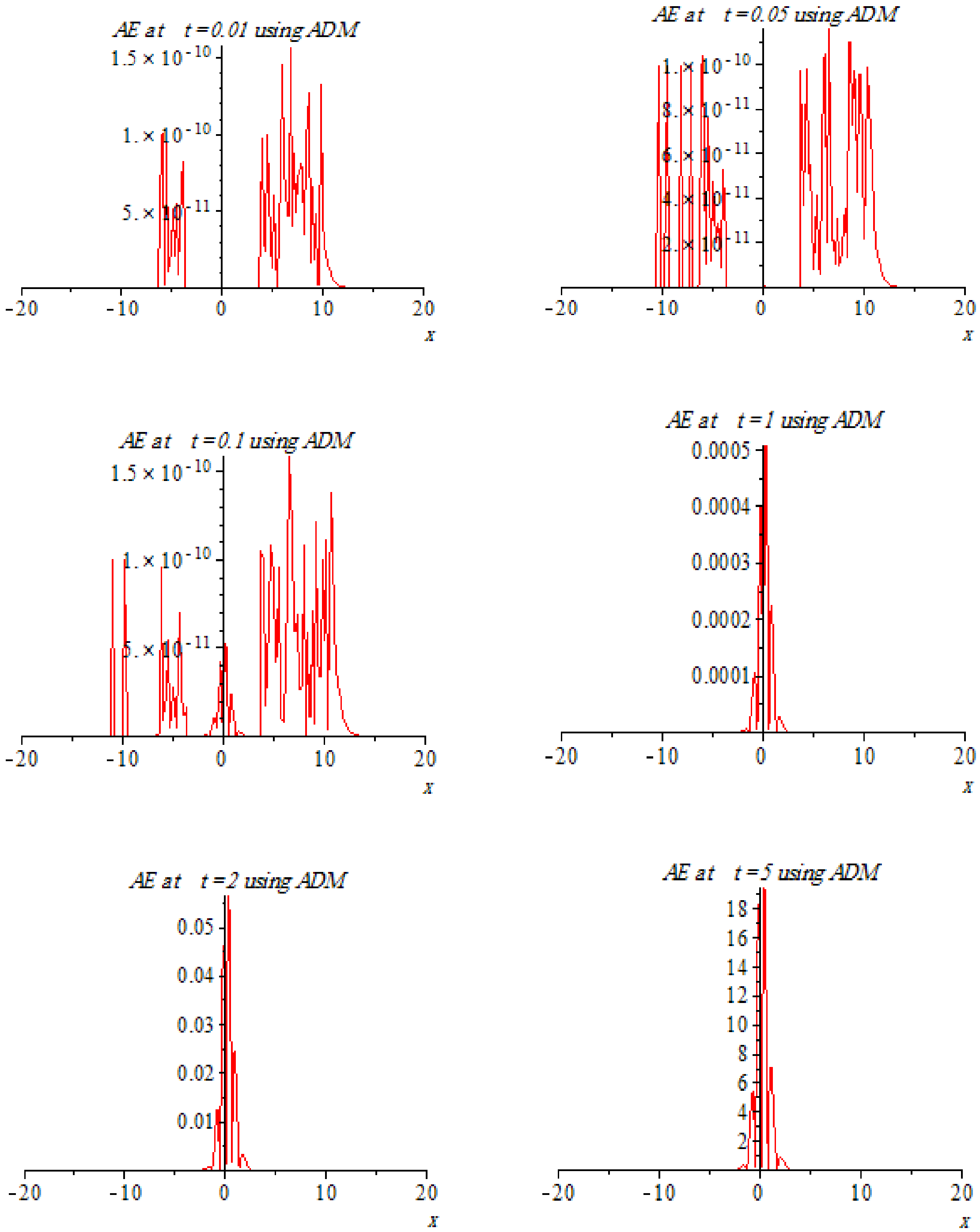


**Fig. 4:** The absolute error between the exact solution $u(x,t)$ and the (ADM) solution using seven terms for KdV-Burger equation at $t = 0.01, 0.05, 0.1, 1, 2$ and 5 where $-20 \leq x \leq 20$.

**Table 1:** The absolute error of MOL and ADM approximation solutions for $N = 500$ and $t = 1$ for KdV-Burger equation.

| t=1 | | | | |
|---|---|---|---|---|
| x | Error of MOLI | Error of MOLII | Error of ADM Using 5 terms | Error of ADM Using 7 terms |
| -6 | 1.95076971*10$^{-8}$ | 1.045180608*10$^{-8}$‘ | 1*10$^{-10}$ | 1*10$^{-10}$ |
| -4.8 | 1.97223468*10$^{-8}$ | 1.840800845*10$^{-8}$ | 5*10$^{-10}$ | 1*10$^{-10}$ |
| -3.6 | 1.951909900*10$^{-8}$ | 5.144083159*10$^{-8}$ | 3.63*10$^{-8}$ | 1.05*10$^{-8}$ |
| -2.4 | 4.6626724703*10$^{-8}$ | 6.023997785*10$^{-8}$ | 3.7610*10$^{-6}$ | 1.0071*10$^{-6}$ |
| -1.2 | 5.940662235*10$^{-7}$ | 5.499337451*10$^{-4}$ | 1.2750*10$^{-5}$ | 2.91644*10$^{-5}$ |
| 0 | 2.669026577*10$^{-7}$ | 3.292312710*10$^{-5}$ | 1.4363*10$^{-5}$ | 6.85968*10$^{-5}$ |
| 1.2 | 3.148278159*10$^{-8}$ | 4.142080438*10$^{-4}$ | 1.9719*10$^{-5}$ | 3.76501*10$^{-5}$ |
| 2.4 | 3.198081237*10$^{-7}$ | 2.668147003*10$^{-5}$ | 5.0092*10$^{-5}$ | 1.64418*10$^{-5}$ |
| 3.6 | 5.789936818*10$^{-9}$ | 9.517125206*10$^{-6}$ | 2.1215*10$^{-8}$ | 2.7171*10$^{-10}$ |
| 4.8 | 8.49272577*10$^{-9}$ | 1.006655001*10$^{-6}$ | 2.0844*10$^{-8}$ | 2.900*10$^{-10}$ |
| 6 | 3.378234497*10$^{-11}$ | 9.394625356*10$^{-8}$ | 1.9060*10$^{-8}$ | 2.690*10$^{-9}$ |
| 7.2 | 5.128405755*10$^{-10}$ | 8.426382432*10$^{-8}$ | 1.750*10$^{-8}$ | 2.601*10$^{-10}$ |
| 8.4 | 4.189192603*10$^{-10}$ | 6.594602121*10$^{-8}$ | 3.30*10$^{-8}$ | 2*10$^{-10}$ |
| 9.6 | 2.438327352*10$^{-10}$ | 1.871565402*10$^{-8}$ | 80*10$^{-8}$ | 7*10$^{-10}$ |
| 10.8 | 2.263137420*10$^{-10}$ | 1.420102646*10$^{-8}$ | 1.30*10$^{-8}$ | 1.3*10$^{-10}$ |
| 12 | 5.151473414*10$^{-10}$ | 5.898389068*10$^{-9}$ | 20.*10$^{-8}$ | 2*10$^{-10}$ |
| 13.2 | 3.799035079*10$^{-10}$ | 5.048499812*10$^{-9}$ | 1*10$^{-8}$ | 1*10$^{-10}$ |
| 14.4 | 4.030135947*10$^{-10}$ | 4.614364446*10$^{-9}$ | 0 | 0 |
| 15.6 | 4.006248145*10$^{-10}$ | 1.144917494*10$^{-9}$ | 0 | 0 |

**Table 2:** The absolute error of MOL and ADM approximation solutions for $N = 500$ and $t = 5$ for KdV-Burger equation.

| t=5 | | | | |
|---|---|---|---|---|
| x | Error of MOLI | Error of MOLII | Error of ADM Using 5 terms | Error of ADM Using 7 terms |
| -6 | 9.811727574*10$^{-8}$ | 3.184064122*10$^{-8}$ | 1.98*10$^{-7}$ | 3.05*10$^{-7}$ |
| -4.8 | 9.793326538*10$^{-8}$ | 3.621059003*10$^{-8}$ | 0.0000023854 | 3.7214*10$^{-6}$ |
| -2.4 | 9.791718291*10$^{-8}$ | 4.904787680*10$^{-8}$ | 0.0305207870 | 0.734788826 |
| -1.2 | 9.561010450*10$^{-7}$ | 1.087383183*10$^{-7}$ | 0.4783218562 | 2.054157460 |
| 0 | 1.278075613*10$^{-8}$ | 1.349134235*10$^{-5}$ | 9.460043003 | 6.074811896 |
| 1.2 | 2.789549492*10$^{-7}$ | 4.013708042*10$^{-5}$ | 2.405908954 | 0.513555601 |
| 2.4 | 3.306383080*10$^{-6}$ | 1.320023576*10$^{-6}$ | 0.4095884839 | 0.024382254 |
| 3.6 | 1.313720809*10$^{-6}$ | 5.262008329*10$^{-5}$ | 0.0532855839 | 4.14292*10$^{-4}$ |
| 4.8 | 6.730153226*10$^{-8}$ | 2.570498745*10$^{-5}$ | 0.0071441455 | 0.000037755 |
| 6 | 2.291066074*10$^{-8}$ | 1.209137799*10$^{-5}$ | 0.0006701499 | 0.000003425 |
| 7.2 | 3.507566520*10$^{-8}$ | 1.745862402*10$^{-6}$ | 0.0000609787 | 0.004340444 |
| 8.4 | 1.207410716*10$^{-9}$ | 1.909985603*10$^{-8}$ | 0.0000055334 | 3.108$*$10$^{-8}$ |
| 9.6 | 1.932325111*10$^{-9}$ | 1.860166550*10$^{-8}$ | 5.016*10$^{-7}$ | 2.821*10$^{-7}$ |
| 10.8 | 1.957402014*10$^{-9}$ | 1.685257301*10$^{-8}$ | $4.555 * 10^{-8}$ | 2.57*10$^{-8}$ |
| 12 | 2.032961226*10$^{-9}$ | 1.914493980*10$^{-8}$ | $4.140 * 10^{-8}$ | 2.7*10$^{-9}$ |
| 13.2 | 1.959853588*10$^{-9}$ | 2.581864930*10$^{-8}$ | $4.100 * 10^{-10}$ | 7*10$^{-10}$ |
| 15.6 | 2.006210853*10$^{-9}$ | 6.316530631*10$^{-8}$ | 0 | 0 |

## 7 Conclusion

In this article, the method of lines and Adomian decomposition method have been implemented for obtaining solutions of the KdV-Burger equations. The results show that the considered methods are powerful mathematical tools for obtaining accurate solutions. A comparison between MOL and ADM shows that the accuracy of the MOL is better than that in the ADM for solutions when the time increase. Moreover, MOL can overcome difficulties arising in the calculation of Adomian's polynomials. Therefore the MOL is more convenient to apply than ADM. we conclude that the nonlinear KdV–

**Table 3:** The absolute error of MOL and ADM approximation solutions for $N = 500$ and $t = 10$ for KdV-Burger equation.

| t=10 | | | | |
|---|---|---|---|---|
| x | Error of MOLI | Error of MOLII | Error of ADM Using 5 terms | Error of ADM Using 7 terms |
| -6 | $1.9560421002*10^{-7}$ | $1.845876784*10^{-7}$ | $7.664*10^{-7}$ | 0.000002454 |
| -4.8 | $1.9584247256*10^{-7}$ | $2.630796691*10^{-7}$ | 0.000093045 | 0.000297790 |
| -2.4 | $1.96117980187*10^{-7}$ | $8.770627557*10^{-7}$ | 1.200017453 | 40.44833669 |
| -1.2 | $1.95971405431*10^{-7}$ | $1.5001417885*10^{-7}$ | 24.03445058 | 41.42833669 |
| 0 | $1.95718343751*10^{-7}$ | 0.00001279501426 | 276.1600000 | 40.44833669 |
| 1.2 | $1.92183368330*10^{-7}$ | 0.00036723414692 | 16.07921173 | 22.73310721 |
| 2.4 | $5.86544390831*10^{-9}$ | 0.00040572874333 | 1.105687563 | 2.955500615 |
| 3.6 | $5.59678794643*10^{-7}$ | 0.00071548482367 | 0.613696532 | 0.434259679 |
| 4.8 | $4.3823289428*10^{-8}$ | $2.4049120831*10^{-5}$ | 0.197800572 | 0.181425669 |
| 6 | $1.9672920730*10^{-8}$ | $1.3278297395*10^{-5}$ | 0.021564028 | 0.0200777332 |
| 7.2 | $9.11246966527*10^{-7}$ | $2.397605292*10^{-6}$ | 0.001991421 | 0.0018565813 |
| 8.4 | $3.04337807585*10^{-8}$ | $3.1510888284*10^{-7}$ | 0.000180951 | 0.0001687194 |
| 9.6 | $3.9962598039*10^{-8}$ | $3.5804745633*10^{-7}$ | 0.000016418 | 0.0000153083 |
| 10.8 | $2.762818601*10^{-9}$ | $3.6336069673*10^{-7}$ | 0.000001489 | 0.0000527848 |
| 12 | $3.87726236579*10^{-9}$ | $3.9331884238*10^{-9}$ | $1.3518*10^{-7}$ | 0.0000013888 |
| 13.2 | $3.90540066477*10^{-9}$ | $1.219562827*10^{-9}$ | $1.8941*10^{-8}$ | $1.2605*10^{-9}$ |
| 15.6 | $4.0757175020*10^{-9}$ | $6.4857230157*10^{-9}$ | $1.02*10^{-9}$ | $9.401*10^{-9}$ |

**Table 4:** The absolute error of MOL and ADM approximation solutions for $N = 500$ and $t = 15$ for KdV-Burger equation.

| t=15 | | | | |
|---|---|---|---|---|
| x | Error of MOLI | Error of MOLII | Error of ADM Using 5 terms | Error of ADM Using 7 terms |
| -6 | $2.6810669007 *10^{-7}$ | $6.5914829150*10^{-7}$ | 0.000006263 | 0.0000304262 |
| -4.8 | $2.7989134165*10^{-7}$ | $1.1667666832*10^{-7}$ | 0.000760262 | 0.0036916033 |
| -2.4 | $2.8726718315*10^{-7}$ | $2.1687496243*10^{-7}$ | 0.091381742 | 0.4412447347 |
| -1.2 | $2.9121456435*10^{-7}$ | $1.6623724619*10^{-7}$ | 9.835546960 | 44.47559886 |
| 0 | $2.9299921155*10^{-7}$ | $1.7657941775*10^{-7}$ | 214.2414628 | 401.2058844 |
| 1.2 | $2.9380452071*10^{-7}$ | $4.7485260168*10^{-7}$ | 1947.360000 | 41.85274520 |
| 2.4 | $2.93992226563*10^{-7}$ | $2.1203759859*10^{-7}$ | 887.9344146 | 3562.564660 |
| 3.6 | $2.93755557767*10^{-7}$ | $1.7390018314*10^{-7}$ | 99.76715580 | 175.5591839 |
| 4.8 | $2.90562275906*10^{-7}$ | $1.2607281341*10^{-7}$ | 11.74070201 | 32.81122881 |
| 6 | $1.5035556899*10^{-7}$ | $9.633360150*10^{-6}$ | 0.2238971429 | 2.267795046 |
| 7.2 | $6.7054763575*10^{-7}$ | $6.0726624081*10^{-5}$ | 0.8501790915 | 0.663658713 |
| 8.4 | 0.0000050517426 | $8.3269207999*10^{-4}$ | 0.8296939158 | 0.8127640893 |
| 9.6 | $1.6956484610*10^{-7}$ | $1.4092512420*10^{-4}$ | 0.0280762015 | 0.0279368659 |
| 10.8 | $3.2263613634*10^{-8}$ | $2.768658380*10^{-6}$ | 0.0025992930 | 0.0025866527 |
| 12 | $3.5331507095*10^{-9}$ | $3.0166396709*10^{-4}$ | 0.0002362413 | 0.0002350946 |
| 13.2 | $3.5331507095*10^{-9}$ | $3.8576692407*10^{-9}$ | 0.0000214347 | 0.0000213307 |
| 15.6 | $5.7036779084*10^{-9}$ | $5.0145934626*10^{-9}$ | $1.7661*10^{-7}$ | $1.7575*10^{-9}$ |

Burgers equation gives soliton solution, which represents an important application in Physics and physical problems. The computations associated here were performed using Maple 15.

## Competing interests

The authors declare that they have no competing interests.

**Table 5:** The absolute error of MOL and ADM approximation solutions for $N = 500$ and $t = 25$ for KdV-Burger equation.

| t=25 | | | | |
|---|---|---|---|---|
| x | Error of MOLI | Error of MOLII | Error of ADM Using 5 terms | Error of ADM Using 7 terms |
| -6 | $2.821226859*10^{-7}$ | $9.39531785*10^{-7}$ | 0.0000856438 | 0.000700732 |
| -4.8 | $3.061088577*10^{-7}$ | $9.39531785*10^{-7}$ | 0.0103965637 | 0.085022365 |
| -2.4 | $3.300522158*10^{-7}$ | $2.038498259*10^{-7}$ | 1.249903314 | 10.16613057 |
| -1.2 | $3.538602846*10^{-7}$ | $2.155697842*10^{-7}$ | 134.8547761 | 1029.221566 |
| 0 | $3.773140159*10^{-7}$ | $8.775313808*10^{-7}$ | 3142.318964 | 7149.615382 |
| 1.2 | $3.999921515*10^{-7}$ | $8.775313800*10^{-7}$ | 23093.26000 | 4253.757852 |
| 2.4 | $4.213395193*10^{-7}$ | $1.003352956*10^{-7}$ | 11341.68709 | 68668.29557 |
| 3.6 | $4.403334442*10^{-7}$ | $1.056396081*10^{-7}$ | 1068.966557 | 2693.452757 |
| 4.8 | $4.560730912*10^{-7}$ | $1.007154359*10^{-7}$ | 138.6426807 | 590.2571954 |
| 6 | $4.685195856*10^{-7}$ | $9.778011733*10^{-7}$ | 12.08368861 | 55.89151403 |
| 7.2 | $4.775559218*10^{-7}$ | $9.65407757*10^{-7}$ | 0.2264402879 | 4.224219303 |
| 8.4 | $4.840431980*10^{-7}$ | $6.316896550*10^{-7}$ | 0.9502332627 | 0.917313296 |
| 9.6 | $4.097090497*10^{-7}$ | $4.952353226*10^{-5}$ | 0.9591052401 | 0.956118795 |
| 10.8 | $5.449300515*10^{-7}$ | $1.892324320*10^{-4}$ | 0.9589183454 | 0.958647421 |
| 12 | $5.711332285*10^{-6}$ | $8.466932619*10^{-5}$ | 0.8905563998 | 0.890531822 |
| 13.2 | $2.695173913*10^{-6}$ | $9.816296780*10^{-5}$ | 0.3422403071 | 0.342238077 |
| 15.6 | $1.959096925*10^{-8}$ | $1.660263860*10^{-5}$ | 0.00388467196 | 0.003884653 |

## Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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