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Solving bi-level multi-objective quadratic fractional programming problems in rough environment through FGP approach

M.S.Osman[1], *O.E.Emam*[2], *KamalR.Raslan*[3], *FarahatA.Farahat* [4*]

[1] El Asher university, Egypt
[2] Department of Information Systems, Faculty of Computers and Information, Helwan University,
[3] Department of Mathematics , Faculty of Science, Al-Azhar University, Nasr-City, Cairo, Egypt
[4] Higher technological institute 10th of Ramadan city, Egypt

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Abstract: This paper presents a fuzzy goal programming (FGP) approach for solving bi-level multi-objective quadratic fractional programming (BL-MOQFP) problem when constraints are rough set. At the first phase of the solution approach and to avoid the complexity of the problem, two crisp (BL-MOQFP) problems will be formulated; the first is a (BL-MOQFPP1) where the set of constraints is the upper approximation set, and the second is a (BL-MOQFPP2) where the set of constraints is the lower approximation set. At the second phase, a fuzzy goal programming model for each problem will be formulated using the procedure introduced in [1, 2]. The proposed methodology is illustrated with two numerical examples in order to support the proposed methodology.

Keywords: KeywordsBi-level programming, Multi-objective programming, Quadratic Fractional programming, Fuzzy goal programming, Rough set.

1 Introduction

A bi-level programming problem (BLPP) is formulated for a problem in which two decision makers (DMs) make decisions successively. For example, in a decentralized firm, top management, an executive or headquarters makes a decision such as a budget of the firm, and then each division determines a production plan in the full knowledge of the budget. Bi-level programming is an effective tool which treating with hierarchical decision making problems which are common in government policies, economic systems, supply chains, agriculture, vehicle path planning problems, and so on [3,4].

In various areas of the real world, the problems are considered a multi-objective programming. Many methodologies have been presented for treating such problems as in [5]. However, choosing a proper method in a given context is still open for further research.

Fractional programming discussed the optimization of one or more ratios of linear or quadratic functions subject to set of constraints. During the past few decades, fractional programming became one of the planning tools. It has been applied in engineering, business, finance, economics and other fields [6]. Computer oriented technique was extended by Helmy et al. [7] to solve a special class of ML-MOFP problems.

* Corresponding author e-mail: pr_dr_othman@yahoo.com

Z. Pawlak introduced the rough set theory (RST) in the early 1980s in [8,9] which is a mathematical theory to deal with the uncertainty. RST deals with uncertainty by giving two boundary regions of a set. The rough set approach plays a major role in different disciplines as, artificial intelligence, decision analysis and machine learning and so on [10]. Osman et al in [11,12] discussed the uncertainty for multi-level non-linear multi-objective programming problems. Roughness is a kind of uncertainty, another kind of uncertainty introduced in [13].

During the past few decades, multi-level and bi-level mathematical programming problems [14,15] have been deeply studied and many methodologies have been established for treating such problems.

Emam [16] introduced a bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level were maximized. It proposed a two planner integer model and a solution method for solving this problem. Therefore Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem [17].

In this paper Bi-level multi-objective quadratic fractional programming problem is considered when the constraints are rough set. The remaining of the paper unfolds as follows: Section 2 introduces problem formulation and solution concept. Section 3, introduces the solution algorithm. In section 5, illustrative numerical examples will be introduced. Finally, in Section 4, conclusion and some open points for future research work are stated in the field of multi-level multi-objective rough fractional programming problems.

2 Problem formulation

Bi-level programming problems have two decision makers. A decision maker is located at each decision level and a vector of linear fractional objective functions needs to be optimized. Consider the hierarchical system be composed of a 2-level decision makers. Let the decision maker at the i^{th} -level denoted by DM_i controls over the decision variable $x_i = (x_{i1}, x_{i2}, \ldots, x_{in_i}) \in R^{n_i},\ i = 1,2$ where $X = (x_1, x_2) \in R^n a$ and $n = n_1 + n_2$ and furthermore assumed that:
$F_i(x_1, x_2) \equiv F_i(X) : R^{n_1} \times R^{n_2} \to R^{m_i},\ i = 1,2$ are the vector of quadratic fractional objective functions for $\mathrm{DM}_i,\ i = 1,2$
Mathematically, the rough bi-level multi objective quadratic fractional programming problem (BL-MORQFPP) with $m_i,\ i = 1,2$ quadratic fractional objective functions can be stated as, **upper Level**

$$\underbrace{max}_{x_1}\ f_1(X) = \underbrace{max}_{x_1}(f_{11}(X),\ f_{12}(X),\ \ldots, f_{1m_1}(X)), \tag{1}$$

where x_2 solves **lower Level**

$$\underbrace{max}_{x_2}\ f_2(X) = \underbrace{max}_{x_2}(f_{21}(X),\ f_{22}(X),\ \ldots, f_{2m_2}(X)), \tag{2}$$

subject to

$$X \in S \tag{3}$$

where

$$S_* \subseteq .S \subseteq S^* \tag{4}$$

$$X = (x_1, x_2) \in R^n, \tag{5}$$

$$S^* = \left\{ X \in R^n; \sum_{j=1}^{n} a_{ij}x_j \leq b_i \,, i = 1,2\ldots,t, X \geq 0 \right\}, \tag{6}$$

$$S_* = \left\{ X \in S^*; \sum_{j=1}^{n} A_{ij}x_j \leq B_i \,, i = 1,2\ldots,T, X \geq 0 \right\}, \tag{7}$$

and

$$f_{ij}(X) = \frac{N_{ij}(X)}{D_{ij}(X)} = \frac{X^T Q^{ij} X + c^{ij} X + \alpha^{ij}}{X^T R^{ij} X + d^{ij} X + \beta^{ij}} \,, i = 1,2, \quad j = 1,2,\ldots,m_i \tag{8}$$

where Q^{ij} is an $n \times n$ negative definite matrix, R^{ij} is an $n \times n$ positive semi-definite matrix c^{ij},d^{ij} are n-vectors. It is customary to assume that $D_{ij}(X) > 0 \; \forall \; X \in S$, also α^{ij} and β^{ij} are constants and,
S is the rough bi-level feasible domain.
S^* is the upper approximation convex set for S .
S_* is the lower approximation convex set for S .

2.1 Formulation of Crisp Set of Constraints and Solution Concept

For solving (BL-MOQRFP) problem (1)-(8), the following two crisp problems (BL-MOQFPP1), (BL-MOFPP2) will be solved simultaneously where,
(BL-MOQFPP$_1$)is the problem for the upper approximation set,
(BL-MOQFPP$_2$)is the problem for the lower approximation set.

BLp-MOQFPP$_1$, upper Level

$$\underbrace{max}_{\mathbf{x_1}} \; f_1(X) = \underbrace{max}_{\mathbf{x_1}} (f_{11}(X), \; f_{12}(X), \; \ldots, f_{1m_1}(X)), \tag{9}$$

where x_2 solves, **lower Level**

$$\underbrace{max}_{\mathbf{x_2}} \; f_2(X) = \underbrace{max}_{\mathbf{x_2}} (f_{21}(X), \; f_{22}(X), \; \ldots, f_{2m_2}(X)), \tag{10}$$

subject to

$$X \in S^* \tag{11}$$

$$X = (x_1, x_2) \in R^n, \tag{12}$$

$$S^* = \left\{ X \in R^n; \sum_{j=1}^{n} a_{ij}x_j \leq b_i \,, i = 1,2\ldots,t, X \geq 0 \right\}, \tag{13}$$

and

$$f_{ij}(X)=\frac{N_{ij}(X)}{D_{ij}(X)}=\frac{X^TQ^{ij}X+\mathbf{c}^{ij}X+\alpha^{ij}}{X^TR^{ij}X+\mathbf{d}^{ij}X+\backslash^{aij}}\ ,\ i=1,2,\ \ j=1,2,\ldots,m_i \tag{14}$$

BL-MOQFPP$_2$, upper Level

$$\underbrace{max}_{\mathbf{x_1}}\ f_1(X)=\underbrace{max}_{\mathbf{x_1}}(f_{11}(X),\ f_{12}(X),\ \ldots,f_{1m_1}(X)), \tag{15}$$

where x_2 solves **lower Level**

$$\underbrace{max}_{\mathbf{x_2}}\ f_2(X)=\underbrace{max}_{\mathbf{x_2}}(f_{21}(X),\ f_{22}(X),\ \ldots,f_{2m_2}(X)), \tag{16}$$

subject to

$$X\in S_* \tag{17}$$

$$X=(x_1,x_2)\in R^n, \tag{18}$$

$$S_*=\left\{X\in S^*;\sum_{j=1}^{n}A_{ij}x_j\leq B_i\ ,i=1,2\ldots,T,X\geq 0\right\}, \tag{19}$$

and

$$f_{ij}(X)=\frac{N_{ij}(X)}{D_{ij}(X)}=\frac{X^TQ^{ij}X+\mathbf{c}^{ij}X+\alpha^{ij}}{X^TR^{ij}X+\mathbf{d}^{ij}X+\beta^{ij}}\ ,\ i=1,2,\ \ j=1,2,\ldots,m_i \tag{20}$$

Definition 1. *The surely pareto optimal solution*
Let (x_1^*, x_2^*, ..., x_n^*) be a pareto optimal solution of problem (9)-(14) if it is belonging to the lower approximation set (S_*) then it is a surely pareto optimal solution

Definition 2. *The possibly pareto optimal solution*
Let (x_1^*, x_2^*, ..., x_n^*) be a pareto optimal solution of problem (9)-(14) if it is not belonging to the lower approximation set (S_*) then it is a possibly pareto optimal solution.

2.2 Non-linear model development of the BL-MOQFP problem

Since the proposed model (BL-MORQFPP) has a vector of quadratic fractional objective functions in each level, which can be solved by adopting the variable transformation parallel to those transformation given by Charnes and Cooper [18] for linear fractional programming problem into linear programming problem. Thus, in contrast to formulation (8) in this paper, the following representation will be employed in order to deal with the BL-MOQFP problem,

$$f_{ij}(X)=\frac{Q_1^{ij}x_1^2+Q_2^{ij}x_2^2+c_1^{ij}x_1+c_2^{ij}x_2+\alpha^{ij}}{R_1^{ij}x_1^2+R_2^{ij}x_2^2+d_1^{ij}x_1+d_2^{ij}x_2+\beta^{ij}}\quad i=1,2,\ \ j=1,2,\ldots,m_i \tag{21}$$

Now, we make further extensions on the article of M. Chakraborty and S. Gupta [2], to develop a methodology for obtaining the non-linear model of the BL-MOQFP problem. Since the MOQFP problem for the i^{th} -level decision maker may be written as

$$\underbrace{max}_{x_i} F_i(x) = \underbrace{max}_{x_i} \left(f_{i1}(x),\ f_{i2}(x),\ \ldots, f_{im_i}(x) \right), \quad i = 1,2 \tag{22}$$

subject to

$$X \in S \tag{23}$$

where

$$S_* \subseteq .S \subseteq S^* \tag{24}$$

$$X = (x_1, x_2) \in R^n, \tag{25}$$

$$S^* = \left\{ X \in R^n; \sum_{j=1}^{n} a_{ij} x_j \leq b_i\ , i = 1,2\ldots,t, X \geq 0 \right\}, \tag{26}$$

$$S_* = \left\{ X \in S^*; \sum_{j=1}^{n} A_{ij} x_j \leq B_i\ , i = 1,2\ldots,T, X \geq 0 \right\}, \tag{27}$$

and

$$f_{ij}(X) = \frac{Q_1^{ij} x_1^2 + Q_2^{ij} x_2^2 + c_1^{ij} x_1 + c_2^{ij} x_2 + \alpha^{ij}}{R_1^{ij} x_1^2 + R_2^{ij} x_2^2 + d_1^{ij} x_1 + d_2^{ij} x_2 + \beta^{ij}}\ i = 1,2, \quad j = 1,2,\ldots,m_i \tag{28}$$

$$\bigcap_{\forall i,j} \frac{1}{R_1^{ij} x_1^2 + R_2^{ij} x_2^2 + d_1^{ij} x_1 + d_2^{ij} x_2 + \beta^{ij}} = t^2, \tag{29}$$

which is equivalent to

$$\frac{1}{R_1^{ij} x_1^2 + R_2^{ij} x_2^2 + d_1^{ij} x_1 + d_2^{ij} x_2 + \beta^{ij}} \geq t^2, \quad and \quad y = tx \tag{30}$$

so each quadratic fractional objective function is transformed into the following quadratic function

$$f_{ij}(y,t) = \left[Q_1^{ij} \frac{y_1^2}{t^2} + Q_2^{ij} \frac{y_2^2}{t^2} + c_1^{ij} \frac{y_1}{t} + c_2^{ij} \frac{y_2}{t} + \alpha^{ij} \right] \times t^2 \tag{31}$$

or

$$\mathrm{f}_{ij}(\mathbf{y},t) = Q_1^{ij} \mathbf{y_1^2} + Q_2^{ij} \mathbf{y_2^2} + \mathbf{c_1^{ij}} t \mathbf{y_1} + \mathbf{c_2^{ij}} t \mathbf{y_2} + \alpha^{ij} t^2 \tag{32}$$

Based on the transformation $y = tx\ (t > 0)$, $y \in R^n$, $t \in R$, and the above inequality (30) therefore, the non-linear model of the MOQFP problem for i^{th} level decision maker is formulated as follows,

$$\underbrace{max}_{y_i} f_{ij}(y,t) = Q_1^{ij} y_1^2 + Q_2^{ij} y_2^2 + c_1^{ij} t y_1 + c_2^{ij} t y_2 + + \alpha^{ij} t^2, \quad i = 1,2, \quad j = 1,2,\ldots,m_i \tag{33}$$

subject to

$$R_1^{ij}y_1^2+R_2^{ij}y_2^2+d_1^{ij}ty_1+d_2^{ij}ty_2+\beta^{ij}t^2\leq 1,\ \ i=1,2 \qquad j=1,\ldots,m_i \tag{34}$$

$$Y\in S \tag{35}$$

where

$$S_*\subseteq S\subseteq S^*, \tag{36}$$

$$Y=(y_1,y_2)\in R^n, \tag{37}$$

$$S^*=\left\{Y\in R^n;\sum_{j=1}^{n}a_{ij}(y_j/t)\leq b_i\ ,i=1,2\ldots,t\right\}, \tag{38}$$

$$S_*=\left\{Y\in S^*;\sum_{j=1}^{n}A_{ij}(y_j/t)\leq B_i\ ,i=1,2\ldots,T\right\}, \tag{39}$$

Following the above discussion thus, the two crisp models of BL-MOQFP problem can formulated as follows, **BL-MOQP1**, **upper Level**

$$\underbrace{max}_{y_1}F_1\left(y\right)=Q_1^{1j}y_1^2+Q_2^{1j}y_2^2+c_1^{1j}ty_1+c_2^{1j}ty_2+\alpha^{1j}t^2,\ j=1,\ldots,m_1 \tag{40}$$

where y_2 solves

$$\underbrace{max}_{y_2}F_2\left(y\right)=Q_1^{2j}y_1^2+Q_2^{2j}y_2^2+c_1^{2j}ty_1+c_2^{2j}ty_2+\alpha^{2j}t^2,\ j=1,\ldots,m_2 \tag{41}$$

subject to

$$R_1^{ij}y_1^2+R_2^{ij}y_2^2+d_1^{ij}ty_1+d_2^{ij}ty_2+\beta^{ij}t^2\leq 1,\quad i=1,2 \qquad j=1,\ldots,m_i \tag{42}$$

$$\sum_{j=1}^{n}a_{ij}\left(y_j/t\right)\leq b_i\ ,i=1,2\ldots,t, \tag{43}$$

BL-MOQP2, **upper Level**

$$\underbrace{max}_{y_1}F_1\left(y\right)=Q_1^{1j}y_1^2+Q_2^{1j}y_2^2+c_1^{1j}ty_1+c_2^{1j}ty_2+\alpha^{1j}t^2,\ j=1,\ldots,m_1 \tag{44}$$

where y_2 solves **lower Level**

$$\underbrace{max}_{y_2}F_2\left(y\right)=Q_1^{2j}y_1^2+Q_2^{2j}y_2^2+c_1^{2j}ty_1+c_2^{2j}ty_2+\alpha^{2j}t^2,\ j=1,\ldots,m_2 \tag{45}$$

subject to

$$R_1^{ij}y_1^2+R_2^{ij}y_2^2+d_1^{ij}ty_1+d_2^{ij}ty_2+\beta^{ij}t^2\le 1,\quad i=1,2 \tag{46}$$

$$\sum_{j=1}^{n}A_{ij}(y_j/t)\le B_i\ ,i=1,2\dots,T, \tag{47}$$

2.3 Fuzzy goal programming approach for ML-MOQFP problem

In the proposed FGP approach, in order to obtain the solution which is a surely or possibly pareto optimal solution. The vector of quadratic objective functions for each DM is formulated as a fuzzy goal characterized by the membership function $\mu_{(f_{ij})}$, $(i=1,2)$, $(j=1,2,\dots,m_i)$.

2.3.1 Characterization of membership functions

To define the membership functions of the fuzzy goals [5,15], each objective function's individual maximum is taken as the corresponding aspiration level, as follows,

$$u_{ij}^{*}=\underbrace{max}_{y\in s^{*}}\ f_{ij}(y),\quad (i=1,2),(j=1,2,\dots,m_i). \tag{48}$$

$$u_{*ij}=\underbrace{max}_{y\in S_{*}}\ f_{ij}(y),\quad (i=1,2),(j=1,2,\dots,m_i). \tag{49}$$

where $u_{ij}^{*},u_{*ij},\ (i=1,2),\quad (j=1,2,\dots,m_i)$, give the upper tolerance limit or aspired level of achievement for the membership function of ij^{th} objective function for the upper and lower approximation set respectively. Similarly, each objective function's individual minimum is taken as the corresponding aspiration level, as follows,

$$g_{ij}^{*}=\underbrace{min}_{y\in s^{*}}\ f_{ij}(y),\quad (i=1,2),(\ j=1,2,\dots,m_i). \tag{50}$$

$$g_{*ij}=\underbrace{min}_{y\in S_{*}}\ f_{ij}(y),\quad (i=1,2),(j=1,2,\dots,m_i). \tag{51}$$

where $g_{ij}^{*},g_{*ij},\ (i=1,2),\ (\ j=1,2,\dots,m_i)$, give the lower tolerance limit or lowest acceptable level of achievement for the membership function of ij^{th} objective function for the upper and lower approximation set respectively. It can be assumed reasonably the values of $f_{ij}(y)\ge u_{ij}^{*},\ f_{ij}(y)\ge u_{*ij}\ \ (i=1,2),\ (\ j=1,2,\dots,m_i)$, are acceptable and all values less than $g_{ij}^{*}=\underbrace{min}_{y\in s^{*}}\ f_{ij}(y),g_{*ij}=\underbrace{min}_{y\in S_{*}}\ f_{ij}(y)$, are absolutely unacceptable. Then, the membership function $\mu_{ij}^{*}(f_{ij}(y))$, $\mu_{*ij}(f_{ij}(y))$ for the ij^{th} fuzzy goal can be formulated as:

$$\mu_{ij}^{*}(f_{ij}(y))=\begin{cases}1, & if\ f_{ij}(y)\ge u_{ij}^{*},\\ \frac{f_{ij}(y)-g_{ij}^{*}}{u_{ij}^{*}-g_{ij}^{*}}, & if\ g_{ij}^{*}\le f_{ij}(y)\le g_{ij}^{*},\\ 0, & if\ f_{ij}(y)\le g_{ij}^{*},\end{cases}\qquad (i=1,2),\ (\ j=1,2,\dots,m_i) \tag{52}$$

$$\mu_{*ij}\left(f_{ij}\left(y\right)\right)=\begin{cases}1, & if\ f_{ij}\left(y\right)\geq u_{*ij},\\ \frac{f_{ij}(y)-g_{*ij}}{u_{*ij}-g_{*ij}}, & if\ g_{*ij}\leq f_{ij}\left(y\right)\leq u_{*ij},\\ 0, & if\ f_{ij}\left(y\right)\leq g_{*ij},\end{cases}\qquad\left(i=1,2\right),\ \left(\ j=1,2,\ldots,m_i\right) \tag{53}$$

Then the FGP models for **BL-MOQP**$_1$and **BL-MOQP**$_2$can be formulated as,

FGP1:

$$min\ Z=\sum_{j=1}^{m_1} w_{1j}^{*-}\ d_{1j}^{*-}+\sum_{j=1}^{m_2} w_{2j}^{*-}d_{2j}^{*-}, \tag{54}$$

subject to

$$\sum_{i=1}^{2} Q_i^{ij}y_i^2+\sum_{i=1}^{2} c_i^{ij}ty_i+\alpha^{ij}t^2+\left(u_{ij}^*-g_{ij}^*\right)d_{ij}^{*-}-\left(u_{ij}^*-g_{ij}^*\right)d_{ij}^{*+}=u_{ij}^*,\ i=1,2\ ,j=1,\ldots,m_i, \tag{55}$$

$$y_{1l}=y_{1l}^*\qquad (l=1,2,..........,n_i), \tag{56}$$

$$R_1^{ij}y_1^2+R_2^{ij}y_2^2+d_1^{ij}ty_1+d_2^{ij}ty_2+\beta^{ij}t^2\leq 1,\qquad i=1,2,\ j=1,\ldots,m_i \tag{57}$$

$$\sum_{j=1}^{n} a_{ij}\left(y_j/t\right)\leq b_i\ ,i=1,2\ldots,t, \tag{58}$$

$$d_{ij}^{*-}\ d_{ij}^{*+}=0,\ and\ d_{ij}^{*-},\ d_{ij}^{*+}\geq 0,\ \ \left(i=1,2\right),\ \ \left(j=1,2,\ldots,m_i\right). \tag{59}$$

where Z represents the achievement function consisting of the weighted under-deviational variables of the fuzzy goals. The numerical weights w_{ij}^{*-} represent the relative importance of achieving the aspired levels of the respective fuzzy goals. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested in [13] is used to assign the values of w_{ij}^{*-}. These values are determined as:

$$w_{ij}^{*-}=\frac{1}{u_{ij}^*-g_{ij}^*},\qquad \left(i=1,2\right),\ \left(\ j=1,2,\ldots,k_i\right), \tag{60}$$

FGP2:

$$min\ Z=\sum_{j=1}^{m_1} w_{*1j}^{-}\ d_{*1j}^{-}+\sum_{j=1}^{m_2} w_{*2j}^{-}d_{*2j}^{-}, \tag{61}$$

subject to

$$\sum_{i=1}^{2} Q_i^{ij}y_i^2+\sum_{i=1}^{2} c_i^{ij}ty_i+\alpha^{ij}t^2+\left(u_{*ij}\ \ --g_{*ij}\right)d_{*ij}^{-}-\left(u_{*ij}\ \ --g_{*ij}\right)d_{*ij}^{+}=u_{*ij},\ i=1,2\ ,j=1,\ldots,m_i, \tag{62}$$

$$y_{1l}=y_{1l}^*,\qquad\qquad (l=1,2,\ldots,n_i) \tag{63}$$

$$R_1^{ij}y_1^2+R_2^{ij}y_2^2+d_1^{ij}ty_1+d_2^{ij}ty_2+\beta^{ij}t^2\leq 1,\qquad i=1,2,\ j=1,\ldots,m_i \tag{64}$$

$$\sum_{j=1}^{n} A_{ij}(y_j/t) \leq B_i \ , i=1,2\ldots,T, \tag{65}$$

$$d^-_{*ij}\ d^+_{*ij}=0,\ and\ d^-_{*ij},\ d^+_{*ij}\geq 0,\ \ (i=1,2),\ \ \ (j=1,2,\ldots,m_i). \tag{66}$$

where, **FGP1** and **FGP2** are the fuzzy goal programming models for **BL-MOQP**$_1$and **BL-MOQP**$_2$ respectively.

3 Solution algorithm

Step (1): Reformulate the (BL MORQFP) (1)-(8)problem into the two crisp problems (15)- (20)
Step (2): Reformulate problems (40) - (43)and (44)- (47) using equations 29, 30, 32.
Step (3): Compute $u^*_{ij}\ and\ g^*_{ij}, w^*_{ij} i=1,2, j=1,\ldots,m_i$.
Step (4): Formulate $\mu^*_{ij}\left(f_{ij}\left(y\right)\right) i=1,2, j=1,\ldots,m_i$.
Step (5): Formulate FGP1 (54)-(59).
Step (6): Solve FGP1 (54)- (59) to find y^*_1, y^*_2, t .
Step (7): Find $x^*_1=\frac{y^*_1}{t}, x^*_2=\frac{y^*_2}{t}$.
Step (8): If the solution of FGP1 (x^*_1, x^*_2) is a surely pareto optimal solution, stop, otherwise go to step (9).
Step (9): Compute $u_{*ij}\ and\ g_{*ij}, w_{*ij},\ i=1,2, j=1,\ldots,m_i$.
Step (10): Formulate $\mu_{*ij}\left(f_{ij}\left(y\right)\right), i=1,2, j=1,\ldots,m_i$.
Step (11): Formulate FGP2 (61)- (66).
Step (12): Solve FGP1 (54)- (59) to find y_{*1}, y_{*2}, t
Step (13): Find $x_{*1}=\frac{y_{*1}}{t},\ x_{*2}=\frac{y_{*2}}{t}$ and then $f_{*ij}\left(X\right)\leq f_{ij}\left(X\right)\leq f^*_{ij}\left(X\right)\ \ \forall i, j$.

4 Illustrative examples:

Illustrative example 1: Consider the following (BL-MORQFP) problem where roughness exists in constraints **upper Level**

$$\underbrace{max}_{x_1}\left(\frac{-{x_1}^2-2{x_2}^2+5x_2+10}{{x_1}^2+3x_2+5},\ \frac{-{x_1}^2-{x_2}^2+5x_1+4}{{x_1}^2+3x_2+5}\right),$$

where x_2 solves **lower Level**

$$\underbrace{max}_{x_1}\left(\frac{-{x_1}^2-2{x_2}^2+3x_1+6}{{x_2}^2+3x_1+1},\ \frac{-{x_1}^2-3{x_2}^2+7x_2+8}{{x_2}^2+3x_1+1}\right),$$

subject to

$$X\in S,$$

where

$$S_*\subseteq S\subseteq S^*$$

$$S^*=\left\{(x_1,x_2)\in R^2\ \begin{array}{c} 2x_1+3x_2\leq 90,\\ 4x_1+x_2\leq 80,\\ x_1,x_2\geq 0.\end{array}\right\},$$

$$S_* = \left\{ (x_1,x_2) \in R^2 \begin{array}{r} x_1+3x_2 \leq 75, \\ 5x_1+x_2 \leq 95, \\ x_1,x_2 \geq 0. \end{array} \right\}$$

.

First, solve the problem for the upper approximation set (BL-MQFPP1), so the following problem will be firstly solved **upper Level**

$$\underbrace{max}_{x_1} \left(\frac{-x_1{}^2-2x_2{}^2+5x_2+10}{x_1{}^2+3x_2+5}, \frac{-x_1{}^2-x_2{}^2+5x_1+4}{x_1{}^2+3x_2+5} \right),$$

where x_2 solves **lower Level**

$$\underbrace{max}_{x_1} \left(\frac{-x_1{}^2-2x_2{}^2+3x_1+6}{x_2{}^2+3x_1+1}, \frac{-x_1{}^2-3x_2{}^2+7x_2+8}{x_2{}^2+3x_1+1} \right),$$

subject to

$$2x_1+3x_2 \leq 90,$$
$$4x_1+x_2 \leq 80,$$
$$x_1,x_2 \geq 0.$$

According to the introduced procedure the (BL-MQFPP$_1$) can be written as **upper Level**

$$\underbrace{max}_{y_1} \left(-y_1^2-2y_2^2+5y_2t+10t^2,\ -y_1^2-y_2^2+5y_1t+4t^2 \right),$$

where y_2 solves **lower Level**

$$\underbrace{max}_{y_2} \left(-y_1^2-2y_2^2+3y_1t+6t^2,\ -y_1^2-3y_2^2+7y_2t+8t^2 \right),$$

subject to

$$2y_1+3y_2-90t \leq 0,$$
$$4y_1+y_2-80t \leq 0,$$
$$y_1{}^2+3y_2t+5t^2 \leq 1,$$
$$y_2{}^2+3y_1t+t^2 \leq 1,$$
$$t > 0.$$

The individual maximum and minimum values are summarized in Table 1.

Table 1: individual maximum, minimum values, and weights w^*_{ij}.

	f_{11}	f_{12}	f_{21}	f_{22}
$max\ (f_{ij}(X))$	2	2.022497	1.778	1.666667
$min\ (f_{ij}(X))$	-1.82019	-0.99445	-1.99112	-2.754717
w^*_{ij}	0.26	0.33	0.265	0.226

The upper level FGP model can be written as:

$$min\ Z = 0.26D_{11}^{*-} + 0.33D_{12}^{*-}$$

subject to

$$-y_1^2 - 2y_2^2 + 5y_2t + 10t^2 + 3.820198D_{11}^{*-} - 3.820198D_{11}^{*+} = 2,$$

$$-y_1^2 - y_2^2 + 5y_1t + 4t^2 + 3.016947D_{12}^{*-} - 3.016947D_{12}^{*+} = 2.022497,$$

$$\begin{gathered} 2y_1 + 3y_2 - 90t \le 0, \\ 4y_1 + y_2 - 80t \le 0, \\ {y_1}^2 + 3y_2t + 5t^2 \le 1, \\ {y_2}^2 + 3y_1t + t^2 \le 1, \\ t > 0. \end{gathered}$$

Solving the above problem using nonlinear techniques or software package, the compromise optimal solution for the upper level problem obtained as:

$$(y_1, y_2) = (0.6381412, 0), t = 0.3443184.$$

The lower level FGP model can be written as:

$$min\ Z = 0.26D_{11}^{*-} + 0.877D_{12}^{*-} + 0.265D_{21}^{*-} + 0.226D_{22}^{*-}$$

subject to

$$-y_1^2 - 2y_2^2 + 5y_2t + 10t^2 + 3.820198D_{11}^{*-} - 3.820198D_{11}^{*+} = 2,$$

$$-y_1^2 - y_2^2 + 5y_1t + 4t^2 + 3.016947D_{12}^{*-} - 3.016947D_{12}^{*+} = 2.022497,$$

$$-y_1^2 - 2y_2^2 + 3y_1t + 6t^2 + 3.769354D_{21}^{*-} - 3.769354D_{21}^{*+} = 1.778,$$

$$-y_1^2 - 3y_2^2 + 7y_2t + 8t^2 + 4.421384D_{12}^{*-} - 4.421384D_{12}^{*+} = 1.666667,$$

$$y_1 = 0.6381412,$$

$$\begin{gathered} 2y_1 + 3y_2 - 90t \le 0, \\ 4y_1 + y_2 - 80t \le 0, \\ {y_1}^2 + 3y_2t + 5t^2 \le 1, \\ {y_2}^2 + 3y_1t + t^2 \le 1, \\ t > 0. \end{gathered}$$

Solving the above problem using nonlinear techniques or software package, the compromise optimal solution for the upper level problem obtained as:

$$(y_1, y_2) = (0.6381412, 0), t = 0.3443184.$$

Then $x_1 = \frac{y_1}{t} = 1.85334621$, $x_2 = \frac{y_2}{t} = 0$, which is a surely pareto optimal solution.
$f_{11} = 0.7783274155$, $f_{12} = 1.165615241$,
$f_{21} = 1.238635103$, $f_{22} = 0.6958965$,

5 Illustrative example 2:

Consider the following (BL-MRQFPP) where roughness exists in constraints **upper Level**

$$\underbrace{max}_{x_1}\left(\frac{-x_1{}^2-2x_2{}^2+3x_2+8}{x_1{}^2+2x_2+4},\ \frac{-x_1{}^2-x_2{}^2+7x_1+5}{x_1{}^2+2x_2+4}\right),$$

where x_2 solves **lower Level**

$$\underbrace{max}_{x_1}\left(\frac{-x_1{}^2-3x_2{}^2+5x_1+9}{x_2{}^2+5x_1+2},\ \frac{-x_1{}^2-2x_2{}^2+4x_2+7}{x_2{}^2+5x_1+2}\right),$$

Subject to $X\in S$, *where*

$$S_*\subseteq S\subseteq S^*$$

$$S^*=\left\{(x_1,x_2)\in R^2\ \begin{matrix}x_1+2x_2\le 56,\\ 10x_1+x_2\le 180,\\ x_1,x_2\ge 0.\end{matrix}\right\}$$

$$S_*=\left\{(x_1,x_2)\in R^2\ \begin{matrix}x_1+x_2\ \le 10,\\ 5x_1-4.8x_2\le 1,\\ x_1,x_2\ge 0.\end{matrix}\right\}$$

.

First, solve the problem for the upper approximation set (BL-MQFPP1), so the following problem will be firstly solved, **upper Level**

$$\underbrace{max}_{x_1}\left(\frac{-x_1{}^2-2x_2{}^2+3x_2+8}{x_1{}^2+2x_2+4},\ \frac{-x_1{}^2-x_2{}^2+7x_1+5}{x_1{}^2+2x_2+4}\right),$$

where x_2 solves **lower Level**

$$\underbrace{max}_{x_1}\left(\frac{-x_1{}^2-3x_2{}^2+5x_1+9}{x_2{}^2+5x_1+2},\ \frac{-x_1{}^2-2x_2{}^2+4x_2+7}{x_2{}^2+5x_1+2}\right),$$

subject to

$$x_1+2x_2\le 56,$$
$$10x_1+x_2\le 180,$$
$$x_1,x_2\ge 0.$$

According to the introduced procedure the (BL-MQFPP$_1$) can be written as:

[Upper Level]

$$\underbrace{max}_{y_1}\left(-y_1^2-2y_2^2+3y_2t+8t^2,\ -y_1^2-y_2^2+7y_1t+5t^2\right),$$

where y_2 solves

[Lower Level]

$$\underbrace{max}_{y_2}\left(-y_1^2-3y_2^2+5y_1t+9t^2,\ -y_1^2-2y_2^2+4y_1t+7t^2\right),$$

subject to

$$y_1+2y_2-56t\le 0,$$
$$10y_1+y_2-180t\le 0,$$
$${y_1}^2+2y_2t+4t^2\le 1,$$
$${y_2}^2+5y_1t+2t^2\le 1,$$
$$t>0.$$

The individual maximum and minimum values are summarized in Table 2.

Table 2: individual maximum, minimum values, and weights w^*_{ij}.

	f_{11}	f_{12}	f_{21}	f_{22}
$max\ (f_{ij}(X))$	2	2.327442	2.715609	2.133951
$min\ (f_{ij}(X))$	-1.877863	-0.991094	-2.980916	-1.986005
w^*_{ij}	0.25787	0.3013377	0.1755	0.2427

The upper level FGP model can be written as:

$$min\ Z = 0.25787D^{*-}_{11}+0.3013377D^{*-}_{12}$$

subject to

$$-y_1^2-2y_2^2+3y_2t+8t^2+3.877863D^{*-}_{11}-3.877863D^{*+}_{11}=2,$$
$$-y_1^2-y_2^2+7y_1t+5t^2+3.3185361D^{*-}_{12}-3.3185361D^{*+}_{12}=2.327442,$$
$$y_1+2y_2-56t\le 0,$$
$$10y_1+y_2-180t\le 0,$$
$${y_1}^2+2y_2t+4t^2\le 1,$$
$${y_2}^2+5y_1t+2t^2\le 1,$$
$$t>0.$$

Solving the above problem using nonlinear techniques or software package, the compromise optimal solution for the upper level problem obtained as:

$$(y_1,y_2)=(0.2141383,0)\,,t=0.4884017.$$

The lower level FGP model can be written as:

$$min\ Z = 0.25787D^{*-}_{11}+0.3013377D^{*-}_{12}+0.1755D^{*-}_{21}+0.2427D^{*-}_{22}$$

subject to

$$-y_1^2-2y_2^2+3y_2t+8t^2+3.877863D^{*-}_{11}-3.877863D^{*+}_{11}=2,$$
$$-y_1^2-y_2^2+7y_1t+5t^2+3.3185361D^{*-}_{12}-3.3185361D^{*+}_{12}=2.327442,$$
$$-y_1^2-3y_2^2+5y_1t+9t^2+5.696525D^{*-}_{21}-5.696525D^{*+}_{21}=2.715609,$$
$$-y_1^2-2y_2^2+4y_1t+7t^2+4.119956D^{*-}_{22}-4.119956D^{*+}_{22}=2.133951,$$
$$y_1=0.2141383,$$

$$\begin{aligned} y_1 + 2y_2 - 56t &\le 0, \\ 10y_1 + y_2 - 180t &\le 0, \\ {y_1}^2 + 2y_2t + 4t^2 &\le 1, \\ {y_2}^2 + 5y_1t + 2t^2 &\le 1, \\ t &> 0. \end{aligned}$$

Solving the above problem using nonlinear techniques or software package, the compromise optimal solution for the upper level problem obtained as

$$(y_1, y_2) = (0.6381412, 0)\,, t = 0.4884017.$$

Then $x_1 = \frac{y_1}{t} = 0.4384470816$, $x_2 = \frac{y_2}{t} = 0$, which is a possibly pareto optimal solution.

$$f_{11}^* = 1.862434379, \qquad f_{12}^* = 1.820193989,$$

$$f_{21}^* = 2.623898352, \qquad f_{22}^* = 2.042240392,$$

Second, solve the problem for the lower approximation set (BL-MQFPP2), so the following problem will be solved,**upper Level**

$$\underbrace{max}_{x_1} \left(\frac{-{x_1}^2-2{x_2}^2+3x_2+8}{{x_1}^2+2x_2+4}, \frac{-{x_1}^2-{x_2}^2+7x_1+5}{{x_1}^2+2x_2+4} \right),$$

where x_2 solves **lower Level**

$$\underbrace{max}_{x_1} \left(\frac{-{x_1}^2-3{x_2}^2+5x_1+9}{{x_2}^2+5x_1+2}, \frac{-{x_1}^2-2{x_2}^2+4x_2+7}{{x_2}^2+5x_1+2} \right),$$

subject to

$$\begin{aligned} x_1 + x_2 &\le 10, \\ 5x_1 - 4.8x_2 &\le 1, \\ x_1, x_2 &\ge 0. \end{aligned}$$

According to the introduced procedure the (BL-MQFPP$_2$) can be written as, **upper Level**

$$\underbrace{max}_{y_1} \left(-y_1^2 - 2y_2^2 + 3y_2t + 8t^2,\ -y_1^2 - y_2^2 + 7y_1t + 5t^2 \right),$$

where y_2 solves **lower Level**

$$\underbrace{max}_{y_2} \left(-y_1^2 - 3y_2^2 + 5y_1t + 9t^2,\ -y_1^2 - 2y_2^2 + 4y_1t + 7t^2 \right),$$

subject to

$$\begin{aligned} y_1 + y_2 - 10t &\le 0, \\ 5y_1 - 4.8y_2 - t &\le 0, \\ {y_1}^2 + 2y_2t + 4t^2 &\le 1, \\ {y_2}^2 + 5y_1t + 2t^2 &\le 1, \\ t &> 0. \end{aligned}$$

Similarly, applying the proposed algorithm to solve the (BL-MQFPP$_2$), then:

$$(y_1, y_2) = (0.2370723, 0.1534293)\,, t = 0.4489008.$$

Then, $x_1 = \frac{y_1}{t} = 0.528117348$, $x_2 = \frac{y_2}{t} = 0.3417888763$,

$$f_{*11} = 1.715434657, \qquad f_{*12} = 1.672769,$$

$$f_{*21} = 2.314542637, \qquad f_{*22} = 1.807691315,$$

Then,

$$f_{11} \in [1.715434657, 1.862434379], f_{12} \in [1.672769, 1.820193989],$$

$$f_{21} \in [2.314542637, 2.623898352], f_{22} \in [1.807691315, 2.042240392].$$

6 Conclusion and future work

This paper introduced an algorithm for solving bi-level multi Objective quadratic fractional programming problem when roughness exists in constraints. Two crisp bi level multi objective programming problems were formulated. For each problem, The FGP algorithm is used to achieve the highest degree (unity) of each of the membership goals by minimizing their deviational variables and thereby obtaining the most satisfactory solution for the two decision makers which can be classified as a surely or possibly pareto optimal solution according to definition(1) and definition(2).
There exist many other open points for future work and research such as:

1. Solving bi-level multi-objective quadratic programming problem when roughness exists in constraints.
2. Solving bi-level multi-objective integer quadratic fractional programming problem when roughness exists in constraints.
3. Solving multi-level multi-objective integer quadratic fractional programming problem when roughness exists in constraints.

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