New Trends in Mathematical Sciences

http://dx.doi.org/10.20852/ntmsci.2018.295

$\alpha_{(\gamma,\gamma')}$-semiregular, $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen Sets and $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi Continuous Functions

Hariwan Z. Ibrahim

Department of Mathematics, Faculty of Education, University of Zakho, Kurdistan-Region, Iraq

Received: 24 January 2018, Accepted: 18 June 2018
Published online: 29 July 2018.

Abstract: The purpose of the present paper is to introduce and study two strong forms of $\alpha_{(\gamma,\gamma')}$-semiopen sets called $\alpha_{(\gamma,\gamma')}$-semiregular sets and $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen sets. And also introduce a new class of functions called $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous functions and obtain several properties of such functions.

Keywords: $\alpha_{(\gamma,\gamma')}$-open sets, $\alpha_{(\gamma,\gamma')}$-semiregular sets, $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen sets, $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous functions.

1 Introduction

Njastad [4] defined α-open sets in a space X and discussed many of its properties. Ibrahim [3] introduced and discussed an operation on a topology $\alpha O(X)$ into the power set $P(X)$ and introduced $\alpha_{(\gamma,\gamma')}$-open sets in topological spaces and studied some of its basic properties [1]. And also in [2] the author introduced the notion of $\alpha_{(\gamma,\gamma')}$-semiopen sets in a topological space and studied some of its properties. In this paper, the author introduce and study the notion of $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed sets, and introduce $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous functions and investigate some important properties.

2 Preliminaries

Throughout the present paper, (X,τ) and (Y,σ) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure and the interior of a subset A of X are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 1.[4] *A subset A of a topological space (X,τ) is called α-open, if $A\subseteq Int(Cl(Int(A)))$.*

The family of all α-open sets in a topological space (X,τ) is denoted by $\alpha O(X,\tau)$ (or $\alpha O(X)$).

Definition 2.[3] *Let (X,τ) be a topological space. An operation γ on the topology $\alpha O(X)$ is a mapping from $\alpha O(X)$ into the power set $P(X)$ of X such that $V\subseteq V^{\gamma}$ for each $V\in\alpha O(X)$, where V^{γ} denotes the value of γ at V. It is denoted by $\gamma:\alpha O(X)\to P(X)$.*

Definition 3.[3] *An operation γ on $\alpha O(X,\tau)$ is said to be α-regular if for every α-open sets U and V containing $x\in X$, there exists an α-open set W of X containing x such that $W^{\gamma}\subseteq U^{\gamma}\cap V^{\gamma}$.*

* Corresponding author e-mail: hariwan_math@yahoo.com

Definition 4.[1] *Let (X,τ) be a topological space and γ,γ' be operations on $\alpha O(X,\tau)$. A subset A of X is said to be $\alpha_{(\gamma,\gamma')}$-open if for each $x\in A$ there exist α-open sets U and V of X containing x such that $U^{\gamma}\cup V^{\gamma'}\subseteq A$. A subset of (X,τ) is said to be $\alpha_{(\gamma,\gamma')}$-closed if its complement is $\alpha_{(\gamma,\gamma')}$-open. The family of all $\alpha_{(\gamma,\gamma')}$-open sets of (X,τ) is denoted by $\alpha O(X,\tau)_{(\gamma,\gamma')}$.*

Proposition 1. [1] Let γ and γ' be α-regular operations. If A and B are $\alpha_{(\gamma,\gamma')}$-open, then $A\cap B$ is $\alpha_{(\gamma,\gamma')}$-open.

Definition 5.[2] *A subset A of X is said to be $\alpha_{(\gamma,\gamma')}$-semiopen, if there exists an $\alpha_{(\gamma,\gamma')}$-open set U of X such that $U\subseteq A\subseteq \alpha_{(\gamma,\gamma')}$-$Cl(U)$. A subset A of X is $\alpha_{(\gamma,\gamma')}$-semiclosed if and only if $X\setminus A$ is $\alpha_{(\gamma,\gamma')}$-semiopen.*

The family of all $\alpha_{(\gamma,\gamma')}$-semiopen sets of a topological space (X,τ) is denoted by $\alpha SO(X,\tau)_{(\gamma,\gamma')}$, the family of all $\alpha_{(\gamma,\gamma')}$-semiopen sets of (X,τ) containing x is denoted by $\alpha SO(X,x)_{(\gamma,\gamma')}$. Also the family of all $\alpha_{(\gamma,\gamma')}$-semiclosed sets of a topological space (X,τ) is denoted by $\alpha SC(X,\tau)_{(\gamma,\gamma')}$.

Definition 6. *Let A be a subset of a topological space (X,τ). Then:*

(1) $\alpha_{(\gamma,\gamma')}$-$Cl(A)=\bigcap\{F: F$ *is* $\alpha_{(\gamma,\gamma')}$*-closed and* $A\subseteq F\}$ *[1].*
(2) $\alpha_{(\gamma,\gamma')}$-$Int(A)=\bigcup\{U: U$ *is* $\alpha_{(\gamma,\gamma')}$*-open and* $U\subseteq A\}$ *[1].*
(3) $\alpha_{(\gamma,\gamma')}$-$sCl(A)=\bigcap\{F: F$ *is* $\alpha_{(\gamma,\gamma')}$*-semiclosed and* $A\subseteq F\}$ *[2].*
(4) $\alpha_{(\gamma,\gamma')}$-$sInt(A)=\bigcup\{U: U$ *is* $\alpha_{(\gamma,\gamma')}$*-semiopen and* $U\subseteq A\}$ *[2].*

3 $\alpha_{(\gamma,\gamma')}$-semiregular Sets and $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen Sets

Definition 7. *A subset A of a topological space (X,τ) is said to be $\alpha_{(\gamma,\gamma')}$-semiregular, if it is both $\alpha_{(\gamma,\gamma')}$-semiopen and $\alpha_{(\gamma,\gamma')}$-semiclosed.*

The family of all $\alpha_{(\gamma,\gamma')}$-semiregular sets in X is denoted by $\alpha SR(X)_{(\gamma,\gamma')}$.

Lemma 1. *The following properties hold for a subset A of a topological space (X,τ):*

(1) *If* $A\in\alpha SO(X)_{(\gamma,\gamma')}$, *then* $\alpha_{(\gamma,\gamma')}$-$sCl(A)\in\alpha SR(X)_{(\gamma,\gamma')}$.
(2) *If* $A\in\alpha SC(X)_{(\gamma,\gamma')}$, *then* $\alpha_{(\gamma,\gamma')}$-$sInt(A)\in\alpha SR(X)_{(\gamma,\gamma')}$.

Proof. (1) Since $\alpha_{(\gamma,\gamma')}$-$sCl(A)$ is $\alpha_{(\gamma,\gamma')}$-semiclosed, we have to show that $\alpha_{(\gamma,\gamma')}$-$sCl(A)\in\alpha SO(X)_{(\gamma,\gamma')}$. Since $A\in\alpha SO(X)_{(\gamma,\gamma')}$, then for $\alpha_{(\gamma,\gamma')}$-open set U of X, $U\subseteq A\subseteq\alpha_{(\gamma,\gamma')}$-$Cl(U)$. Therefore we have, $U\subseteq\alpha_{(\gamma,\gamma')}-sCl(U)\subseteq\alpha_{(\gamma,\gamma')}-sCl(A)\subseteq\alpha_{(\gamma,\gamma')}-sCl(\alpha_{(\gamma,\gamma')}-Cl(U))=\alpha_{(\gamma,\gamma')}-Cl(U)$ or $U\subseteq\alpha_{(\gamma,\gamma')}$-$sCl(A)\subseteq\alpha_{(\gamma,\gamma')}$-$Cl(U)$ and hence $\alpha_{(\gamma,\gamma')}$-$sCl(A)\in\alpha SO(X)_{(\gamma,\gamma')}$.
(2) This follows from (1).

Definition 8. *A point $x\in X$ is said to be $\alpha_{(\gamma,\gamma')}$-$\theta$-semiadherent point of a subset A of X if $\alpha_{(\gamma,\gamma')}$-$sCl(U)\cap A\neq\phi$ for every $\alpha_{(\gamma,\gamma')}$-semiopen set U containing x. The set of all $\alpha_{(\gamma,\gamma')}$-$\theta$-semiadherent points of A is called the $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosure of A and is denoted by $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. A subset A is called $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed if $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)=A$. A subset A is called $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen if and only if $X\setminus A$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed.*

Definition 9. *A point $x\in X$ is said to be $\alpha_{(\gamma,\gamma')}$-$\theta$-adherent point of a subset A of X if $\alpha_{(\gamma,\gamma')}$-$Cl(U)\cap A\neq\phi$ for every $\alpha_{(\gamma,\gamma')}$-open set U containing x. The set of all $\alpha_{(\gamma,\gamma')}$-$\theta$-adherent points of A is called the $\alpha_{(\gamma,\gamma')}$-$\theta$-closure of A and is denoted by $\alpha_{(\gamma,\gamma')}$-$Cl_\theta(A)$. A subset A is called $\alpha_{(\gamma,\gamma')}$-$\theta$-closed if $\alpha_{(\gamma,\gamma')}$-$Cl_\theta(A)=A$. The complement of an $\alpha_{(\gamma,\gamma')}$-$\theta$-closed set is called an $\alpha_{(\gamma,\gamma')}$-$\theta$-open set.*

Corollary 1. *Let $x\in X$ and $A\subseteq X$. If $x\in\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$, then $x\in\alpha_{(\gamma,\gamma')}$-$Cl_\theta(A)$.*

Proof. Let $x \in \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. Then, $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap A \neq \phi$ for every $\alpha_{(\gamma,\gamma')}$-semiopen set U containing x. Since $\alpha_{(\gamma,\gamma')}$-$sCl(U) \subseteq \alpha_{(\gamma,\gamma')}$-$Cl(U)$, so we have $\phi \neq \alpha_{(\gamma,\gamma')}$-$sCl(U) \cap A \subseteq \alpha_{(\gamma,\gamma')}$-$Cl(U) \cap A$. Hence, $\alpha_{(\gamma,\gamma')}$-$Cl(U) \cap A \neq \phi$ for every $\alpha_{(\gamma,\gamma')}$-open set U containing x. Therefore, $x \in \alpha_{(\gamma,\gamma')}$-$Cl_\theta(A)$.

Lemma 2. *The following properties hold for a subset A of a topological space (X,τ):*

(1) *If $A \in \alpha SO(X)_{(\gamma,\gamma')}$, then $\alpha_{(\gamma,\gamma')}$-$sCl(A) = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$.*
(2) *If $A \in \alpha SR(X)_{(\gamma,\gamma')}$ if and only if A is both $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed and $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen.*
(3) *If $A \in \alpha O(X)_{(\gamma,\gamma')}$, then $\alpha_{(\gamma,\gamma')}$-$Cl(A) = \alpha_{(\gamma,\gamma')}$-$Cl_\theta(A)$.*

Proof. (1) Clearly $\alpha_{(\gamma,\gamma')}$-$sCl(A) \subseteq \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. Suppose that $x \notin \alpha_{(\gamma,\gamma')}$-$sCl(A)$. Then, for some $\alpha_{(\gamma,\gamma')}$-semiopen set U containing x, $A \cap U = \phi$ and hence $A \cap \alpha_{(\gamma,\gamma')}$-$sCl(U) = \phi$, since $A \in \alpha SO(X)_{(\gamma,\gamma')}$. This shows that $x \notin \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. Therefore $\alpha_{(\gamma,\gamma')}$-$sCl(A) = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$.
(2) Let $A \in \alpha SR(X)_{(\gamma,\gamma')}$. Then, $A \in \alpha SO(X)_{(\gamma,\gamma')}$, by (1), we have $A = \alpha_{(\gamma,\gamma')}$-$sCl(A) = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. Therefore, A is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed. Since $X \setminus A \in \alpha SR(X)_{(\gamma,\gamma')}$, by the argument above, $X \setminus A$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed and hence A is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen. The converse is obvious.
(3) This similar to (1).

Theorem 1. *Let (X,τ) be a topological space and $A \subseteq X$. Then, A is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen in X if and only if for each $x \in A$ there exists $U \in \alpha SO(X,x)_{(\gamma,\gamma')}$ such that $\alpha_{(\gamma,\gamma')}$-$sC(U) \subseteq A$.*

Proof. Let A be $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen and $x \in A$. Then, $X \setminus A$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed and $X \setminus A = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(X \setminus A)$. Hence, $x \notin \alpha_{(\gamma,\gamma')}$-$sCl_\theta(X \setminus A)$. Therefore, there exists $U \in \alpha SO(X,x)_{(\gamma,\gamma')}$ such that $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap (X \setminus A) = \phi$ and so $\alpha_{(\gamma,\gamma')}$-$sCl(U) \subseteq A$.

Conversely, let $A \subseteq X$ and $x \in A$. From hypothesis, there exists $U \in \alpha SO(X,x)_{(\gamma,\gamma')}$ such that $\alpha_{(\gamma,\gamma')}$-$sCl(U) \subseteq A$. Therefore, $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap (X \setminus A) = \phi$. Hence, $X \setminus A = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(X \setminus A)$ and A is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen.

Theorem 2. *For a subset A of a topological space (X,τ), we have $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A) = \cap\{V : A \subseteq V$ and $V \in \alpha SR(X)_{(\gamma,\gamma')}\}$.*

Proof. Let $x \notin \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. Then, there exists an $\alpha_{(\gamma,\gamma')}$-semiopen set U containing x such that $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap A = \phi$. Then $A \subseteq X \setminus \alpha_{(\gamma,\gamma')}$-$sCl(U) = V$ (say). Thus $V \in \alpha SR(X)_{(\gamma,\gamma')}$ such that $x \notin V$. Hence $x \notin \cap\{V : A \subseteq V$ and $V \in \alpha SR(X)_{(\gamma,\gamma')}\}$. Again, if $x \notin \cap\{V : A \subseteq V$ and $V \in \alpha SR(X)_{(\gamma,\gamma')}\}$, then there exists $V \in \alpha SR(X)_{(\gamma,\gamma')}$ containing A such that $x \notin V$. Then $(X \setminus V)$ $(= U$, say) is an $\alpha_{(\gamma,\gamma')}$-semiopen set containing x such that $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap V = \phi$. This shows that $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap A = \phi$, so that $x \notin \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$.

Corollary 2. *A subset A of X is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed if and only if $A = \cap\{V : A \subseteq V \in \alpha SR(X)_{(\gamma,\gamma')}\}$.*

Proof. Obvious.

Theorem 3. *Let A and B be any two subsets of a space X. Then, the following properties hold:*

(1) *$x \in \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$ if and only if $U \cap A \neq \phi$ for each $U \in \alpha SR(X)_{(\gamma,\gamma')}$ containing x.*
(2) *If $A \subseteq B$, then $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A) \subseteq \alpha_{(\gamma,\gamma')}$-$sCl_\theta(B)$.*

Proof. Clear.

Theorem 4. *For any subset A of X, $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)) = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$.*

Proof. Obviously, $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A) \subseteq \alpha_{(\gamma,\gamma')}$-$sCl_\theta(\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A))$. Now, let $x \in \alpha_{(\gamma,\gamma')}$-$sCl_\theta(\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A))$ and $U \in \alpha SO(X,x)_{(\gamma,\gamma')}$. Then, $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A) \neq \phi$. Let $y \in \alpha_{(\gamma,\gamma')}$-$sCl(U) \cap \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$. Since $\alpha_{(\gamma,\gamma')}$-$sCl(U) \in \alpha SO(X,y)_{(\gamma,\gamma')}$, then $\alpha_{(\gamma,\gamma')}$-$sCl(\alpha_{(\gamma,\gamma')}$-$sCl(U)) \cap A \neq \phi$, that is $\alpha_{(\gamma,\gamma')}$-$sCl(U) \cap A \neq \phi$. Thus, $x \in \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$.

Corollary 3. *For any $A \subseteq X$, $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed.*

Proof. Obvious.

Theorem 5. *Intersection of arbitrary collection of $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed sets in X is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed.*

Proof. Let $\{A_i : i \in I\}$ be any collection of $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed sets in a topological space (X,τ) and $A = \cap_{i\in I}A_i$. Now, using Definition 8, $x \in \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$, in consequence, $x \in \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A_i)$ for all $i \in I$. Follows that $x \in A_i$ for all $i \in I$. Therefore, $x \in A$. Thus, $A = \alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$.

Corollary 4. *For any $A \subseteq X$, $\alpha_{(\gamma,\gamma')}$-$sCl_\theta(A)$ is the intersection of all $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed sets each containing A.*

Proof. Obvious.

Corollary 5. *Let A and A_i $(i \in I)$ be any subsets of a space X. Then, the following properties hold:*

(1) *A is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen in X if and only if for each $x \in A$ there exists $U \in \alpha SR(X)_{(\gamma,\gamma')}$ such that $x \in U \subseteq A$.*
(2) *If A_i is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen in X for each $i \in I$, then $\cup_{i\in I}A_i$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen in X.*

Proof. Obvious.

Remark. The following example shows that the union of $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed sets may fail to be $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed.

Example 1. Let $X = \{1,2,3\}$ and $\tau = \{\phi,\{1\},\{3\},\{1,3\},X\}$ be a topology on X. For each $A \in \alpha O(X,\tau)$, we define two operations γ and γ', respectively, by $A^\gamma = Int(Cl(A))$ and

$$A^{\gamma'} = \begin{cases} X, & \text{if } A = \{1,3\} \\ A, & \text{if } A \neq \{1,3\}. \end{cases}$$

Then, the subsets $A = \{1\}$ and $B = \{3\}$ are $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed, but their union $\{1,3\} = A \cup B$ is not $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed.

Example 2. Let $X = \{1,2,3\}$ and $\tau = \{\phi,\{1\},\{3\},\{1,3\},X\}$ be a topology on X. For each $A \in \alpha O(X,\tau)$, we define two operations γ and γ', respectively, by

$$A^\gamma = \begin{cases} A, & \text{if } A \neq \{1,3\} \\ X, & \text{if } A = \{1,3\}, \end{cases}$$

and $A^{\gamma'} = A$. The subsets $\{2\}$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed, but not $\alpha_{(\gamma,\gamma')}$-semiregular.

Remark. From Lemma 2 (2), we have $\alpha_{(\gamma,\gamma')}$-semiregular set is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed set. In the above example, $\{2\}$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-semiclosed, but not $\alpha_{(\gamma,\gamma')}$-semiregular.

Remark. For a subset A, we always have $A \subseteq \alpha_{(\gamma,\gamma')}$-$sCl(A) \subseteq \alpha_{(\gamma,\gamma')}$-$sCl_\theta$(A). Therefore, every $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen set is $\alpha_{(\gamma,\gamma')}$-semiopen. The following example shows that the converse is not true in general.

Example 3. Let $X = \{1,2,3\}$ and $\tau = \{\phi,\{1\},\{2\},\{1,2\},\{2,3\},X\}$ be a topology on X. For each $A \in \alpha O(X,\tau)$, we define two operations γ and γ', respectively, by

$$A^\gamma = \begin{cases} A, & \text{if } A \neq \{1\} \\ X, & \text{if } A = \{1\}, \end{cases}$$

and

$$A^{\gamma'} = \begin{cases} A, & \text{if } A \neq \{2\} \\ X, & \text{if } A = \{2\}. \end{cases}$$

Then, $\{1,2\}$ is $\alpha_{(\gamma,\gamma')}$-semiopen set but not an $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen set.

Remark. The notions $\alpha_{(\gamma,\gamma')}$-openness and $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopenness are independent. In Example 2, $\{1,2\}$ is an $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen set but not an $\alpha_{(\gamma,\gamma')}$-open set, whereas in Example 3, $\{1,2\}$ is an $\alpha_{(\gamma,\gamma')}$-open set but not an $\alpha_{(\gamma,\gamma')}$-$\theta$-semiopen set.

Remark. Every $\alpha_{(\gamma,\gamma')}$-$\theta$-open set is $\alpha_{(\gamma,\gamma')}$-open.

4 $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous

Throughout this section, let $f:(X,\tau)\to(Y,\sigma)$ be a function and $\gamma,\gamma^{'}:\alpha O(X,\tau)\to P(X)$ be operations on $\alpha O(X,\tau)$ and $\beta,\beta^{'}:\alpha O(Y,\sigma)\to P(Y)$ be operations on $\alpha O(Y,\sigma)$.

Definition 10. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous if for each point $x\in X$ and each $\alpha_{(\beta,\beta')}$-semiopen set V of Y containing $f(x)$, there exists an $\alpha_{(\gamma,\gamma')}$-open set U of X containing x such that $f(U)\subseteq\alpha_{(\beta,\beta')}$-$sCl(V)$.*

Example 4. Let $X=\{r,m,n\}$, $Y=\{1,2,3\}$, $\tau=\{\phi,\{r\},\{m\},\{r,m\},\{r,n\},X\}$ and $\sigma=\{\phi,\{3\},\{1,2\},Y\}$. For each $A\in\alpha O(X,\tau)$ and $B\in\alpha O(Y,\sigma)$, we define the operations $\gamma:\alpha O(X,\tau)\to P(X)$, $\gamma^{'}:\alpha O(X,\tau)\to P(X)$, $\beta:\alpha O(Y,\sigma)\to P(Y)$ and $\beta^{'}:\alpha O(Y,\sigma)\to P(Y)$, respectively, by

$$A^{\gamma}=\begin{cases}A, & \text{if } n\notin A\\ X, & \text{if } n\in A,\end{cases}$$

$$A^{\gamma'}=\begin{cases}A, & \text{if } m\in A \text{ or } A=\phi\\ A\cup\{m\}, & \text{if } m\notin A,\end{cases}$$

$$B^{\beta}=\begin{cases}Y, & \text{if } 2\notin B\\ B, & \text{if } 2\in B \text{ or } B=\phi,\end{cases}$$

and

$$B^{\beta'}=\begin{cases}Y, & \text{if } 1\notin B\\ B, & \text{if } 1\in B \text{ or } B=\phi.\end{cases}$$

Define a function $f:(X,\tau)\to(Y,\sigma)$ as follows:

$$f(x)=\begin{cases}1, & \text{if } x=r\\ 1, & \text{if } x=m\\ 3, & \text{if } x=n.\end{cases}$$

Clearly, $\alpha O(X,\tau)_{(\gamma,\gamma')}=\{\phi,\{m\},\{r,m\},X\}$ and $\alpha SO(Y,\sigma)_{(\beta,\beta')}=\{\phi,\{1,2\},Y\}$. Then, f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Theorem 6. *The following statements are equivalent for a function $f:(X,\tau)\to(Y,\sigma)$:*

(1) *f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous.*
(2) *For each $x\in X$ and $V\in\alpha SR(Y)_{(\beta,\beta')}$ containing $f(x)$, there exists an $\alpha_{(\gamma,\gamma')}$-open set U containing x such that $f(U)\subseteq V$.*
(3) *$f^{-1}(V)$ is $\alpha_{(\gamma,\gamma')}$-clopen (That is, $\alpha_{(\gamma,\gamma')}$-open as well as $\alpha_{(\gamma,\gamma')}$-closed) in X for every $V\in\alpha SR(Y)_{(\beta,\beta')}$.*
(4) *$f^{-1}(V)\subseteq\alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V)))$ for every $V\in\alpha SO(Y)_{(\beta,\beta')}$.*
(5) *$\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(V)))\subseteq f^{-1}(V)$ for every $\alpha_{(\beta,\beta')}$-semiclosed set V of Y.*
(6) *$\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(V))\subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$ for every $V\in\alpha SO(Y)_{(\beta,\beta')}$.*

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and $V \in \alpha SR(Y)_{(\beta,\beta')}$ containing $f(x)$. By (1), there exists an $\alpha_{(\gamma,\gamma')}$-open set U containing x such that $f(U) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V) = V$.
(2) $\Rightarrow$ (3): Let $V \in \alpha SR(Y)_{(\beta,\beta')}$ and $x \in f^{-1}(V)$. Then, $f(U) \subseteq V$ for some $\alpha_{(\gamma,\gamma')}$-open set U of X containing x, hence $x \in U \subseteq f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\alpha_{(\gamma,\gamma')}$-open in X. Since $Y \setminus V \in \alpha SR(Y)_{(\beta,\beta')}$, $f^{-1}(Y \setminus V)$ is also $\alpha_{(\gamma,\gamma')}$-open and hence $f^{-1}(V)$ is $\alpha_{(\gamma,\gamma')}$-clopen in X.
(3) $\Rightarrow$ (4): Let $V \in \alpha SO(Y)_{(\beta,\beta')}$. Since $V \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$ and by Lemma 1, we have $\alpha_{(\beta,\beta')}$-$sCl(V) \in \alpha SR(Y)_{(\beta,\beta')}$. By (3), we have $f^{-1}(V) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$ and $f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$ is $\alpha_{(\gamma,\gamma')}$-open in X. Therefore, we obtain $f^{-1}(V) \subseteq \alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V)))$.
(4) $\Rightarrow$ (5): Let V be an $\alpha_{(\beta,\beta')}$-semiclosed subset of Y. By (4), we have $f^{-1}(Y \setminus V) \subseteq \alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(Y \setminus V))) = \alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(Y \setminus \alpha_{(\beta,\beta')}$-$sInt(V))) = X \setminus \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(V)))$. Therefore, we obtain $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(V))) \subseteq f^{-1}(V)$.
(5) $\Rightarrow$ (6): Let $V \in \alpha SO(Y)_{(\beta,\beta')}$. By Lemma 1, $\alpha_{(\beta,\beta')}$-$sCl(V) \in \alpha SR(Y)_{(\beta,\beta')}$. By (5), we obtain $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(V)) \subseteq \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))) = \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V)))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$.
(6) $\Rightarrow$ (1): Let $x \in X$ and $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$. By Lemma 1, we have $\alpha_{(\beta,\beta')}$-$sCl(V) \in \alpha SR(Y)_{(\beta,\beta')}$ and $f(x) \notin Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V) = \alpha_{(\beta,\beta')}$-$sCl(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V))$. Thus, by (6) we obtain $x \notin \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V)))$. There exists an $\alpha_{(\gamma,\gamma')}$-open set U of X containing x such that $U \cap f^{-1}(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V)) = \phi$. Therefore, we have $f(U) \cap (Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V)) = \phi$ and hence $f(U) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$. This shows that f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Proposition 2. *The following statements are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(1) *f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.*
(2) *For each $x \in X$ and $V \in \alpha SR(Y)_{(\beta,\beta')}$ containing $f(x)$, there exists an $\alpha_{(\gamma,\gamma')}$-clopen set U containing x such that $f(U) \subseteq V$.*
(3) *For each $x \in X$ and $V \in \alpha SO(Y)_{(\beta,\beta')}$ containing $f(x)$, there exists an $\alpha_{(\gamma,\gamma')}$-open set U containing x such that $f(\alpha_{(\gamma,\gamma')}$-$Cl(U)) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$.*

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and $V \in \alpha SR(Y)_{(\beta,\beta')}$ containing $f(x)$. By Theorem 6, $f^{-1}(V)$ is $\alpha_{(\gamma,\gamma')}$-clopen in X. Put $U = f^{-1}(V)$, then $x \in U$ and $f(U) \subseteq V$.
(2) $\Rightarrow$ (3): Let $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$. By Lemma 1, we have $\alpha_{(\beta,\beta')}$-$sCl(V) \in \alpha SR(Y)_{(\beta,\beta')}$ and by (2), there exists an $\alpha_{(\gamma,\gamma')}$-clopen set U containing x such that $f(\alpha_{(\gamma,\gamma')}$-$Cl(U)) = f(U) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$.
(3) $\Rightarrow$ (1): Let $x \in X$ and $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$. By (3), there exists an $\alpha_{(\gamma,\gamma')}$-open set U containing x such that $f(\alpha_{(\gamma,\gamma')}$-$Cl(U)) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$ implies that $f(U) \subseteq f(\alpha_{(\gamma,\gamma')}$-$Cl(U)) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$. This shows that f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Theorem 7. *The following statements are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(1) *f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.*
(2) *$\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(B)) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$ for every subset B of Y.*
(3) *$f(\alpha_{(\gamma,\gamma')}$-$Cl(A)) \subseteq \alpha_{(\beta,\beta')}$-$sCl_\theta(f(A))$ for every subset A of X.*
(4) *$f^{-1}(F)$ is $\alpha_{(\gamma,\gamma')}$-closed in X for every $\alpha_{(\beta,\beta')}$-θ-semiclosed set F of Y.*
(5) *$f^{-1}(V)$ is $\alpha_{(\gamma,\gamma')}$-open in X for every $\alpha_{(\beta,\beta')}$-θ-semiopen set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y and $x \notin f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$. Then, $f(x) \notin \alpha_{(\beta,\beta')}$-$sCl_\theta(B)$ and there exists $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$ such that $\alpha_{(\beta,\beta')}$-$sCl(V) \cap B = \phi$. By (1), there exists an $\alpha_{(\gamma,\gamma')}$-open set U containing x such that $f(U) \subseteq \alpha_{(\beta,\beta')}$-$sCl(V)$. Hence, $f(U) \cap B = \phi$ and $U \cap f^{-1}(B) = \phi$. Consequently, we obtain $x \notin \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(B))$.
(2) $\Rightarrow$ (3): Let A be any subset of X. By (2), we have $\alpha_{(\gamma,\gamma')}$-$Cl(A) \subseteq \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(f(A))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(f(A)))$ and hence $f(\alpha_{(\gamma,\gamma')}$-$Cl(A)) \subseteq \alpha_{(\beta,\beta')}$-$sCl_\theta(f(A))$.

(3) $\Rightarrow$ (4): Let F be any $\alpha_{(\beta,\beta')}$-θ-semiclosed set of Y. Then, by (3), we have $f(\alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(F))) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl_\theta(f(f^{-1}(F))) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl_\theta(F) = F$. Therefore, we have $\alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(F)) \subseteq f^{-1}(F)$ and hence $\alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is $\alpha_{(\gamma,\gamma')}$-closed in X.
(4) $\Rightarrow$ (5): Obvious.
(5) $\Rightarrow$ (1): Let $x \in X$ and $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$. By Lemmas 1 and 2 (2), $\alpha_{(\beta,\beta')}$-$sCl(V)$ is $\alpha_{(\beta,\beta')}$-θ-semiopen in Y. Put $U = f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(V))$. Then by (5), U is $\alpha_{(\gamma,\gamma')}$-open containing x and $f(U) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl(V)$. Thus, f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Proposition 3. *The following statements are equivalent for a function $f : (X,\tau) \to (Y,\sigma)$:*

(1) *f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.*
(2) *$\alpha_{(\gamma,\gamma')}\text{-}Cl_\theta(f^{-1}(B)) \subseteq f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl_\theta(B))$ for every subset B of Y.*
(3) *$f(\alpha_{(\gamma,\gamma')}\text{-}Cl_\theta(A)) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl_\theta(f(A))$ for every subset A of X.*
(4) *$f^{-1}(F)$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-closed in X for every $\alpha_{(\beta,\beta')}$-θ-semiclosed set F of Y.*
(5) *$f^{-1}(V)$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-open in X for every $\alpha_{(\beta,\beta')}$-θ-semiopen set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y and $x \notin f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl_\theta(B))$. Then, $f(x) \notin \alpha_{(\beta,\beta')}\text{-}sCl_\theta(B)$ and there exists $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$ such that $\alpha_{(\beta,\beta')}\text{-}sCl(V) \cap B = \phi$. By Proposition 2 (3), there exists an $\alpha_{(\gamma,\gamma')}$-open set U containing x such that $f(\alpha_{(\gamma,\gamma')}\text{-}Cl(U)) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl(V)$. Hence, $f(\alpha_{(\gamma,\gamma')}\text{-}Cl(U)) \cap B = \phi$ and $\alpha_{(\gamma,\gamma')}\text{-}Cl(U) \cap f^{-1}(B) = \phi$. Consequently, we obtain $x \notin \alpha_{(\gamma,\gamma')}\text{-}Cl_\theta(f^{-1}(B))$.
(2) $\Rightarrow$ (3): Let A be any subset of X. By (2), we have $\alpha_{(\gamma,\gamma')}$ $\text{-}Cl_\theta(A) \subseteq \alpha_{(\gamma,\gamma')}$ $\text{-}Cl_\theta(f^{-1}(f(A))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$ $\text{-}sCl_\theta(f(A)))$ and hence $f(\alpha_{(\gamma,\gamma')}$ $\text{-}Cl_\theta(A)) \subseteq \alpha_{(\beta,\beta')}$ $\text{-}sCl_\theta(f(A))$.
(3) $\Rightarrow$ (4): Let F be any $\alpha_{(\beta,\beta')}$-θ-semiclosed set of Y. Then, by (3), we have $f(\alpha_{(\gamma,\gamma')}\text{-}Cl_\theta(f^{-1}(F))) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl_\theta(f(f^{-1}(F))) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl_\theta(F) = F$. Therefore, we have $\alpha_{(\gamma,\gamma')}\text{-}Cl_\theta(f^{-1}(F)) \subseteq f^{-1}(F)$ and hence $\alpha_{(\gamma,\gamma')}\text{-}Cl_\theta(f^{-1}(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is $\alpha_{(\gamma,\gamma')}$-$\theta$-closed in X.
(4) $\Rightarrow$ (5): Obvious.
(5) $\Rightarrow$ (1): Let $x \in X$ and $V \in \alpha SO(Y, f(x))_{(\beta,\beta')}$. By Lemmas 1 and 2 (2), $\alpha_{(\beta,\beta')}$-$sCl(V)$ is $\alpha_{(\beta,\beta')}$-θ-semiopen in Y. Put $U = f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(V))$. Then, by (5), U is $\alpha_{(\gamma,\gamma')}$-$\theta$-open containing x and by Remark 3, U is $\alpha_{(\gamma,\gamma')}$-open such that $f(U) \subseteq \alpha_{(\beta,\beta')}\text{-}sCl(V)$. Thus, f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Proposition 4. *The following statements are equivalent for a function $f : (X,\tau) \to (Y,\sigma)$:*

(1) *f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.*
(2) *$\alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(\alpha_{(\beta,\beta')}\text{-}sInt(\alpha_{(\beta,\beta')}\text{-}sCl(B)))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(B))$, for every subset B of Y.*
(3) *$f^{-1}(\alpha_{(\beta,\beta')}\text{-}sInt(B)) \subseteq \alpha_{(\gamma,\gamma')}\text{-}Int(f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(\alpha_{(\beta,\beta')}\text{-}sInt(B))))$, for every subset B of Y.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Then, $\alpha_{(\beta,\beta')}$-$sCl(B)$ is $\alpha_{(\beta,\beta')}$-semiclosed in Y and by Theorem 6 (5), we have $\alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(\alpha_{(\beta,\beta')}\text{-}sInt(\alpha_{(\beta,\beta')}\text{-}sCl(B)))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(B))$.
(2) $\Rightarrow$ (3): Let B be any subset of Y and $x \in f^{-1}(\alpha_{(\beta,\beta')}\text{-}sInt(B))$. Then, we have $x \in X \setminus f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(Y \setminus B))$. So, $x \notin f^{-1}(\alpha_{(\beta,\beta')}$ $\text{-}sCl(Y \setminus B))$ and by (2), we have $x \in X \setminus \alpha_{(\gamma,\gamma')}$ $\text{-}Cl(f^{-1}(\alpha_{(\beta,\beta')}$ $\text{-}sInt(\alpha_{(\beta,\beta')}$ $\text{-}sCl(Y \setminus B)))) = \alpha_{(\gamma,\gamma')}\text{-}Int(f^{-1}(\alpha_{(\beta,\beta')}$ $\text{-}sCl(\alpha_{(\beta,\beta')}\text{-}sInt(B))))$.
(3) $\Rightarrow$ (1): Let V be any $\alpha_{(\beta,\beta')}$-semiopen set of Y. Suppose that $z \notin f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(V))$. Then, $f(z) \notin \alpha_{(\beta,\beta')}\text{-}sCl(V)$ and there exists an $\alpha_{(\beta,\beta')}$-semiopen set W containing $f(z)$ such that $W \cap V = \phi$ and hence $\alpha_{(\gamma,\gamma')}\text{-}sCl(W) \cap V = \phi$. By (3), we have $z \in \alpha_{(\gamma,\gamma')}\text{-}Int(f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(W)))$ and hence there exists $U \in \alpha O(X)_{(\gamma,\gamma')}$ such that $z \in U \subseteq f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl(W))$. Since $\alpha_{(\beta,\beta')}\text{-}sCl(W) \cap V = \phi$, then $U \cap f^{-1}(V) = \phi$ and so, $z \notin \alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(V))$. Therefore, $\alpha_{(\gamma,\gamma')}\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(\alpha_{(\beta,\beta')}\text{-}sCl((V)))$ for every $V \in \alpha SO(Y)_{(\beta,\beta')}$. Hence, by Theorem 6, f is $(\alpha_{(\gamma,\gamma')}, \alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Proposition 5. *The following statements are equivalent for a function $f : (X,\tau) \to (Y,\sigma)$:*

(1) f *is* $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$*-*$\theta$*-semi continuous.*
(2) $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl_\theta(B)))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$*, for every subset* B *of* Y*.*
(3) $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(B)))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$*, for every subset* B *of* Y*.*
(4) $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(O)))) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(O))$*, for every* $\alpha_{(\beta,\beta')}$*-semiopen set* O *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Then, $\alpha_{(\beta,\beta')}$-$sCl_\theta(B)$ is $\alpha_{(\beta,\beta')}$-semiclosed in Y. Then by Theorem 6 (5), if $x \in \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))))$, then $x \in f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$.
(2) $\Rightarrow$ (3): This is obvious since $\alpha_{(\beta,\beta')}$-$sCl(B) \subseteq \alpha_{(\beta,\beta')}$-$sCl_\theta(B)$ for every subset B.
(3) $\Rightarrow$ (4): By Lemma 2 (1), we have $\alpha_{(\beta,\beta')}$-$sCl(O) = \alpha_{(\beta,\beta')}$-$sCl_\theta(O)$ for every $\alpha_{(\beta,\beta')}$-semiopen set O.
(4) $\Rightarrow$ (1): Let V be any $\alpha_{(\beta,\beta')}$-semiopen set of Y and $x \in \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(V))$. Then, V is $\alpha_{(\beta,\beta')}$-semiopen and $x \in \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))))$. By (4), $x \in f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$. It follows from Theorem 6, that f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Corollary 6. *A function* $f:(X,\tau)\to(Y,\sigma)$ *is* $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$*-*$\theta$*-semi continuous if and only if* $f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$ *is* $\alpha_{(\gamma,\gamma')}$*-open set in* X*, for each* $\alpha_{(\beta,\beta')}$*-semiopen set* V *in* Y*.*

Proof. Since, if $V \in \alpha SO(Y)_{(\beta,\beta')}$, then, $\alpha_{(\beta,\beta')}$-$sCl(V) \in \alpha SR(Y)_{(\beta,\beta')}$, so by Theorem 6 (3), $f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$ is $\alpha_{(\gamma,\gamma')}$-clopen, which is $\alpha_{(\gamma,\gamma')}$-open.

Conversely, if $V \in \alpha SO(Y)_{(\beta,\beta')}$, then by hypothesis, $f^{-1}(V) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$ -$sCl(V)) = \alpha_{(\gamma,\gamma')}$ -$sInt(f^{-1}(\alpha_{(\beta,\beta')}$ -$sCl(V)))$, so by Theorem 6 (4), f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$ -θ-semi continuous.

Corollary 7. *A function* $f:(X,\tau)\to(Y,\sigma)$ *is* $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$ *-*θ*-semi continuous if and only if* $f^{-1}(\alpha_{(\beta,\beta')}$ -$sInt(F))$ *is an* $\alpha_{(\gamma,\gamma')}$*-closed set in* X*, for each* $\alpha_{(\beta,\beta')}$*-semclosed set* F *of* Y*.*

Proof. It follows from Corollary 6.

Corollary 8. *Let* $f:(X,\tau)\to(Y,\sigma)$ *be a function. If* $f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$ *is* $\alpha_{(\gamma,\gamma')}$*-closed in* X *for every subset* B *of* Y*, then* f *is* $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$*-*$\theta$*-semi continuous.*

Proof. Let $B \subseteq Y$. Since $f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$ is $\alpha_{(\gamma,\gamma')}$-closed in X, then $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(B)) \subseteq \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))) = f^{-1}(\alpha_{(\beta,\beta')}$-$sCl_\theta(B))$. By Theorem 7, f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Proposition 6. *The following statements are equivalent for a function* $f:(X,\tau)\to(Y,\sigma)$*:*

(1) f *is* $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$*-*$\theta$*-semi continuous.*
(2) $\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(V)) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$*, for every* $V \subseteq \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))$*.*
(3) $f^{-1}(V) \subseteq \alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V)))$*, for every* $V \subseteq \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))$*.*

Proof. (1) $\Rightarrow$ (2): Let $V \subseteq \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))$ such that $x \in \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(V))$. Suppose that $x \notin f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$. Then there exists an $\alpha_{(\beta,\beta')}$-semopen set W containing $f(x)$ such that $W \cap V = \phi$. Hence, we have $W \cap \alpha_{(\beta,\beta')}$-$sCl(V) = \phi$ and hence $\alpha_{(\beta,\beta')}$-$sCl(W) \cap \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V)) = \phi$. Since $V \subseteq \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))$ and we have $V \cap \alpha_{(\beta,\beta')}$-$sCl(W) = \phi$. Since f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous at $x \in X$ and W is an $\alpha_{(\beta,\beta')}$-semopen set containing $f(x)$, there exists $U \in \alpha O(X)_{(\gamma,\gamma')}$ containing x such that $f(U) \subseteq \alpha_{(\beta,\beta')}$-$sCl(W)$. Then, $f(U)\cap V = \phi$ and hence $U \cap f^{-1}(V) = \phi$. This shows that $x \notin \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(V))$. This is a contradiction. Therefore, we have $x \in f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V))$.
(2) $\Rightarrow$ (3): Let $V \subseteq \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))$ and $x \in f^{-1}(V)$. Then, we have $f^{-1}(V) \subseteq f^{-1}(\alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))) = X \setminus f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V)))$. Therefore, $x \notin f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V)))$. Then by (2), $x \notin \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V)))$. Hence, $x \in X \setminus \alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(Y \setminus \alpha_{(\beta,\beta')}$-$sCl(V))) = \alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(V)))$.
(3) $\Rightarrow$ (1): Let V be any $\alpha_{(\beta,\beta')}$-semiopen set of Y. Then, $V = \alpha_{(\beta,\beta')}$-$sInt(V) \subseteq \alpha_{(\beta,\beta')}$-$sInt(\alpha_{(\beta,\beta')}$-$sCl(V))$. Hence, by (3) and Theorem 6, f is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous.

Proposition 7. *If $f:(X,\tau)\rightarrow(Y,\sigma)$ is $(\alpha_{(\gamma,\gamma')},\alpha_{(\beta,\beta')})$-$\theta$-semi continuous at $x\in X$, then for each $\alpha_{(\beta,\beta')}$-semiopen set B containing $f(x)$ and each $\alpha_{(\gamma,\gamma')}$-open set A containing x, there exists a nonempty $\alpha_{(\gamma,\gamma')}$-open set $U\subseteq A$ such that $U\subseteq\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(B)))$. Where γ and γ' are α-regular operations.*

Proof. Let B be any $\alpha_{(\beta,\beta')}$-semiopen set containing $f(x)$ and A be an $\alpha_{(\gamma,\gamma')}$-open set of X containing x. By Lemma 1 and Theorem 6, $x\in\alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(B)))$, then $A\cap\alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(B)))\neq\phi$. Take $U=A\cap\alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(B)))$. Thus, U is a nonempty $\alpha_{(\gamma,\gamma')}$-open set by Proposition 1, and hence $U\subseteq A$ and $U\subseteq\alpha_{(\gamma,\gamma')}$-$Int(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(B)))\subseteq\alpha_{(\gamma,\gamma')}$-$Cl(f^{-1}(\alpha_{(\beta,\beta')}$-$sCl(B)))$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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