Journal of Abstract and Computational Mathematics

http://www.ntmsci.com/jacm

Some oscillation criteria for nonlinear conformable fractional differential equations

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Received: 15 January 2020, Accepted: 20 February 2020
Published online: 15 March 2020.

Abstract: This paper concerns the oscillation problem of a general class of fractional differential equations. New oscillation criteria for a class of fractional nonlinear differential equations with damping and forcing terms have been established by using the classical Riccati technique.

Keywords: Interval criteria, conformable derivative, oscillation

1 Introduction and Preliminaries

Fractional differential (or difference) equations are a more general form of differential equations with integer order. And there is an increasing interest in the study of them due to some important contributions in science and engineering see [1, 2]. For the many theories and applications of fractional differential equations, we refer the monographs [3,4,5].

In recent years, research on oscillation of various equations, including differential equations, fractional differential equations, difference equations, fractional difference equations, partial differential equations, fractional partial differential equations and dynamic equations on time scales has been a hot topic in the literature, and much effort has already been put into establishing new oscillation criteria for these equations; see [6,7,8,9,10,11,12,13,14,15,16,17,18, 19,20,21,22,23,24,25,26,27,28].

In this study, with the definition of conformable fractional derivative given by R. Khalil [29], we study the oscillation properties of the solutions of following conformable fractional differential equations

$$T_\alpha\left[a(t)\,\psi(x(t))\,f(T_\alpha x(t))\right]+h(t)\,f(T_\alpha x(t))+q(t)\,g(x(t))=Q(t,T_\alpha x(t),x(t)) \tag{1}$$

where $a\in C^\alpha([t_0,\infty),\mathbb{R})$, $h,q\in C([t_0,\infty),\mathbb{R})$, $\psi,f\in C^\alpha(\mathbb{R},\mathbb{R})$, $g\in C(\mathbb{R},\mathbb{R})$, $Q\in C\left([t_0,\infty)\times\mathbb{R}^2,\mathbb{R}\right)$, T_α denotes the operator called conformable fractional derivative of order α, C^α denotes continuous function with fractional derivative of order α. We shall make use of the following conditions in our results:

(C1) $a(t)>0,\ t\geq 0$;
(C2) $0<k_1\leq\psi(x)\leq k_2$, for all $x\neq 0$;
(C3) there exists a constant $l>0$ such that $f^2(y)\leq lyf(y)$ for all $y\in\mathbb{R}$;
(C4) $Q(t,y,x)/g(x)\leq p(t)$ for $t\in[t_0,\infty)$, $x,y\in\mathbb{R}$, $x\neq 0$ and $p\in C([t_0,\infty),\mathbb{R})$;

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(C5) $g'(x) \geq k > 0$ for all $x \neq 0$;
(C6) g satisfies $g(x)/x \geq k^* > 0$ for all $x \neq 0$ and $q(t) - p(t) \geq 0$ for $t \geq t_0$.

Definition 1. [29] *Given a function $f : [0,\infty) \to \mathbb{R}$. Then the "conformable fractional derivative" of f of order α is defined by*

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0} \frac{f\left(t+\varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon}, \ \ \forall t > 0, \ \alpha \in (0,1).$$

If f is $\alpha-$differentiable in some $(0,a)$, $a > 0$, and $\lim_{t\to 0^+} f^{\alpha}(t)$ exists, then define

$$f^{(\alpha)}(0) = \lim_{t\to 0^+} f^{(\alpha)}(t).$$

Definition 2. [29] *Let $\alpha \in (n, n+1]$, and f be an $n-$differentiable at t, where $t > 0$. Then the conformable fractional derivative of f of order α is defined as*

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0} \frac{f^{(\lceil\alpha\rceil-1)}\left(t+\varepsilon t^{(\lceil\alpha\rceil-\alpha)}\right) - f^{(\lceil\alpha\rceil-1)}(t)}{\varepsilon}, \ \ \forall t > 0, \ \alpha \in (0,1)$$

where $\lceil\alpha\rceil$ is the smallest integer greater than or equal to α.

Definition 3. [29] *Let $\alpha \in (0,1]$ and $0 \leq a < b$. A function $f : [a,b] \to \mathbb{R}$ is $\alpha-$fractional integrable on $[a,b]$ if the integral*

$$\int_a^b f(x)\, d_\alpha x = \int_a^b f(x)\, x^{\alpha-1} dx$$

exists and is finite.

Theorem 1. [29] *Let $\alpha \in (0,1]$ and f, g be $\alpha-$differentiable at a point t, then:*

(1) $T_\alpha(af+bg) = aT_\alpha(f) + bT_\alpha(g)$ for all $a, b \in \mathbb{R}$.
(2) $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.
(3) $T_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in \mathbb{R}$.
(4) $T_\alpha\left(\frac{f}{g}\right) = \frac{fT_\alpha(g)-gT_\alpha(f)}{g^2}$.
(5) $T_\alpha(c) = 0$ where c is constant.
If, in addition, f is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha}\frac{df}{dt}$.

Theorem 2. [30] *Let $f, g : [a,\infty] \to \mathbb{R}$ be two functions such that fg is differentiable. Then*

$$\int_a^b f(x)\, T_\alpha^a(g)(x)\, d_\alpha x = fg|_a^b - \int_a^b g(x)\, T_\alpha^a(f)(x)\, d_\alpha x$$

where $T_\alpha^a(.)$ represent the conformable fractional derivative of order α.

Theorem 3. [30,31,32] *Assume functions $f, g : (0,\infty) \to \mathbb{R}$ be $\alpha-$differentiable functions, where $0 < \alpha \leq 1$. Then the following rule is obtained*

$$T_\alpha^a(f \circ g)(t) = t^{1-\alpha} g'(t)\, f'(g(t)).$$

In the proof of our results we define following a useful operator, as an important tool. Firstly, we introduce a class of functions Y. We say that a function $\Phi(t,s,r)$ belongs to the function class Y denoted by $\Phi \in Y$, if $\Phi \in C(E,\mathbb{R})$, where $E = \{(t,s,r) : t_0 \leq r \leq s \leq t < \infty\}$ which satisfies $\Phi(t,t,r) = 0$, $\Phi(t,r,r) = 0$, $\Phi(t,s,r) \neq 0$ for $r < s < t$, and has the partial derivative $\partial^{\alpha}\Phi/\partial s^{\alpha}$ on E such that $\partial^{\alpha}\Phi/\partial s^{\alpha}$ is locally $\alpha-$ integrable with respect to s in E and satisfies

$$\frac{\partial^{\alpha}\Phi(t,s,r)}{\partial s^{\alpha}} = \phi(t,s,r)\,\Phi(t,s,r). \tag{2}$$

Next, we define the operator

$$A(b;r,t) = \int_r^t \Phi^2(t,s,r)\,b(s)\,d_{\alpha}s \text{ for } t \geq s \geq r \geq t_0 \text{ and } b \in C([0,\infty),\mathbb{R}). \tag{3}$$

It is easy to verify that linear operator $A(.;r,t)$ satisfies

$$A(T_{\alpha}(b);r,t) = -2A(\phi b;r,t) \text{ for } b \in C^{\alpha}([0,\infty),\mathbb{R}). \tag{4}$$

2 Main Results

Theorem 4. *Let assumption (C_1)–(C_5) be fulfilled. If there exist functions $\Phi \in Y$, $\eta \in C^{\alpha}([t_0,\infty),(0,\infty))$ such that*

$$\limsup_{t\to\infty} A\left[\eta(s)H(s) - \frac{2lk_2}{k}\eta(s)a(s)\phi^2;r,t\right] > 0, \tag{5}$$

where $r \geq t_0$, the operator A is defined by (3), the function $\phi = \phi(s,r,t)$ is defined by (2) and

$$H(t) = q(t) - p(t) - \frac{l}{4k}\left(\frac{1}{k_1} - \frac{1}{k_2}\right)\frac{h^2(t)}{a(t)} - \frac{lk_2}{2k}a(t)\left(\frac{T_{\alpha}\eta(t)}{\eta(t)} - \frac{h(t)}{k_2 a(t)}\right)^2,$$

then every solution of (1) is oscillatory.

Proof. Let x be a nonoscillatory solution of (1). Without loss of generality, we assume that $x(t) > 0$ for $t \geq t_1 \geq t_0$. A similar argument holds for the case when x is eventually negative. Define

$$\begin{aligned}
\vartheta(t) =& \eta(t)\frac{a(t)\psi(x(t))f(T_{\alpha}x(t))}{g(x(t))},\\
T_{\alpha}\vartheta(t) =& \eta(t)T_{\alpha}\frac{a(t)\psi(x(t))f(T_{\alpha}x(t))}{g(x(t))} + \frac{a(t)\psi(x(t))f(T_{\alpha}x(t))}{g(x(t))}T_{\alpha}\eta(t)\\
=& \eta(t)\left(\frac{T_{\alpha}[a(t)\psi(x(t))f(T_{\alpha}x(t))]g(x(t)) - a(t)\psi(x(t))f(T_{\alpha}x(t))T_{\alpha}g(x(t))}{g^2(x(t))}\right)\\
&+ \frac{a(t)\psi(x(t))f(T_{\alpha}x(t))}{g(x(t))}T_{\alpha}\eta(t)\\
=& \eta(t)\frac{T_{\alpha}[a(t)\psi(x(t))f(T_{\alpha}x(t))]}{g(x(t))} - \eta(t)\frac{a(t)\psi(x(t))f(T_{\alpha}x(t))T_{\alpha}x(t)g'(x(t))}{g^2(x(t))}\\
&+ \frac{a(t)\psi(x(t))f(T_{\alpha}x(t))}{g(x(t))}T_{\alpha}\eta(t)
\end{aligned}$$

and then we have

$$\begin{aligned}
T_{\alpha}\vartheta(t) = \eta(t)&\left(\frac{Q(t,T_{\alpha}x(t),x(t))}{g(x(t))} - \frac{h(t)f(T_{\alpha}x(t))}{g(x(t))} - q(t)\right)\\
&- \eta(t)\frac{a(t)\psi(x(t))f(T_{\alpha}x(t))T_{\alpha}x(t)g'(x(t))}{g^2(x(t))} + \frac{T_{\alpha}\eta(t)}{\eta(t)}\vartheta(t).
\end{aligned}$$

Using (C_3) and (C_4), we get

$$\begin{aligned}
T_\alpha \vartheta(t) &\leq \eta(t)\left(p(t)-\frac{h(t)f(T_\alpha x(t))}{g(x(t))}-q(t)\right)-\frac{k}{l}\eta(t)\frac{a(t)\psi(x(t))f^2(T_\alpha x(t))}{g^2(x(t))}+\frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t)\\
&=\frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t)+\eta(t)(p(t)-q(t))+\frac{l\eta(t)h^2(t)}{4ka(t)\psi(x(t))}\\
&-\frac{\eta(t)}{\psi(x(t))}\left(\sqrt{\frac{ka(t)}{l}}\frac{\psi(x(t))f(T_\alpha x(t))}{g(x(t))}+\frac{h(t)}{2}\sqrt{\frac{l}{ka(t)}}\right)^2.
\end{aligned}$$

By assuming (C_2), we obtain

$$\begin{aligned}
T_\alpha\vartheta(t) &\leq \frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t)+\eta(t)(p(t)-q(t))+\frac{l\eta(t)h^2(t)}{4kk_1a(t)}-\frac{\eta(t)}{k_2}\left(\sqrt{\frac{ka(t)}{l}}\frac{\vartheta(t)}{\eta(t)a(t)}+\frac{h(t)}{2}\sqrt{\frac{l}{ka(t)}}\right)^2\\
&=\eta(t)\left(p(t)-q(t)+\frac{l}{4k}\left(\frac{1}{k_1}-\frac{1}{k_2}\right)\frac{h^2(t)}{a(t)}\right)+\left(\frac{T_\alpha\eta(t)}{\eta(t)}-\frac{h(t)}{k_2a(t)}\right)\vartheta(t)-\frac{k}{lk_2\eta(t)a(t)}\vartheta^2(t).
\end{aligned}$$

Using the following inequality

$$-az^2+bz\leq-\frac{a}{2}z^2+\frac{b^2}{2a},\ \ a>0,\ b,z\in\mathbb{R},$$

we have

$$T_\alpha\vartheta(t)\leq-\eta(t)H(t)-\frac{k\vartheta^2(t)}{2lk_2\eta(t)a(t)}. \tag{6}$$

Applying $A[\cdot;r,t]$ to (6) and using (4) we have

$$A[\eta(s)H(s);r,t]\leq A\left[2|\phi||\vartheta(s)|-\frac{k\vartheta^2(s)}{2lk_2\eta(s)a(s)};r,t\right]. \tag{7}$$

Set

$$F(v)=2|\phi|v-\frac{k}{2lk_2\eta a}v^2,\ \ v>0.$$

We have $F'(v^*)=0$ and $F''(v^*)<0$ where $v^*=\frac{2lk_2}{k}\eta a|\phi|$, which implies that F obtains its maximum at v^*. So we have

$$F(v)\leq F(v^*)=\frac{2lk_2}{k}\eta a\phi^2. \tag{8}$$

Then by using (8) in (7) we get

$$A[\eta(s)H(s);r,t]\leq A\left[\frac{2lk_2}{k}\eta(s)a(s)\phi^2;r,t\right]$$

which contradicts (5). Thus, the proof is complete.

Theorem 5. *Let assumption (C_1)–(C_4) and (C_6) be fulfilled. If there exist functions $\Phi\in Y$, $\eta\in C^\alpha([t_0,\infty),(0,\infty))$ such that*

$$\limsup_{t\to\infty}A\left[\eta(s)H^*(s)-2lk_2\eta(s)a(s)\phi^2;r,t\right]>0, \tag{9}$$

where $r\geq t_0$, the operator A is defined by (3), the function $\phi=\phi(s,r,t)$ is defined by (2) and

$$H^*(t)=k^*(q(t)-p(t))-\frac{l}{4}\left(\frac{1}{k_1}-\frac{1}{k_2}\right)\frac{h^2(t)}{a(t)}-\frac{lk_2}{2}a(t)\left(\frac{T_\alpha\eta(t)}{\eta(t)}-\frac{h(t)}{k_2a(t)}\right)^2$$

then every solution of (1) *is oscillatory.*

Proof. Let x be a nonoscillatory solution of (1). Without loss of generality, we assume that $x(t) > 0$ for $t \geq t_1 \geq t_0$. A similar argument holds for the case when x is eventually negative. Define

$$\begin{aligned}
\vartheta(t) &= \eta(t)\frac{a(t)\psi(x(t))f(T_\alpha x(t))}{x(t)},\\
T_\alpha\vartheta(t) &= \eta(t)T_\alpha\frac{a(t)\psi(x(t))f(T_\alpha x(t))}{x(t)} + \frac{a(t)\psi(x(t))f(T_\alpha x(t))}{x(t)}T_\alpha\eta(t)\\
&= \eta(t)\left(\frac{T_\alpha\left[a(t)\psi(x(t))f(T_\alpha x(t))\right]x(t) - a(t)\psi(x(t))f(T_\alpha x(t))T_\alpha x(t)}{x^2(t)}\right)\\
&+ \frac{a(t)\psi(x(t))f(T_\alpha x(t))}{g(x(t))}T_\alpha\eta(t)\\
&= \eta(t)\frac{T_\alpha\left[a(t)\psi(x(t))f(T_\alpha x(t))\right]}{x(t)} - \eta(t)\frac{a(t)\psi(x(t))f(T_\alpha x(t))T_\alpha x(t)}{x^2(t)}\\
&+ \frac{a(t)\psi(x(t))f(T_\alpha x(t))}{g(x(t))}T_\alpha\eta(t)
\end{aligned}$$

and then we deduce

$$\begin{aligned}
T_\alpha\vartheta(t) = \eta(t)&\left(\frac{Q(t,T_\alpha x(t),x(t))}{x(t)} - \frac{h(t)f(T_\alpha x(t))}{x(t)} - \frac{q(t)g(x(t))}{x(t)}\right)\\
&- \eta(t)\frac{a(t)\psi(x(t))f(T_\alpha x(t))T_\alpha x(t)}{x^2(t)} + \frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t).
\end{aligned}$$

Using (C_3) and (C_4), we have

$$\begin{aligned}
T_\alpha\vartheta(t) &\leq \eta(t)\left((p(t)-q(t))\frac{g(x(t))}{x(t)} - \frac{h(t)f(T_\alpha x(t))}{x(t)}\right) - \eta(t)\frac{a(t)\psi(x(t))f(T_\alpha x(t))T_\alpha x(t)}{x^2(t)} + \frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t)\\
&\leq \eta(t)\left((p(t)-q(t))k^* - \frac{h(t)f(T_\alpha x(t))}{x(t)}\right) - \eta(t)\frac{a(t)\psi(x(t))f^2(T_\alpha x(t))}{lx^2(t)} + \frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t),
\end{aligned}$$

i.e.

$$\begin{aligned}
T_\alpha\vartheta(t) \leq &\frac{T_\alpha\eta(t)}{\eta(t)}\vartheta(t) + k^*\eta(t)(p(t)-q(t)) - \eta(t)\frac{h(t)f(T_\alpha x(t))}{x(t)} + \frac{l\eta(t)h^2(t)}{4k_1a(t)}\\
&- \frac{\eta(t)}{k_2}\left(\sqrt{\frac{a(t)}{l}}\frac{\psi(x(t))f(T_\alpha x(t))}{x(t)} + \frac{h(t)}{2}\sqrt{\frac{l}{a(t)}}\right)^2.
\end{aligned}$$

By assuming (C_2), we obtain

$$T_\alpha\vartheta(t) \leq \eta(t)\left(k^*(p(t)-q(t)) + \frac{l}{4}\left(\frac{1}{k_1}-\frac{1}{k_2}\right)\frac{h^2(t)}{a(t)}\right) + \left(\frac{T_\alpha\eta(t)}{\eta(t)} - \frac{h(t)}{k_2a(t)}\right)\vartheta(t) - \frac{1}{lk_2\eta(t)a(t)}\vartheta^2(t).$$

Using the following inequality

$$-az^2 + bz \leq -\frac{a}{2}z^2 + \frac{b^2}{2a}, \quad a > 0,\ b,z \in \mathbb{R},$$

we have

$$T_\alpha\vartheta(t) \leq -\eta(t)H^*(t) - \frac{\vartheta^2(t)}{2lk_2\eta(t)a(t)}.$$

The rest of the proof is similar to that of Theorem 1, hence is omitted.

Theorem 6. *Let assumption* (C_1)*–*(C_5) *be fulfilled. If there exist functions* $\Phi \in Y$, $\eta \in C^{\alpha}\left([t_0,\infty),(0,\infty)\right)$ *such that for any sufficiently large* $T \geq t_0$, *there exist* c,d *with* $T \leq c < d$ *satisfying*

$$A\left[\eta(s)H(s) - \frac{2lk_2}{k}\eta(s)a(s)\phi^2; c,d\right] > 0, \tag{10}$$

where A, ϕ *and* H *are the same as defined in Theorem 4, then every solution of* (1) *is oscillatory.*

Theorem 7. *Let assumption* (C_1)*–*(C_4) *and* (C_6) *be fulfilled. If there exist functions* $\Phi \in Y$, $\eta \in C^{\alpha}\left([t_0,\infty),(0,\infty)\right)$ *such that for any sufficiently large* $T \geq t_0$, *there exist* c,d *with* $T \leq c < d$ *satisfying*

$$A\left[\eta(s)H^*(s) - 2lk_2\eta(s)a(s)\phi^2; c,d\right] > 0, \tag{11}$$

where A, ϕ *and* H^* *are the same as defined in Theorem 5, then every solution of* (1) *is oscillatory*

3 Conclusion

In this study, we are concerned with the oscillatory behavior of solutions of a class of conformable fractional differential equations. By using the properties of conformable fractional derivative and the classical Riccati technique, we present some new oscillation criteria for Eq. (1). We note that the approach in establishing the main theorems above can be generalized to research oscillation of fractional differential equations with more complicated forms, which are expected to research further.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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