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Delay-independent stability criteria for Riemann Liouville's nonlinear fractional neutral systems: A descriptor system approach

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Abstract: In this study, some novel approaches related to the delay-independent stability for Riemann Liouville's nonlinear fractional neutral systems (RL-NFNSs) are presented. These approaches are based on the Lyapunov functional method and the linear matrix inequality (LMI) technique. By constructing a meaningful Lyapunov functional associated with fractional integral and derivative terms, several sufficient conditions to derive delay-independent asymptotically stability of the equilibrium point are constructed. Illustrative examples demonstrate the validity and effectiveness of the proposed theoretical results. MATLAB-Simulink is applied to illustrate the behaviors of the paths of solutions of the considered system for a special case. All these results are expected to be used in the study of nonlinear fractional neutral systems.

Keywords: Delay-independent stability, Riemann Liouville's nonlinear fractional neutral systems (RL-NFNSs), Lyapunov functional, LMI.

1 Introduction

Fractional calculus is concerned with the derivatives and integrals of optional non-integer order. Fractional calculus has piqued the interest of many researchers as an extension of integer-order differentiation and integration. To investigate the stability of fractional neutral systems is more difficult than the stability of integer-order neutral systems. Therefore, investigating the stability of fractional neutral systems, which plays an important role in both theory and applications, is necessary and interesting. Initially, fractional order derivatives as pure mathematical theory are studied only by mathematicians. In the last few decades, many researchers pointed that fractional order calculus, that is, non-integer order calculus is also applicable to many fields, such as control systems [1,2], economics [3], diffusion [4], biological systems [5,6] , HIV infection models [7], viscoelastic materials [8], neural networks [9], and so on. Fractional dynamical systems in both Caputo's and RL sense have recently gained prominence as a dominant method for depicting and modeling certain physical processes with heredity and memory characteristics [10].

Investigating the stability of non-integer order systems is more complex than the integer-order systems. When the relevant literature is reviewed, various approaches to the stability of fractional order linear and nonlinear systems have attracted attention in recent years. The authors used a variety of techniques to conduct their research, including the Lyapunov functional method, LMI, integral inequalities, perturbation techniques, model transformations, and so on [11-27]. In particular, the Lyapunov functional method presents a very powerful approach to analyzing the qualitative behaviors of fractional order systems. However, the non-integer derivative of the Lyapunov functionals is computationally quite difficult. That is the main reason why there are very few studies for stability of delayed fractional

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systems.

This research paper, which is based on the above discusses, deals with the asymptotically stability of RL-NFNSs. When compared to integer-order neutral systems with delay, it is seen that the works related to the stability of delayed fractional order, that is, non-integer order neutral systems are still in the process of benefiting. The main purpose and contribution of this study can be summarized as follows:

NFNS has a more general structure than linear fractional systems. It is not easy to calculate fractional derivatives of Lyapunov functionals constructed for these systems. To overcome the difficulty arising from fractional derivatives and time delays, we derived an appropriate Lyapunov functional that includes the terms fractional derivative and integral. The proposed method avoids calculating non-integer order derivative of Lyapunov functionals. Therefore, the method used in the study provides an advantage in terms of directly calculating integer ordered derivatives of Lyapunov functions.

The purpose of this study is to search the delay-independent asymptotically stability of RL-NFNSs with a descriptive system approach. For this, some basic inequalities, Lyapunov functional method and LMI technique were used.

Two examples are presented to demonstrate the applicability of the proposed theoretical results for the system considered in this study. MATLAB-Simulink was used to illustrate the behaviors of the solutions of the systems discussed in the examples.

Finally, we consider that the theoretical findings of this study would add to the existing literature and studies on the qualitative properties of NFNS.

2 Preliminaries

In this study, which motivated by above discussions, we consider the following RL-NFNS with delay

$$ {}_{t_0}^{RL}D_t^q x(t) = -Ax(t) + Bx(t-\delta(t)) + C\,{}_{t_0}^{RL}D_t^q x(t-\delta(t)) + Ff(x(t) + Gg(x(t-\delta(t)), \tag{1} $$

with the given initial condition

$$ {}_{t_0}^{RL}D_t^{q-1} x(t) = \vartheta(t),\ t \in [-\delta_M,\ 0], \tag{2} $$

where $x(t) \in \Re^n$ is state vector; ${}_{t_0}^{RL}D_t^q x(\cdot)$ states a q order RL derivative of $x(\cdot)$ with $q \in (0,\ 1)$; $A \in \Re^{n\times n}$ is a diagonal matrix and $B,\ C,\ F,\ G\ \in \Re^{n\times n}$ are constant matrices with $\|C\| < 1$;f and $g \in \Re^n$ represent the nonlinear terms of system (1) with respect to $x(t)$ and $x(t-\delta(t))$,respectively, which satisfy that

$$ \|f(x(t))\| \le \lambda_1 \|x(t)\|, \quad \|g(x(t-\delta(t)))\| \le \lambda_2 \|x(t-\delta(t))\|, \tag{3} $$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are given positive constants.

Constraint (3) can be rewritten as

$$ f^T(x(t))f(x(t)) \le \lambda_1^2 x^T(t)x(t), \quad g^T(x(t-\delta(t)))g(x(t-\delta(t))) \le \lambda_2^2 x^T(t-\delta(t))x(t-\delta(t)). \tag{4} $$

Moreover, the differentiable function $\delta(t)$ is a variable delay and for positive constants δ_M and δ_d,

$$0 \leq \delta(t) \leq \delta_M, \quad \dot{\delta}(t) \leq \delta_d < 1 . \tag{5}$$

To prove our main result, we need the following property, basic definition and lemma.

Definition 1. *([21]) The RL fractional integral and RL fractional derivative are defined as, respectively*

$$ {}_{t_0}D_t^{-q}x(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} x(s)ds, \ (q>0) ,$$

$$ {}_{t_0}D_t^{q}x(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_{t_0}^{t} \frac{x(s)}{(t-s)^{q+1-n}} ds, \ (n-1 \leq q < n) .$$

Proposition 1. *([21]) For $x(t) \in \Re^n$, if $p > q > 0$, then the following relation holds*

$$ {}_{t_0}D_t^{q}({}_{t_0}D_t^{-p}x(t)) = {}_{t_0}D_t^{q-p}x(t).$$

Lemma 1. *([18]) For a vector of differentiable function $x(t) \in \Re^n$ and constant matrix $W = W^T (\geq 0) \in \Re^{n\times n}$, then*

$$\frac{1}{2} {}_{t_0}D_t^{q}\left\{x^T(t)Wx(t)\right\} \leq x^T(t)W {}_{t_0}D_t^{q}\left\{x(t)\right\}, \ q \in (0,1),$$

for all $t \geq t_0$.

3 Asymptotic Stability Results

We will conduct a delay-independent stability analysis of uncertain RL-NFNS with time varying delay and nonlinear uncertainties defined by (1) in this section. We can reconstruct system (1) using the descriptor system below:

$$\begin{aligned} {}^{RL}_{t_0}D_t^{q}x(t) &= y(t) \\ y(t) &= -Ax(t) + Bx(t-\delta(t)) + C {}^{RL}_{t_0}D_t^{q}x(t-\delta(t)) + Ff(x(t) + Gg(x(t-\delta(t)) \end{aligned} \tag{6}$$

Theorem 1. *Let $\|C\| < 1$. The RL-NFNS described by (1) with (2) is asymptotically stable, if there exist real matrices K_2, K_3 and symmetric matrices $K_1 > 0, L_1 > 0, L_2 > 0$ and scalars $\varepsilon_i, \lambda_i \geq 0, \ (i = 1,2)$ such that the following LMI holds*

$$\Omega = \left(\Omega_{jk}\right) < 0, \tag{7}$$

where Ω is a 6×6 symmetric matrix with the elements $\Omega_{11} = -(K_2^T A + AK_2) + L_1 + \varepsilon_1\lambda_1^2 I, \Omega_{12} = K_1 - K_2^T - AK_3, \Omega_{13} = K_2^T B, \Omega_{14} = K_2^T C, \Omega_{15} = K_2^T F, \Omega_{16} = K_2^T G, \Omega_{22} = -(K_3 + K_3^T) + L_2, \Omega_{23} = K_3^T B, \Omega_{24} = K_3^T C, \Omega_{25} = K_3^T F, \Omega_{26} = K_3^T G, \Omega_{33} = -(1-\delta_d)L_1 + \varepsilon_2\lambda_2^2 I, \Omega_{34} = \Omega_{35} = \Omega_{36} = 0, \Omega_{44} = -(1-\delta_d)L_2, \Omega_{45} = \Omega_{46} = 0, \Omega_{55} = -\varepsilon_1 I, \Omega_{56} = 0, \Omega_{66} = -\varepsilon_2 I.$

Proof. For this theorem, we define the following descriptor type Lyapunov functional including the fractional derivative and integral terms

$$W(t) = W_1(t) + W_2(t), \tag{8}$$

where

$$W_1(t) = {}_{t_0}^{RL}D_t^{q-1}[(x^T(t)\ y^T(t))\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} K_1 & 0 \\ K_2 & K_3 \end{bmatrix}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}],$$

$$W_2(t) = \int_{t-\delta(t)}^{t} x^T(s)L_1x(s)ds + \int_{t-\delta(t)}^{t} y^T(s)L_2y(s)ds.$$

From Definition 1, we know that $W_1(t)$ and $W_2(t)$ are positive-definite functionals. Here, the functional $W(t)$ corresponds to the descriptor system and the delay-independent stability with respect to the discrete delays (see, [28]). By Property 1 and Lemma 1, computing the differential of $W(t)$ along the solutions of RL-NFNS in (1)

$$\begin{aligned} \dot{W}_1(t) =& {}_{t_0}^{RL}D_t^{q}[(x^T(t)\ y^T(t))\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} K_1 & 0 \\ K_2 & K_3 \end{bmatrix}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}] \\ \leq& 2x^T(t)K_1 {}_{t_0}^{RL}D_t^{q}(x(t)) \\ =& 2(x^T(t)\ y^T(t))\begin{bmatrix} K_1 & K_2^T \\ 0 & K_3^T \end{bmatrix}\begin{pmatrix} {}_{t_0}^{RL}D_t^{q}(x(t)) \\ 0 \end{pmatrix}. \end{aligned}$$

From descriptor system (6), we have

$$\begin{aligned} \dot{W}_1(t) \leq& 2(x^T(t)\ y^T(t))\begin{bmatrix} K_1 & K_2^T \\ 0 & K_3^T \end{bmatrix}\begin{pmatrix} y(t) \\ 0 \end{pmatrix} \\ =& 2(x^T(t)\ y^T(t))\begin{bmatrix} K_1 & K_2^T \\ 0 & K_3^T \end{bmatrix}\begin{pmatrix} y(t) \\ -y(t)-Ax(t)+Bx(t-\delta(t))+Cy(t-\delta(t)) \end{pmatrix} \\ &+2(x^T(t)\ y^T(t))\begin{bmatrix} K_1 & K_2^T \\ 0 & K_3^T \end{bmatrix}\begin{pmatrix} 0 \\ Ff(x(t))+Gg(x(t-\delta(t))) \end{pmatrix} \\ =& -x^T(t)(K_2^TA+AK_2)x(t)+2x^T(t)(K_1-K_2^T-AK_3)y(t) \\ &-y^T(t)(K_3+K_3^T)y(t)+2x^T(t)K_2^TBx(t-\delta(t)) \\ &+2y^T(t)K_3^TBx(t-\delta(t))+2x^T(t)K_2^TCy(t-\delta(t)) \\ &+2y^T(t)K_3^TCy(t-\delta(t))+2x^T(t)K_2^TFf(x(t)) \\ &+2x^T(t)K_2^TGg(x(t-\delta(t)))+2y^T(t)K_3^TFf(x(t)) \\ &+2y^T(t)K_3^TGg(x(t-\delta(t))). \end{aligned} \tag{9}$$

Computing the differential of $W_2(t)$, we obtained

$$\begin{aligned} \dot{W}_2(t) =& x^T(t)L_1x(t)-(1-\dot{\delta}(t))x^T(t-\delta(t))L_1x(t-\delta(t)) \\ &+y^T(t)L_2y(t)-(1-\dot{\delta}(t))y^T(t-\delta(t))L_2y(t-\delta(t)) \\ \leq& x^T(t)L_1x(t)-(1-\delta_d)x^T(t-\delta(t))L_1x(t-\delta(t)) \\ &+y^T(t)L_2y(t)-(1-\delta_d)y^T(t-\delta(t))L_2y(t-\delta(t)). \end{aligned} \tag{10}$$

Therefore, according to (9) and (10), we can conclude that

$$\dot{W}(t) = \dot{W}_1(t) + \dot{W}_2(t) \leq \eta^T(t)\Omega_0\eta(t),$$

where

$$\eta^T = \left[x^T(t)\ y^T(t)\ x^T(t-\delta(t))\ y^T(t-\delta(t))\ f^T(x(t))\ g^T(x(t-\delta(t)))\right],$$

and

$$\Omega_0 = \begin{bmatrix} -(K_2^TA + A^TK_2) + L_1 & K_1 - K_2^T - AK_3 & K_2^TB & K_2^TC & K_2^TF & K_2^TG \\ * & -(K_3 + K_3^T) + L_2 & K_3^TB & K_3^TC & K_3^TF & K_3^TG \\ * & * & -(1-\delta_d)L_1 & 0 & 0 & 0 \\ * & * & * & -(1-\delta_d)L_2 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}.$$

Existence of real matrices K_2, K_3 and symmetric matrices $K_1 > 0, L_1 > 0$ and $L_2 > 0$ is a necessary condition for asymptotically stability of system (1) such that

$$\eta^T(t)\Omega_0\eta(t) \leq 0, \tag{11}$$

for all $\eta(t) \neq 0$, where (11) means that it is semi negative definite whenever neither $x(t)$ nor $x(t-\delta(t))$is zero. Note that for any $\varepsilon_1, \varepsilon_2 \geq 0$,it follows from (4) that

$$\eta^T(t)\Omega_0\eta(t) + \varepsilon_1(\lambda_1^2x^T(t)x(t) - f^T(x(t))f(x(t))) + \varepsilon_2(\lambda_2^2x^T(t-\delta(t))x(t-\delta(t))$$
$$- g^T(x(t-\delta(t)))g(x(t-\delta(t)))) < 0,$$

for $\forall\eta(t) \neq 0$. Thus, if there exist real matrices K_2, K_3 and symmetric matrices $K_1 > 0, L_1 > 0, L_2 > 0$ and scalar $\varepsilon_1, \varepsilon_2 \geq 0$ such that LMI (7) is satisfied, then the fractional system (1) is asymptotically stable.

Remark 1. If the time delay $\delta(t) = \delta$ is a constant, the following corollary is easily obtained.

Corollary 1. *Let $\|C\| < 1$. The RL-NFNS defined by (1) with (2) and $\delta(t) = \delta$(constant) is asymptotically stable, if there exist real matrices K_2, K_3 and symmetric matrices $K_1 > 0, L_1 > 0, L_2 > 0$ and scalars $\varepsilon_i, \lambda_i \geq 0$, $(i = 1,2)$ such that the following LMI holds*

$$\Psi = \left(\Psi_{jk}\right) < 0,$$

where Ψ is a 6×6 symmetric matrix with the elements $\Psi_{11} = -(K_2^TA + AK_2) + L_1 + \varepsilon_1\lambda_1^2 I, \Psi_{12} = K_1 - K_2^T - AK_3, \Psi_{13} = K_2^TB, \Psi_{14} = K_2^TC, \Psi_{15} = K_2^TF, \Psi_{16} = K_2^TG, \Psi_{22} = -(K_3 + K_3^T) + L_2, \Psi_{23} = K_3^TB$, $\Psi_{24} = K_3^TC$, $\Psi_{25} = K_3^TF$, $\Psi_{26} = K_3^TG$, $\Psi_{33} = -L_1 + \varepsilon_2\lambda_2^2 I$, $\Psi_{34} = \Psi_{35} = \Psi_{36} = 0, \Psi_{44} = -L_2$, $\Psi_{45} = \Psi_{46} = 0$, $\Psi_{55} = -\varepsilon_1 I$, $\Psi_{56} = 0$, $\Psi_{66} = -\varepsilon_2 I$.

Remark 2. Consider system (1) with $C = 0$;

$${}_{t_0}^{RL}D_t^q x(t) = -Ax(t) + Bx(t-\delta(t)) + Ff(x(t) + Gg(x(t-\delta(t)), \tag{12}$$

with the given initial condition

$$ {}_{t_0}^{RL}D_t^{q-1}x(t) = \vartheta(t),\ t \in [-\delta_M,\ 0], \tag{13} $$

where $x(t) \in \Re^n$ is state vector; ${}_{t_0}D_t^q x(\cdot)$states a q order RL derivative of $x(\cdot)$ with $q \in (0,\ 1)$; $A \in \Re^{n\times n}$is a diagonal matrix and $B,\ F,\ G\ \in \Re^{n\times n}$ are constant matrices;f and $g \in \Re^n$ satisfying the conditions in (4) represent the nonlinear terms of system (12) with respect to $x(t)$ and $x(t-\delta(t))$,respectively. Here, the differentiable function $\delta(t)$ is a variable delay that satisfies the conditions in (5).

Corollary 2. *The RL nonlinear fractional system defined by (12) with (13) is asymptotically stable, if there exist real matrices K_2, K_3 and symmetric matrices $K_1 > 0, L_1 > 0, L_2 > 0$ and scalars $\varepsilon_i, \lambda_i \geq 0,\ (i = 1,2)$ such that the following LMI holds*

$$ \tilde{\Psi} = \left(\tilde{\Psi}_{jk}\right) < 0, \tag{14} $$

where Ω is a 6×6 symmetric matrix with the elements $\tilde{\Psi}_{11} = -(K_2^T A + AK_2) + L_1 + \varepsilon_1\lambda_1^2 I, \tilde{\Psi}_{12} = K_1 - K_2^T - AK_3,\ \tilde{\Psi}_{13} = K_2^T B,\ \tilde{\Psi}_{14} = 0,\ \tilde{\Psi}_{15} = K_2^T F,\ \tilde{\Psi}_{16} = K_2^T G,\ \tilde{\Psi}_{22} = -(K_3 + K_3^T) + L_2, \tilde{\Psi}_{23} = K_3^T B,\ \tilde{\Psi}_{24} = 0,\ \tilde{\Psi}_{25} = K_3^T F,\ \tilde{\Psi}_{26} = K_3^T G,\ \tilde{\Psi}_{33} = -(1-\delta_d)L_1 + \varepsilon_2\lambda_2^2 I,\ \tilde{\Psi}_{34} = \tilde{\Psi}_{35} = \tilde{\Psi}_{36} = 0, \tilde{\Psi}_{44} = -(1-\delta_d)L_2,\ \tilde{\Psi}_{45} = \tilde{\Psi}_{46} = 0,\ \tilde{\Psi}_{55} = -\varepsilon_1 I,\ \tilde{\Psi}_{56} = 0,\ \tilde{\Psi}_{66} = -\varepsilon_2 I.$

4 Illustrative Examples with Numeric Simulations

To show the usefulness of the employed method, we present the following examples with simulation results.

Example 1. For $n = 2$, as a particular state of (1), we consider the following RL-NFNS

$$ {}_{t_0}^{RL}D_t^q x(t) = -Ax(t) + Bx(t-\delta(t)) + C\,{}_{t_0}^{RL}D_t^q x(t-\delta(t)) + Ff(x(t) + Gg(x(t-\delta(t)) \tag{15} $$

where $0 < q \leq 1,\ \delta(t) = 0.2$,

$$ A = \begin{bmatrix} 6.92 & 0 \\ 0 & 6.92 \end{bmatrix},\ B = \begin{bmatrix} 0.6 & -1.2 \\ -1.1 & 1.3 \end{bmatrix},\ C = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix},\ F = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.06 \end{bmatrix},\ G = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}. $$

Let us choose $\varepsilon_1 = 1.2$,$\varepsilon_2 = 1.4$,$\lambda_1 = 0.3$,$\lambda_2 = 0.4$,

$$ K_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 1.6 \end{bmatrix},\ K_2 = \begin{bmatrix} 1.8 & 0 \\ 0 & 1.8 \end{bmatrix},\ K_3 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}, $$
$$ L_1 = \begin{bmatrix} 4 & 0.1 \\ 0.1 & 2 \end{bmatrix} \text{ and } L_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}. $$

By MATLAB-Simulink, under the above assumptions, we can easily obtain that all the eigenvalues in LMI defined in (7) are $\lambda_{\max}(\Omega) \leq -0.0076$. Thus, according to Theorem 1, this shows that the origin of system (15) is asymptotically stable.

Example 2. For $n = 2$, as a particular state of (1) with $C = 0$, we consider the following RL nonlinear fractional system

$$ {}_{t_0}^{RL}D_t^q x(t) = -Ax(t) + Bx(t-\delta(t)) + Ff(x(t) + Gg(x(t-\delta(t)), \tag{16} $$

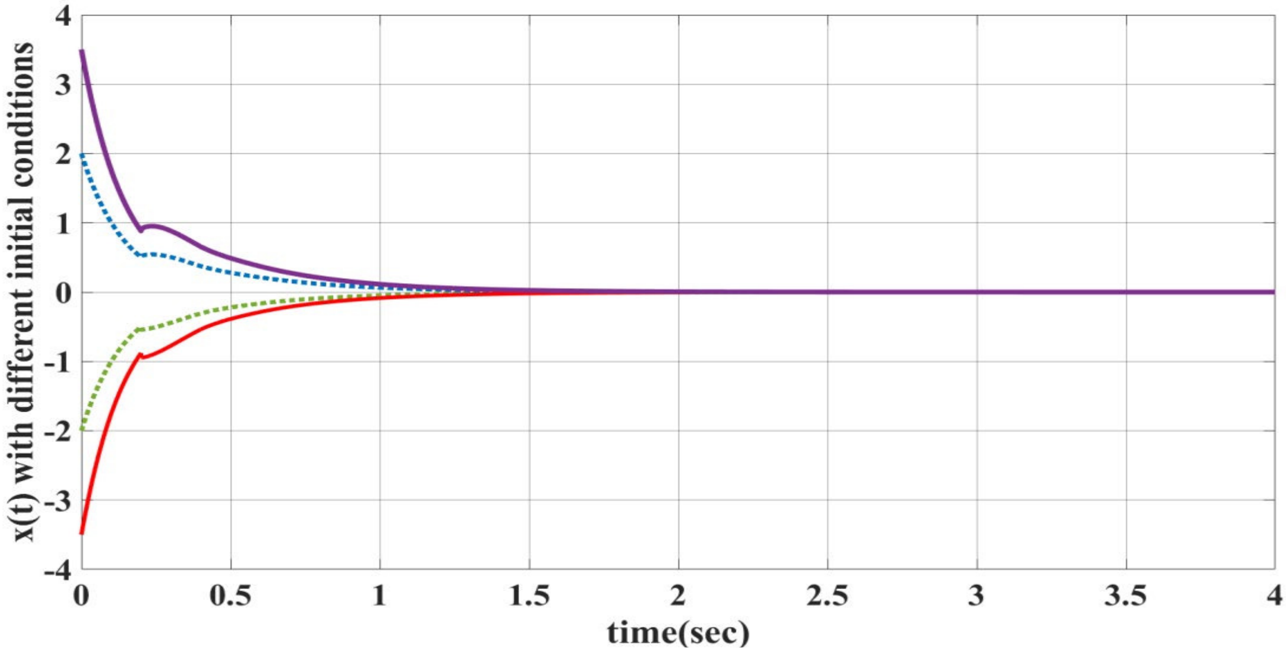

Fig. 1: The behavior of orbits of RL-NFNS (15) for $\delta(t) = 0.2$.

where $0 < q \leq 1,\ \delta(t) = 0.3$,

$$A = \begin{bmatrix} 5.2 & 0 \\ 0 & 5.2 \end{bmatrix},\ B = \begin{bmatrix} 0.5 & -0.2 \\ -0.1 & 1.2 \end{bmatrix},\ F = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix},\ G = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Let us choose $\varepsilon_1 = 2.6$, $\varepsilon_2 = 0.6$, $\lambda_1 = 0.5$, $\lambda_2 = 0.7$,

$$K_1 = \begin{bmatrix} 2.7 & 0 \\ 0 & 2.7 \end{bmatrix},\ K_2 = \begin{bmatrix} 6.4 & 0 \\ 0 & 6.4 \end{bmatrix},\ K_3 = \begin{bmatrix} 1.12 & 0 \\ 0 & 1.12 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 8 & 0.2 \\ 0.2 & 6 \end{bmatrix} \text{ and } L_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

By MATLAB-Simulink, under the above assumptions, we can easily obtain that all the eigenvalues in LMI defined in (14) are $\lambda_{\max}(\tilde{\Psi}) \leq -0.14$. Thus, according to Corollary 1, this shows that the origin of system (16) is asymptotically stable.

5 Conclusion

In this paper, some new sufficient conditions concerning the delay-independent asymptotically stability of RL-NFNS are derived. These stability criteria have been derived by constructing an appropriate Lyapunov functional and expressing in terms of LMI to derive delay-independent asymptotically stability of the equilibrium point. The employed method avoids computing fractional-order derivative of Lyapunov functionals. The proposed method is suitable for general RL-NFNS. Also, two examples are given to illustrate the usefulness of the theoretical results of addressed RL-NFNS. Moreover, the theoretical results of this paper are supported by the numerical simulations in Figure 1 and Figure2. Finally, the presented results provide contribution to the control and design of RL-NFNSs.

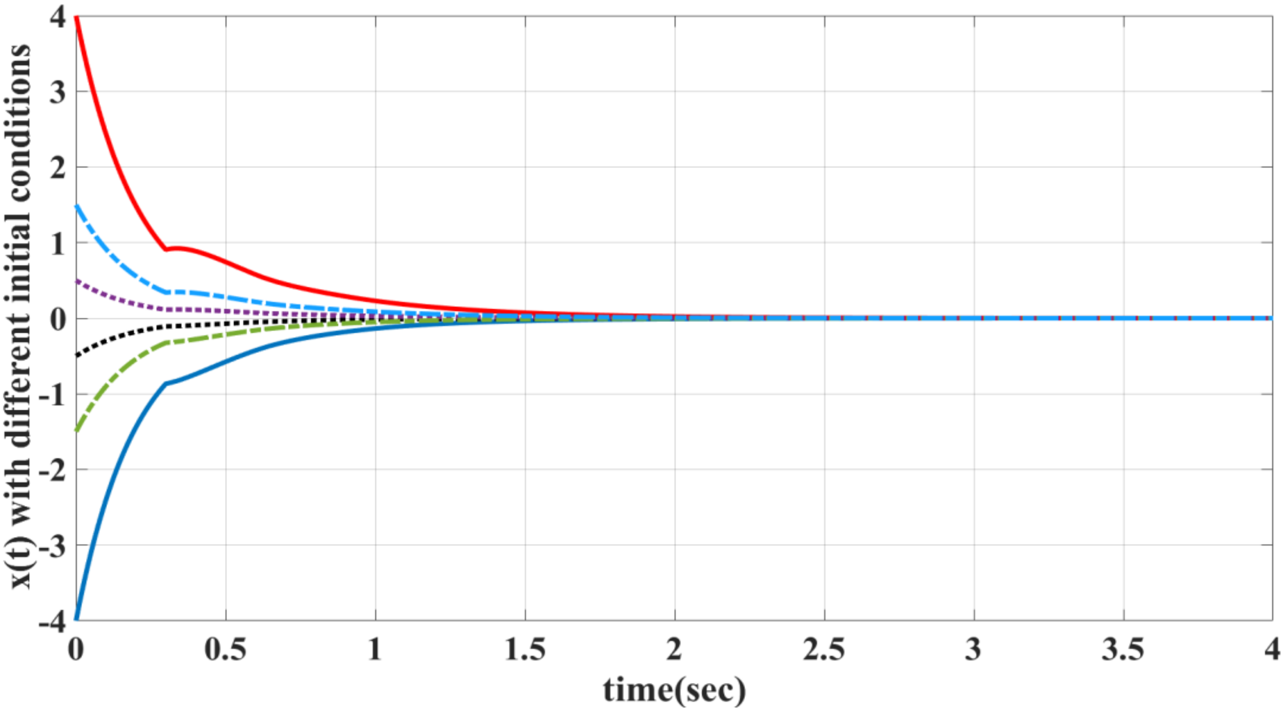

Fig. 2: The behavior of orbits of RL-NFNS (16) for $\delta(t) = 0.3$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

References

[1] Sabatier, J., Agrawal, O.P. and Machado, J.A.T. Advances in fractional calculus. Springer, Dordrecht, 2007.

[2] Baleanu, D., Machado, J.A.T. and Luo, A.C.J. Fractional dynamics and control. Springer, Berlin, 2012.

[3] Rostek, S. and Schobel, R . A note on the use of fractional Brownian motion for financial modeling. Econ Model., 30:30-35, 2013.

[4] Metzler, R. and Klafter, J. The random walk?s guide to anomalous diffusion: a fractional Dynamics approach. Phys. Rep., 339, 1-77, 2000.

[5] Magin, R. Fractional calculus models of complex dynamics in biological tissues. Comput. Math. Appl., 59, 1586-1593, 2010.

[6] Xu, C. and Yu, Y. Stability analysis of time delayed fractional order preda- tor-prey system with Crowley-Martin functional response. J. Appl. Anal. Comput. 9:928-942, 2019.

[7] Dasbasi, B. Stability analysis of the HIV model through incommensurate frac- tional-order nonlinear system. Chaos Solitons and Fractals, 137,1-12, 2020

[8] Shimizu, N. and Zhang, W. Fractional calculus approach to dynamic problems of viscoelastic materials. JSME Int. J. Ser. C.,42, 825-837, 1999.

[9] Jia, J., Huang, X., Li, Y., Cao, J. and Ahmed, A. Global stabilization of fractional-order memristor-based neural networks with time delay. IEEE Trans. Neural Netw. Learn Syst., 31,997-1009, 2020.

[10] Qian, D. Li, C. Agarwal, R.P. and Wong P.J.Y. Stability analysis of fractional differential system with Riemann?Liouville derivative. Math. Comput. Model., 52, 862-874, 2010.

[11] Altun Y. Further results on the asymptotic stability of Riemann?Liouville fractional neutral systems with variable delays. Adv. Difference Equ., 437, 1-13, 2019.

[12] Altun Y. Stability conditions for fractional differential and neutral systems with discrete and distributed constant delay components. Journal— MESA, 12, 219-230, 2021.

[13] Arslan, D. A novel hybrid method for singularly perturbed delay differential equations. Gazi University Journal of Science, 32,217-223, 2019.

[14] Cakir, M. and Arslan, D. The Adomian decomposition method and the differential transform method for numerical solution of multi-pantograph delay differential equations. Applied Mathematics, 6, 1332-1343, 2015.

[15] Chen, L. He, Y. and Chai, Y. Wu, R. New results on stability stabilization of a class of nonlinear fractional-order systems. Nonlinear Dynam., 75(4), 633-641, 2014.

[16] Celik E. and Karabacak, M. The numerical solution of fractional differential algebraic equations. New Trends in Mathematical Sciences, 1, 1-6, 2013.

[17] Liu, S. Zhou, X.F. Li, X. and Jiang, W. Stability of fractional nonlinear singular systems its applications in synchronization of complex dynamical networks. Nonlinear Dynam., 84, 2377-2385, 2016.

[18] Liu S., Zhou X.F., Li X. and Jiang W. Asymptotical stability of Riemann?Liouville fractional singular systems with multiple time-varying delays. Appl. Math. Lett., 65, 32-39, 2017.

[19] Lu, J.G. and Chen, G. Robust stability and stabilization of fractional-order interval systems: An LMI approach. IEEE Trans. Automat. Control, 54(6), 1294-1299, 2009.

[20] Lu, Y.F. Wu, R.C. and Qin, Z.Q. Asymptotic stability of nonlinear fractional neutral singular systems. J. Appl. Math. Comput., 45, 351-364, 2014.

[21] Podlubny I. Fractional Differential Equations. Academic Press., New York, USA, 1999.

[22] Sabatier, J. Moze, M. and Farges, C. LMI stability conditions for fractional order systems. Comput. Math. Appl., 59(5), 1594-1609, 2010.

[23] Sefidgar, E., Celik E. and Shiri, B. Numerical solution of fractional differential equation in a model of HIV infection of CD4 (+) T Cells. International Journal of Applied Mathematics and Statistics, 56, 23-32, 2017.

[24] Wang, J. Lv, L. and Zhou, Y. New concepts and results in stability of fractional differential equations. Commun. Nonlinear Sci., 17(6), 2530-2538, 2012.

[25] Zhou, X. F. Hu, L.G. Liu, S. and Jiang, W. Stability criterion for a class of nonlinear fractional differential systems. Appl. Math. Lett., 28, 25-29, 2014.

[26] Zhang, H. Ye, R. Cao, J. and Alsaedi, A. Delay-independent stability of Riemann-Liouville fractional neutral-type delayed neural networks. Neural Proces. Lett., 47, 427-442, 2018.

[27] Zhang, H. Ye, R. Liu, S. Cao, J. Alsaedi, A. and Li, X. LMI-based approach to stability analysis for fractional-order neural networks with discrete and distributed delays. Int. J. Syst. Sci., 49, 537-545, 2018.

[28] Fridman E. New Lyapunov?Krasovskii functionals for stability of linear retarded and neutral type systems. Syst. Control Lett., 43, 309-319, 2001.