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Stabilization and hybrid synchronization via the adaptive control method of non-identical 4-D financial hyper-chaotic systems

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Abstract: In this work, we have considered the stabilization and the hybrid synchronization by the adaptive control method for two non-identical 4-D financial hyper-chaotic systems with unknown parameters, based on Lyapunov's stability theory and the theory of adaptive control. We derive new control results via adaptive control method based on Lyapunov stability theory and adaptive control theory for globally synchronizing two non-identical 4-D financial hyper-chaotic systems with unknown system parameters.The results are validated by numerical simulation using Matlab.

Keywords: 4-D financial hyper-chaotic system, Lyapunov exponent, stabilization, synchronization.

1 Description and stabilization of a new hyper-chatic system

We consider the financial hyperchaotic system described by the following differential equation model:

$$\begin{cases} \frac{dx_1}{dt} = & -a(x_2+x_1)+x_4 \\ \frac{dx_2}{dt} = -x_2 - ax_1x_3 \\ \frac{dx_3}{dt} = b + ax_1x_2 \\ \frac{dx_4}{dt} = -dx_4 - cx_1x_3 \end{cases} \tag{1}$$

such that x_1,x_2,x_3,x_4 are the state variables and a,b,c,d are the system parameters respectively. For $a=3$; $b=15$; $c=0,2$ and $d=0,12$, the Lyapunov exponents of the system (1) are:

$$L_1 = 0.7083,\ L_2 = 0.032247\ , L_3 = 0\ ,\ L_4 = -4.7139 \tag{2}$$

So, the system (1) has two positive Lyapunov exponents L_1et L_2, which shows that the system is hyperchaotic and the Kaplan-york dimension for this hyperchaotic system is calculated as:

$$D_{KY} = 3 + \frac{L_1+L_2+L_3}{|L_4|} = 3.15709 \tag{3}$$

1.1 Stabiliziting hyperchaotic system (1) via adaptive control:

In this section, we will build an adaptive controller to globally stabilize the new hyperchaotic system (1), with undefined parameters.

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The controlled hyperchaotic system of system (1) is:

$$\begin{cases} \frac{dx_1}{dt} = & -a(x_2+x_1)+x_4+u_1 \\ \frac{dx_2}{dt} = & -x_2-ax_1x_3+u_2 \\ \frac{dx_3}{dt} = & b+ax_1x_2+u_3 \\ \frac{dx_4}{dt} = & -dx_4-cx_1x_3\ +u_4 \end{cases} \tag{4}$$

tel que x_1,x_2,x_3,x_4 the state variables, a,b,c,d are constants and unknown parameters and u_1,u_2,u_3,u_4 are adaptive controls to be designed using estimates $\hat{a}(t),\hat{b}(t),\hat{c}(t)$ et $\hat{d}(t)$ which are unknown parameters.

Consider the adaptive controller defined as follows:

$$\begin{cases} u_1(t) = \hat{a}(t)(x_2+x_1)-x_4-k_1x_1 \\ u_2(t) = x_2+\hat{a}(t)x_1x_3-k_2x_2 \\ u_3(t) = -\hat{b}(t)-\hat{a}(t)x_1x_2-k_3x_3 \\ u_4(t) = \hat{d}(t)x_4+\hat{c}(t)x_1x_3\ -k_4x_4 \end{cases} \tag{5}$$

where k_1,k_2,k_3,k_4, are positive constants and $\hat{a}(t),\hat{b}(t),\hat{c}(t),\hat{d}(t)$ are the undefined parameters a,b,c,d respectively.

Introducing (5) into (4), we get:

$$\begin{cases} \frac{dx_1}{dt} = & -(a-\hat{a}(t))(x_2+x_1)-k_1x_1 \\ \frac{dx_2}{dt} = & -(a-\hat{a}(t))x_1x_3-k_2x_2 \\ \frac{dx_3}{dt} = & b-\hat{b}(t)+(a-\hat{a}(t))x_1x_2\ -k_3x_3 \\ \frac{dx_4}{dt} = & -\left(d-\hat{d}(t)\right)x_4-(c-\hat{c}(t))x_1x_3\ -k_4x_4 \end{cases} \tag{6}$$

Now, let's define the error estimation parameters as:

$$\begin{cases} e_a(t) = a-\hat{a}(t) \\ e_b(t) = b-\hat{b}(t) \\ e_c(t) = c-\hat{c}(t) \\ e_d(t) = d-\hat{d}(t) \end{cases} \tag{7}$$

using (7), then introducing in (6), and from the simplification we obtain the following equations:

$$\begin{cases} \frac{dx_1}{dt} = & -e_a(t)(x_2+x_1)-k_1x_1 \\ \frac{dx_2}{dt} = & -e_a(t)x_1x_3-k_2x_2 \\ \frac{dx_3}{dt} = & e_b(t)+e_a(t)x_1x_2\ -k_3x_3 \\ \frac{dx_4}{dt} = & -e_d(t)x_4-e_c(t)x_1x_3\ -k_4x_4 \end{cases} \tag{8}$$

Deffirentiating (7) with respect to t and we obtain:

$$\begin{cases} \frac{de_a(t)}{dt} = -\frac{d\hat{a}(t)}{dt} \\ \frac{de_b(t)}{dt} = -\frac{d\hat{b}(t)}{dt} \\ \frac{de_c(t)}{dt} = -\frac{d\hat{c}(t)}{dt} \\ \frac{de_d(t)}{dt} = -\frac{d\hat{d}(t)}{dt} \end{cases} \tag{9}$$

We use Lyapunov's stability theory to find a law for updating the parameters of the estimates.

Consider a quadratic Lyapunov function given by:

$$V = 1/2\left(x_1^2+x_2^2+x_3^2+x_4^2+e_a^2+e_b^2+e_c^2+e_d^2\right). \tag{10}$$

Deffirentiating V along the trajectories of systems (8) and (9) and introducing into $\dot{V}$, we obtain:

$$\dot{V} = -k_1x_1^2-k_2x_2^2-k_3x_3^2-k_4x_4^2+e_a\left[-(x_2+x_1)x_1-\frac{d\hat{a}(t)}{dt}\right]+e_b\left[x_3-\frac{d\hat{b}(t)}{dt}\right]$$
$$+e_c\left[-x_1x_3x_4-\frac{d\hat{c}(t)}{dt}\right]+e_d\left[-x_4^2-\frac{d\hat{d}(t)}{dt}\right] \tag{11}$$

Then from (11), we take:

$$\begin{cases} \frac{d\hat{a}(t)}{dt} = -(x_2+x_1)x_1 \\ \frac{d\hat{b}(t)}{dt} = x_3 \\ \frac{d\hat{c}(t)}{dt} = -x_1x_3x_4 \\ \frac{d\hat{b}(t)}{dt} = -x_4^2 \end{cases} \tag{12}$$

then;

$$\dot{V} = \sum_{i=1}^{4} -k_ix_i^2 \tag{13}$$

The hyperchaotic system (1) is globally stabilized by the adaptive control law (5) and the updating law (12), such that k_1, k_2, k_3, k_4, are positive constants.

2 Hybrid synchronization of non-identical hyper-chaotic systems

For this synchronization, as a master system, we choose the financial hyper-chaotic system

$$\begin{cases} \dot{x_1} = x_3+(x_2-a)x_1+x_4 \\ \dot{x_2} = 1-bx_2-x_1^2 \\ \dot{x_3} = -x_1-cx_3 \\ \dot{x_4} = -dx_1x_2-kx_4 \end{cases} \tag{14}$$

and the slave system we used is the controlled hyperchaotic system of system (1) given by:

$$\begin{cases} \frac{dy_1}{dt} = -\alpha(y_2+y_1)+y_4+u_1 \\ \frac{dy_2}{dt} = -y_2-\alpha y_1y_3+u_2 \\ \frac{dy_3}{dt} = \beta+\alpha y_1y_2+u_3 \\ \frac{dy_4}{dt} = -\delta y_4-\gamma y_1y_3+u_4 \end{cases} \tag{15}$$

with $\alpha, \beta, \gamma, \delta$ are the parameters of the system. The error of hybrid synchronization between the two hyper-chaotic systems (14) and (15) is:

$$\begin{cases} e_1 = y_1-x_1 \\ e_2 = y_2+x_2 \\ e_3 = y_3-x_3 \\ e_4 = y_4+x_4 \end{cases} \tag{16}$$

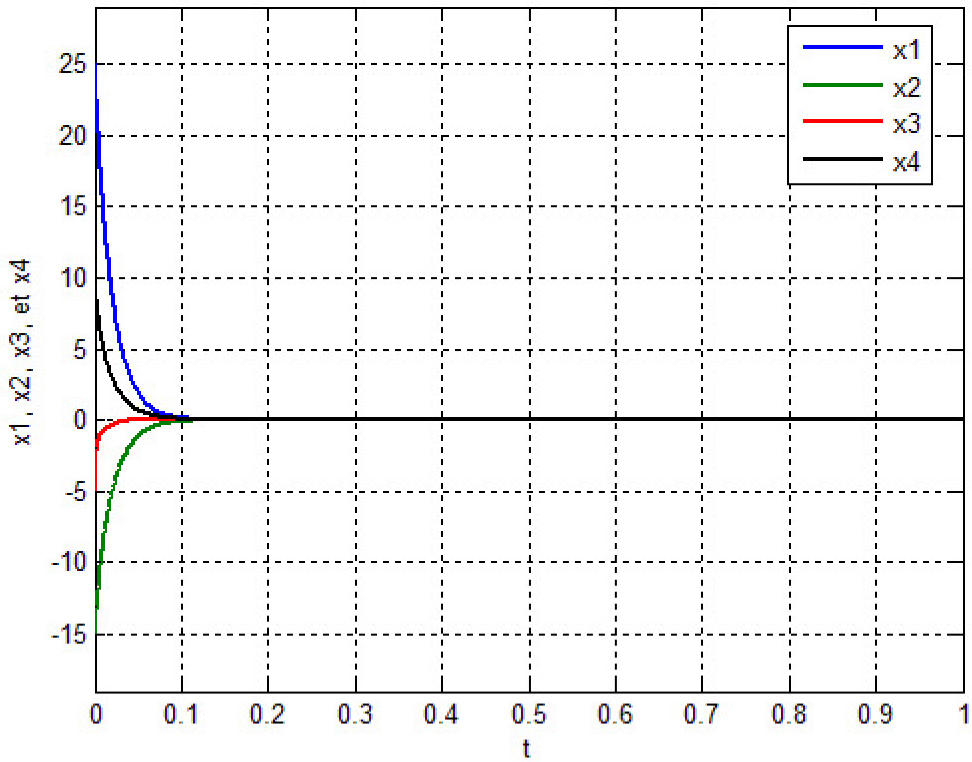

Fig. 1: History of the controlled states x_1, x_2, x_3, x_4 of the system (1).

then

$$\begin{cases} \dot{e}_1 = \dot{y}_1 - \dot{x}_1 \\ \dot{e}_2 = \dot{y}_2 + \dot{x}_2 \\ \dot{e}_3 = \dot{y}_3 - \dot{x}_3 \\ \dot{e}_4 = \dot{y}_4 + \dot{x}_4 \end{cases} \tag{17}$$

Then the dynamic error of hybrid synchronization is:

$$\begin{cases} \dot{e}_1 = -\alpha(y_2 + y_1) + y_4 - (x_3 + (x_2 - a)x_1 + x_4\ ) + u_1 \\ \dot{e}_2 = -y_2 - \alpha y_1 y_3 + \left(1 - bx_2 - x_1^2\right) + u_2 \\ \dot{e}_3 = \beta + \alpha y_1 y_2 - (-x_1 - cx_3) + u_3 \\ \dot{e}_4 = -\delta y_4 - \gamma y_1 y_3 + (-dx_1 x_2 - kx_4 \quad) + u_4. \end{cases} \tag{18}$$

Consider the law of adaptive control:

$$\begin{cases} u_1(t) = \hat{\alpha}(t)(y_2 + y_1) - y_4 + (x_3 + (x_2 - \hat{a}(t))x_1 + x_4\ ) - k_1 e_1 \\ u_2(t) = y_2 + \hat{\alpha}(t) y_1 y_3 - \left(1 - \hat{b}(t)x_2 - x_1^2\right) - k_2 e_2 \\ u_3(t) = -\hat{\beta}(t) - \hat{\alpha}(t) y_1 y_2 + (-x_1 - \hat{c}(t)x_3) - k_3 e_3 \\ u_4(t) = \hat{\delta}(t) y_4 + \hat{\gamma}(t) y_1 y_3 - \left(-\hat{d}(t)x_1 x_2 - \hat{k}(t)x_4 \quad\right) - k_4 e_4 \quad . \end{cases} \tag{19}$$

Putting (19) in (18), the system (18) becomes:

$$\begin{cases} \dot{e}_1 = -(\alpha-\hat{\alpha})(y_2+y_1)+(a-\hat{a}(t))x_1-k_1e_1 \\ \dot{e}_2 = -(\alpha-\hat{\alpha}(t))y_1y_3-\left(b-\hat{b}(t)\right)x_2-k_2e_2 \\ \dot{e}_3 = \left(\beta-\hat{\beta}(t)\right)+(\alpha-\hat{\alpha}(t))y_1y_2+(c-\hat{c}(t))x_3-k_3e_3 \\ \dot{e}_4 = -\left(\delta-\hat{\delta}(t)\right)y_4-(\gamma-\hat{\gamma}(t))y_1y_3-\left(d-\hat{d}(t)\right)x_1x_2-\left(k-\hat{k}\right)x_4\ -k_4e_4 \end{cases} \tag{20}$$

Then the parameters of the error estimates are:

$$\begin{cases} e_\alpha=(\alpha-\hat{\alpha});e_\delta=(\delta-\hat{\delta});e_\beta=\left(\beta-\hat{\beta}\right);e_\theta=(\gamma-\hat{\gamma}). \\ e_a=(a-\hat{a});e_b=(b-\hat{b});e_c=(c-\hat{c});e_d=(d-\hat{d});e_k=\left(k-\hat{k}\right). \end{cases} \tag{21}$$

it implies that:

$$\begin{cases} \frac{de_\alpha}{dt}=-\frac{d\hat{\alpha}}{dt};\frac{de_\delta}{dt}=-\frac{d\hat{\delta}}{dt};\frac{de_\beta}{dt}=-\frac{d\hat{\beta}}{dt};\frac{de_\gamma}{dt}=-\frac{d\hat{\gamma}}{dt}. \\ \frac{de_a}{dt}=-\frac{d\hat{a}}{dt};\frac{deb}{dt}=-\frac{d\hat{b}}{dt};\frac{de_c}{dt}=-\frac{d\hat{c}}{dt};\frac{de_d}{dt}=-\frac{d\hat{d}}{dt};\frac{de_k}{dt}=-\frac{d\hat{k}}{dt}. \end{cases} \tag{22}$$

Replacing (21) in (20), so the dynamic error becomes:

$$\begin{cases} \dot{e}_1=-e_\alpha(y_2+y_1)+e_ax_1-k_1e_1 \\ \dot{e}_2=-e_\alpha y_1y_3-e_bx_2-k_2e_2 \\ \dot{e}_3=e_\beta+e_\alpha y_1y_2+e_cx_3-k_3e_3 \\ \dot{e}_4=-e_\delta y_4-e_\gamma y_1y_3-e_dx_1x_2-e_kx_4\ -k_4e_4 \end{cases} \tag{23}$$

Consider the Lyapunov function:

$$V=\frac{\left(e_1^2+e_2^2+e_3^2+e_4^2+e_a^2+e_b^2+e_c^2+e_d^2+e_\alpha^2+e_\beta^2+e_\delta^2+e_\gamma^2+e_k^2\right)}{2}. \tag{24}$$

V is a positive definite function in $\mathbb{R}^{13}$. Differentiating the function of Lyapunov V and obtaining the following result:

$$\begin{cases} \dot{V}=-k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2+e_a[x_1e_1-\frac{d\hat{a}}{dt}] \\ +e_b[-x_2e_2-\frac{d\hat{b}}{dt}]+e_c[x_3e_3-\frac{d\hat{c}}{dt}]+e_d[-x_1x_2e_4-\frac{d\hat{d}}{dt}]+e_\alpha[-(y_2+y_1)e_1-y_1y_3e_2+y_1y_2e_3-\frac{d\hat{\alpha}}{dt}] \\ +e_\beta\left[e_3-\frac{d\hat{\beta}}{dt}\right]+e_\delta[-y_4e_4-\frac{d\hat{\delta}}{dt}]+e_\gamma[-y_1y_3e_4-\frac{d\hat{\gamma}}{dt}]+e_k[-x_4e_4-\frac{d\hat{k}}{dt}]. \end{cases} \tag{25}$$

From (25), we take the parameters of the updating law as follows:

$$\begin{cases} \frac{d\hat{a}}{dt}=x_1e_1 \\ \frac{d\hat{b}}{dt}=-x_2e_2 \\ \frac{d\hat{c}}{dt}=x_3e_3 \\ \frac{d\hat{d}}{dt}=-x_1x_2e_4 \\ \frac{d\hat{\alpha}}{dt}=-(y_2+y_1)e_1-y_1y_3e_2+y_1y_2e_3 \\ \frac{d\hat{\beta}}{dt}=e_3 \\ \frac{d\hat{\delta}}{dt}=-y_4e_4 \\ \frac{d\hat{\gamma}}{dt}=-y_1y_3e_4 \\ \frac{d\hat{k}}{dt}=-x_4e_4 \end{cases} \tag{26}$$

Replacing (26) in (25), the function $\dot{V}$ become:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2. \tag{27}$$

$\dot{V}$ is a negative definite function in $\mathbb{R}^4$. Therefore, we have proved the following result.

Theorem 1. *The non-identical hyperchaotic systems (14), (15) with unknown parameters are globally synchronized by the adaptive control law (19), and the updating law for the estimation parameters (26).*

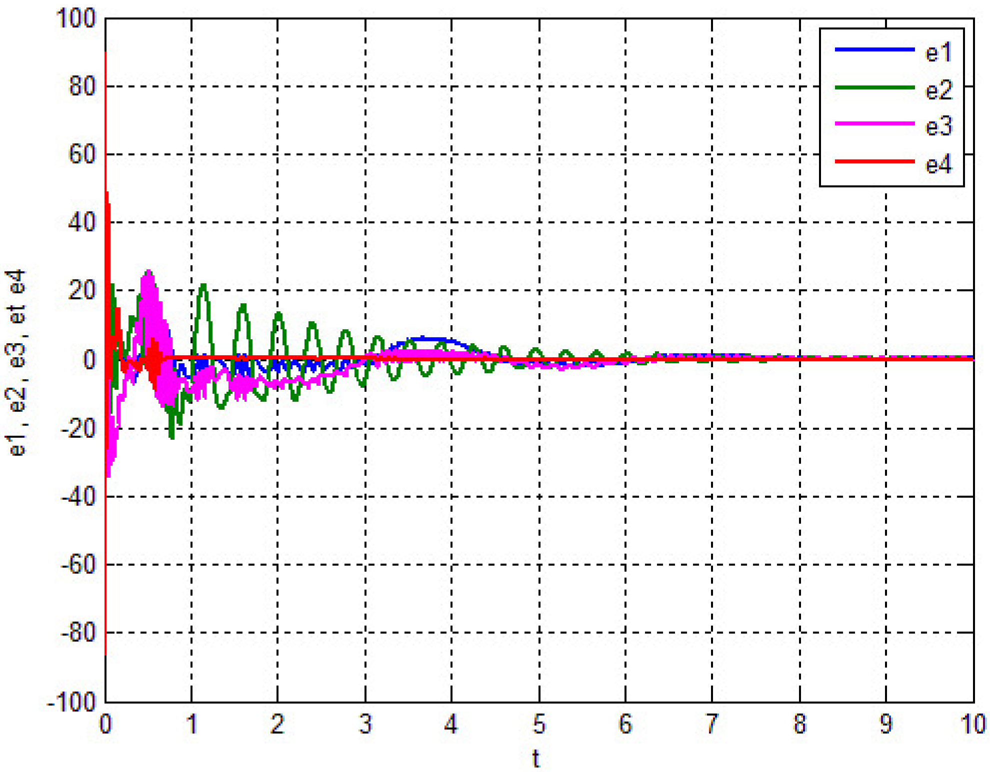

Fig. 2: Temporal history of the hybrid synchronization error of non identical systems (14) and (15).

3 Conclusion

In this paper, we have considered the stabilization and the hybrid synchronization for some non-identical financial hyperchaotic systems with continuous time according to the adaptive control method with unknown parameters. Numerical simulations are used to validate the obtained results via the approach considered for this kind of synchronization.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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