

# Comparative study of different-fault-tolerant control strategies for three-phase induction motor

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**Abstract:** In this paper, we have studied a different fault tolerant control (FTC) strategies for a three-phase induction motor (3p-IM). Further we introduce Backstepping controller (BC) and Input-output linearization controller (IOLC). To provide a direct comparison between these FTCs approaches, the performances are evaluated using the control of 3p-IM under failures, variable speed, and variable parameters. A comparison between the two control strategies is proposed to prove the most robust one. The simulation results show the robustness and good performance of the fault tolerant control with Input-output linearization controller compared to one with Backstepping controller. The FTC with IOLC is more stable and robust against failures, load torque perturbation and speed reversion.

**Keywords:** Three-phase induction motor (3p-IM), Fault Tolerant Control (FTC), Input-output linearization controller, Backstepping controller.

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## 1 Introduction

With the development of power electronics technology and control technology, three-phase induction motor has been widely applied in various industrial sectors and airline industry [1]. Due to its compact size, cost and reliability. 3p-IMs are subjected to various faults, such as stator short circuits, broken bars or rings, eccentricity, sensor and actuator faults,...etc. As it is the case in ([2,3]).

In recent years, fault-tolerant control (FTC) has begun to concern a wider range of industrial applications such as aerospace, automotive, nuclear power, manufacturing, etc. [4-6]. The passive FTC approach considers fault as a special kind of uncertainties, and consequently controllers are fixed and designed to be robust against a class of presumed faults. The remainder of this paper is organized as follows. Section 2 describes the mathematical faulty model of induction motor. In Section 3 and Section 4 the objective and design philosophy passive based FTCs strategies are described. Simulation results describe similarities and differences between them are summarized in Section 5. Finally, the conclusions are drawn in Section 6.

## 2 Mathematic faulty model of IM and problem formulation

The dynamic model of an induction motor in the stator reference frame can be described as:

$$\dot{x} = Ax + Bu \tag{1}$$

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where  $B$  is a constant matrix and  $A$  is a nonlinear matrix.

$$\begin{cases} x = [x_1 x_2 x_3 x_4]^T = [i_{s\alpha} i_{s\beta} \Phi_{r\alpha} \Phi_{r\beta}]^T \\ u = [u_1 u_2]^T = [v_{s\alpha} v_{s\beta}]^T \end{cases} \quad (2)$$

The objective of this section is to provide a robust PFTC fault design for the induction motor system affected by sensor faults and unknown bounded disturbances.

Let us consider the IM model given by the equations (1) and affected by sensor faults as [7]:

$$\dot{x} = Ax + Bu + D_f \mathcal{F} \quad (3)$$

where  $\mathcal{F} \in \mathcal{R}$  is the sensor fault vector. Matrix  $D_f$  is of appropriate dimension.

$$\begin{cases} D_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T \\ \mathcal{F} = [\mathcal{F}_1 \quad \mathcal{F}_2]^T \end{cases} \quad (4)$$

The presence of electrical and/or mechanical faults generates asymmetry of the IM yielding some slot harmonics in the stator winding:

$$\begin{cases} x_1 \rightarrow x_1 + \mathcal{F}_{h1} \\ x_2 \rightarrow x_2 + \mathcal{F}_{h2} \end{cases} \quad (5)$$

where:

$$\begin{cases} \mathcal{F}_{h1} = \sum_i^{n_f} A_i \sin(\omega_i t + \varphi_i) \\ \mathcal{F}_{h2} = \sum_i^{n_f} A_i \cos(\omega_i t + \varphi_i) \\ \omega_i = 2\pi f_i + 2\pi f_a = 2\pi(f_i + f_a) \end{cases} \quad (6)$$

with:  $n_f$  is faults number,  $f_i$  is the characteristic frequency of the fault and  $f_a$  is the fundamental frequency.

### 3 Passive FTC based backstepping controller

Backstepping control is an efficient method for nonlinear system [8]. This approach is presented in the form of steps for the determination of the control law given by the stator voltages of the IM [9-11]. The backstepping technique has been widely used in the design of speed controllers for induction motors [12-14].

#### 3.1 Speed controller design methodology

The controller for the speed state can be designed in three steps.

**Step 1.** To solve speed tracking problem, the state tracking error variable can be defined as:

$$\begin{cases} e_1 = \omega_{ref} - \omega_r \\ e_2 = x_{ref} - x_r \end{cases} \quad (7)$$

where  $\omega_{ref}$  and  $x_{ref}$  are respectively rotor speed and rotor flux reference,  $\omega_r$  is the actual rotor speed,  $x_r$  is the actual rotor flux. For stabilizing the speed component, the derivative of (7) is given by:

$$\begin{cases} \dot{e}_1 = \dot{\omega}_{ref} - \frac{\rho}{J} (x_2x_3 - x_1x_4) + \frac{T_L}{J} \\ \dot{e}_2 = \dot{x}_{ref} + 2T_r x_r - 2MT_r(x_1x_3 + x_2x_4) \end{cases} \quad (8)$$

**Step 2.** Choose the following candidate Lyapunov function as:

$$V_1 = \frac{1}{2}(e_1^2 + e_2^2) \quad (9)$$

**Step 3.** The time derivative of Lyapunov function (9) can be obtained as:

$$\begin{aligned} \dot{V}_1 &= (e_1\dot{e}_1 + e_2\dot{e}_2) \\ &= e_1 \left( \dot{\omega}_{ref} - \frac{\rho}{J} (x_2x_3 - x_1x_4) + \frac{T_L}{J} \right) + e_2 (\dot{x}_{ref} + 2T_r x_r - 2T_r M(x_1x_3 + x_2x_4)) \end{aligned} \quad (10)$$

Thus, the tracking objectives will be satisfied if we choose:

$$\begin{cases} x_{1ref} = -\frac{J}{\rho x_4} \left( k_1 e_1 + \dot{\omega}_{ref} + \frac{T_L}{J} \right) + \frac{1}{2T_r M x_3} (k_2 e_2 + \dot{x}_r + 2T_r x_r) \\ x_{2ref} = \frac{J}{\rho x_3} \left( k_1 e_1 + \dot{\omega}_{ref} + \frac{T_L}{J} \right) + \frac{1}{2T_r M x_4} (k_2 e_2 + \dot{x}_r + 2T_r x_r) \end{cases} \quad (11)$$

Therefore, (10) can be rewritten as:

$$\dot{V}_1 = (-k_1 e_1^2 - k_2 e_2^2) < 0 \quad (12)$$

where  $k_1$  and  $k_2$  are positive design constants that determine the closed loop dynamics.

According to (12), the controls  $x_{1ref}$  and  $x_{2ref}$  in (11) are asymptotically stabilizing.

### 3.2 Current controller design methodology

**Step 1.** In this step, we define other errors  $e_3$  and  $e_4$  in the components of the stator currents and their references.

$$\begin{cases} e_3 = x_{2ref} - x_2 \\ e_4 = x_{1ref} - x_1 \end{cases} \quad (13)$$

So the dynamics of  $e_1$  and  $e_2$  can be obtained as:

$$\begin{cases} \dot{e}_1 = -k_1 e_1 - \frac{\rho}{J} (e_4 x_4 - e_3 x_3) \\ \dot{e}_2 = -k_2 e_2 + 2MT_r (e_4 x_3 - e_3 x_4) \end{cases} \quad (14)$$

The derivative errors dynamics in (13) are given by:

$$\begin{cases} \dot{e}_3 = \dot{x}_{2ref} - \delta_1 - b_2 u_2 \\ \dot{e}_4 = \dot{x}_{1ref} - \delta_2 - b_1 u_1 \end{cases} \quad (15)$$

with:

$$\begin{cases} \delta_1 = -a_1 x_2 - a_3 \omega_r x_3 + a_3 T_r x_4 \\ \delta_2 = -a_1 x_1 - a_3 T_r x_3 + a_3 \omega_r x_4 \end{cases} \quad (16)$$

**Step 2.** In this step we define the control laws, while final Lyapunov function based on all the errors of the speed, the rotor flux and the stator currents, such as:

$$V_2 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (17)$$

**Step 3.** The time derivative of  $V_2$  is given by:

$$\dot{V}_2 = (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4) \quad (18)$$

According to (13) and (14), this equation can be rewritten as follows:

$$\dot{V}_2 = -k_1e_1^2 - k_2e_2^2 + e_3(\dot{x}_{2ref} - \delta_1 - b_2u_2 - 2MT_r x_4 + \frac{\rho}{J}x_3e_1) + e_4(\dot{x}_{1ref} - \delta_2 - b_1u_1 + 2MT_r x_3e_2 - \frac{\rho}{J}x_4e_1) \quad (19)$$

where  $u_1$  and  $u_2$  are the actual control variables. If the stabilizing control law is defined as:

$$\begin{cases} u_1 = \frac{1}{b_1} (\dot{x}_{1ref} + k_4e_4 - \delta_2 + 2MT_r x_3e_2 - \frac{\rho}{J}x_4e_1) \\ u_2 = \frac{1}{b_2} (\dot{x}_{2ref} + k_3e_3 - \delta_1 + 2MT_r x_4e_2 + \frac{\rho}{J}x_3e_1) \\ k_3 > 0, k_4 > 0 \end{cases} \quad (20)$$

Then, (15) can be expressed as:

$$\begin{cases} \dot{e}_3 = -k_3e_3 - \frac{\rho}{J}x_3e_1 - 2MT_r x_4e_2 \\ \dot{e}_4 = -k_4e_4 + \frac{\rho}{J}x_4e_1 - 2MT_r x_3e_2 \end{cases} \quad (21)$$

The choice of  $k_3$  and  $k_4$  as positive parameters can made  $\dot{V}_2 < 0$ , which indicates the tracking error will converge asymptotically to zero. The objective of BC for IM is completed.

#### 4 Passive FTC based input-output linearization controller

For the induction motor case, by choosing the output functions as the flux square and rotor speed respectively as showing bellow:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \varphi_r \alpha^2 + \varphi_r \beta^2 = \varphi_r = x_r \\ \omega \end{bmatrix} \quad (22)$$

Therefore, the output dynamics could be easily written as follows:

$$y_1 : \begin{cases} \dot{h}_1(x) = L_f h_1(x) = 2a_5 L_m f_1 - 2a_5 x_r \\ \ddot{h}_1(x) = L_f^2 h_1(x) + L_{G_\alpha} L_f h_1(x) u_1 + L_{G_\beta} L_f h_1(x) u_2 \end{cases} \quad (23)$$

with:

$$\begin{cases} L_f^2 h_1(x) = 2a_4 a_5 L_m f_3 - (2a_1 a_4 + 6a_4 a_5) f_1 + 2a_4 \omega f_2 + \left(\frac{4}{T_r} a_5 + 2a_2 a_4\right) x_r \\ L_{G_\alpha} L_f h_1(x) = 2a_2 L_r x_3 \\ L_{G_\beta} L_f h_1(x) = 2a_2 L_r x_4 \end{cases} \quad (24)$$

$$y_2 : \begin{cases} \dot{h}_2(x) = L_f h_2(x) = \frac{\rho L_m}{J L_r} f_2 - \frac{C_f}{J} \\ \ddot{h}_2(x) = L_f^2 h_2(x) + L_{G_\alpha} L_f h_2(x) u_1 + L_{G_\beta} L_f h_2(x) u_2 \end{cases} \quad (25)$$

with:

$$\begin{cases} L_f^2 h_2(x) = \frac{pL_m}{JL_r} \omega f_1 - \frac{pL_m}{JL_r} (a_1 + a_5) f_2 - \frac{pL_m^2}{JL_r^2} \frac{\omega}{\sigma L_s} x_r \\ L_{G_\alpha} L_f h_2(x) = -\frac{pL_m}{JL_r} \frac{1}{\sigma L_s} x_4 \\ L_{G_\beta} L_f h_2(x) = \frac{pL_m}{JL_r} \frac{1}{\sigma L_s} x_3 \end{cases} \quad (26)$$

where:  $f_1 = x_1 x_3 + x_2 x_4$ ,  $f_2 = x_2 x_3 - x_1 x_4$ ,  $f_3 = x_1^2 + x_2^2$ ,  $a_1 = \frac{-R_t}{\sigma L_s}$ ,  $a_2 = \frac{L_m}{\sigma L_s L_r T_r}$ ,  $a_3 = \frac{L_m}{\sigma L_s L_r}$ ,  $a_4 = \frac{L_m}{T_r}$ ,  $a_5 = \frac{1}{T_r}$

Although the system dynamics order is five, the output dynamics have the order of four which implies the presence of an internal dynamics and the corresponding stability could be easily proven.

Using input-output feedback linearization, only the derivatives of the outputs are considered, we obtain:

$$\begin{bmatrix} \ddot{h}_1 \\ \ddot{h}_2 \end{bmatrix} = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{bmatrix} + E(x) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (27)$$

with:

$$E(x) = \begin{bmatrix} L_{G_\alpha} L_f h_1(x) & L_{G_\beta} L_f h_1(x) \\ L_{G_\alpha} L_f h_2(x) & L_{G_\beta} L_f h_2(x) \end{bmatrix} = \begin{bmatrix} 2a_2 L_r x_3 & 2a_2 L_r x_4 \\ -\frac{pL_m}{JL_r} \frac{1}{\sigma L_s} x_4 & \frac{pL_m}{JL_r} \frac{1}{\sigma L_s} x_3 \end{bmatrix} \quad (28)$$

$Det [E(x)] = 2 \frac{a_2 L_r}{\sigma L_s} x_r$ . If  $x_r \neq 0$ , the matrix  $E(x)$  is non-singular. By defining  $v$  as the new control input for linear system of:

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{bmatrix} + E(x) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (29)$$

Main control equation can be defined as:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = E(x)^{-1} \left( \begin{bmatrix} -L_f^2 h_1(x) \\ -L_f^2 h_2(x) \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right) \quad (30)$$

As a result, the system control effort would be simplified to:

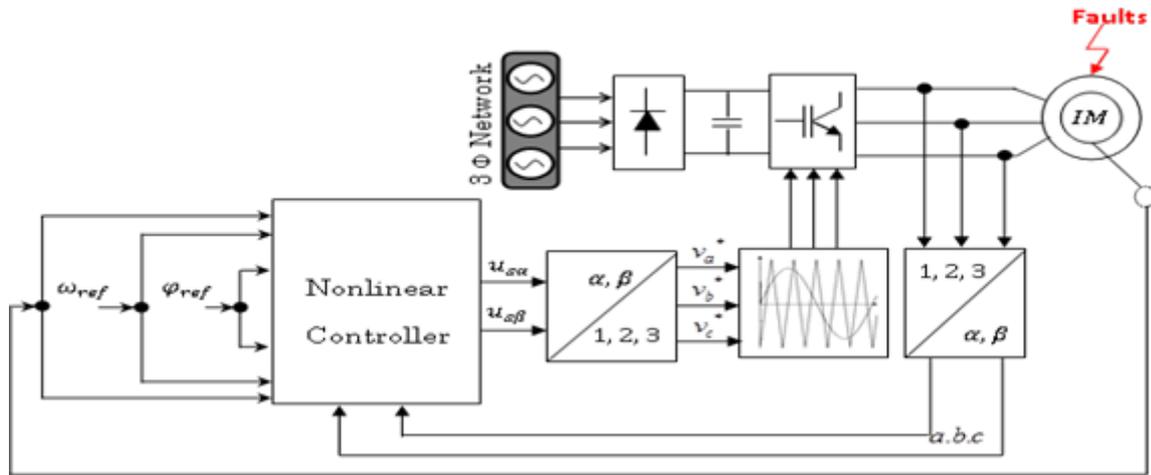
$$v_1 = \ddot{h}_1 = \frac{d^2 x_r}{dt^2} = k_{\phi 1} (x_{ref} - x_r) + k_{\phi 2} \left( \frac{dx_{ref}}{dt} - \frac{dx_r}{dt} \right) + \frac{d^2 x_{ref}}{dt^2} \quad (31)$$

$$v_2 = \ddot{h}_2 = \frac{d^2 \omega}{dt^2} = k_{\omega 1} (\omega_{ref} - \omega) + k_{\omega 2} \left( \frac{d\omega_{ref}}{dt} - \frac{d\omega}{dt} \right) + \frac{d^2 \omega_{ref}}{dt^2} \quad (32)$$

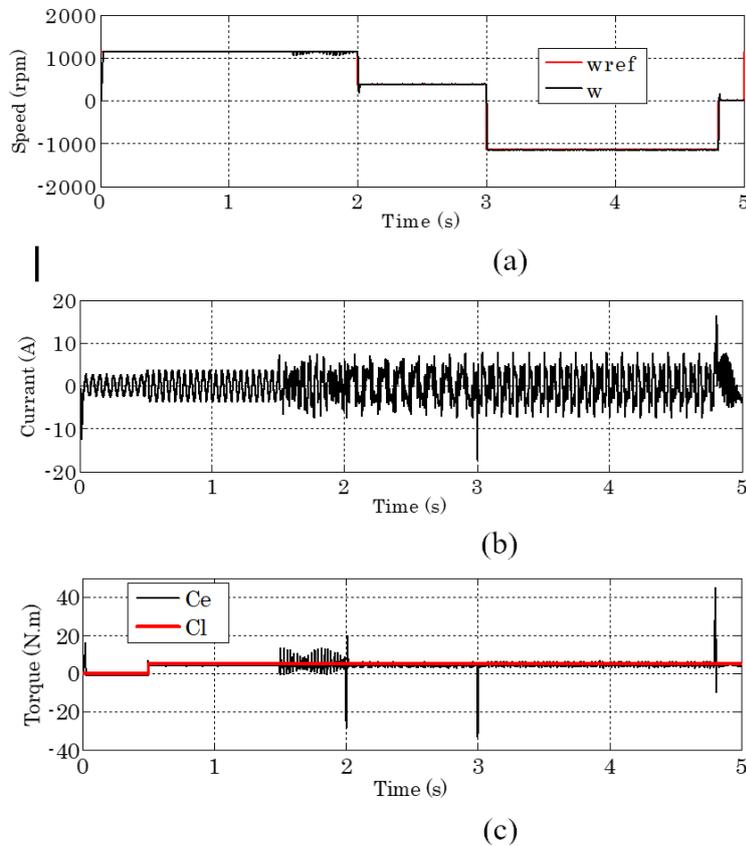
The gains  $k_{\phi 1}$ ,  $k_{\phi 2}$ ,  $k_{\omega 1}$  and  $k_{\omega 2}$  are chosen by identification with a second order system by using the pole placement method.

## 5 Simulation results

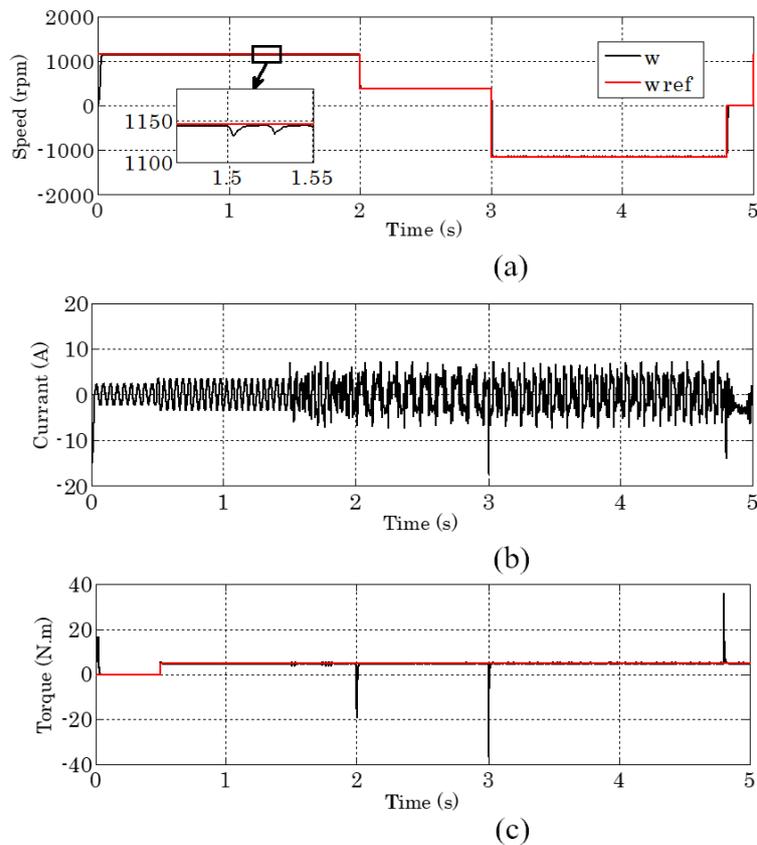
In this section, some numerical simulations have been performed to validate the proposed FTC scheme (see. Figure.1).The induction motor parameters are given in the appendix. The closed-loop simulation results are reported in Figures below under both healthy and faulty conditions. On the other hand variations of 50% of the stator resistance ( $R_s$ ) and rotor resistance ( $R_r$ ) between the time  $t = 3$  s and  $t = 4$  s with variable speed reference are introduced. After the faults occurred at  $t = 1.5$  s. Two levels of simulation results will be presented here: firstly, Figure (2) show that the simulation results of PFTC based on Backstepping control, secondly, Figure (3) show that the simulation results of PFTC based on Input-output linearization controller.



**Fig. 1:** Block diagram of the proposed passive fault tolerant



**Fig. 2:** PFTC strategy using Backstepping control: (a) Motor speed, (b) Stator Current, (c) Motor Torque (load torque is applied at  $t = 0.5$  s and the actuator faults occurred at  $t = 1.5$  s).



**Fig. 3:** PFTC strategy using Input-output linearization controller: (a) Motor speed, (b) Stator Current, (c) Motor Torque (load torque is applied at  $t = 0.5$  s and the actuator faults occurred at  $t = 1.5$  s).

## 6 Conclusion

This paper dealt with a comparative study of fault tolerant control strategies, namely PFTC, Backstepping Controller (BC) and Input-output linearization controller (IOLC), for an induction motor. The study indicates that each approach has its own advantages and limitations. A passive FTCs approach is more flexible to deal with different types of faults, including failure scenarios beyond the design basis faults. For the PFTC, based IOLC simulation results demonstrate a good performance in the presence of fault. The global system drive is stable and robust against load perturbation, speed reversion, parameter variations and fault rejection. On conclusion, we can say that the PFTC based IOLC is more superior to PFTC based BC.

## Competing interests

The authors declare that they have no competing interests.

## Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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