Some optical soliton solutions for the generalized Fokas-Lenells equation

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Abstract: In this article, we have considered the Fokas-Lenells equation with full nonlinearity. By the help of three types Jacobi elliptic functions, we have obtained bright, dark and singular optical soliton solutions of the Fokas-Lenells equation. We have also presented the constraint conditions which guarantee the existence of obtained solutions.

Keywords: Fokas Lenells equation, optical solitons, Jacobi elliptic functions.

1 Introduction

Soliton propagation studies in optical fibers have been flourishing because of the rich features of models describing these phenomena over the past few decades. In the field of optics, by careful selection of higher order nonlinear and linear effects, Fokas-Lenells equation(FLE) emerges as a model governing femtosecond pulse propagation through single mode silica optical fiber. Accordingly it is a useful model to understand the physical insight of ultra short pulses in media. It is one of the models that govern pulse transmissions technology first appeared about a decade ago. The existence of solitons in the nonlinear dispersive media has been studied within the framework of the FLE. Recently some research work have been conducted on exact solutions of this equation [1,2,3,4,5,6,7,8,9,10,11]. Here, the model is considered in the presence of perturbation terms that provide a picture much closer to reality. The FLE is studied with full nonlinearity in the generalized form using Jacobi elliptic functions in this work. It is obtained Jacobi elliptic function solutions and bright, dark and singular optical soliton solutions are also attained with the help of modulus limit of Jacobi elliptic functions for FLE. These solitons appear with constraint conditions on their parameters and they are also presented

2 Mathematical analysis

In the presence of dual dispersion, the dimensionless form of FLE with perturbation is given as

$$iq_t + a_1 q_{xx} + a_2 q_{xt} + |q|^2 (bq + i\sigma q_x) = i \left[\alpha q_x + \lambda \left(|q|^{2m} q \right)_x + \mu \left(|q|^{2m} \right)_x q \right]. \tag{1}$$

In (1), $q(x,t)$ describes the profile of optical pulse and the first term is the temporal evolution of the pulses.The coefficients a, b are real valued constants describing group velocity dispersion (GVD) and spatio-temporal dispersion (STD) respectively.

$$q(x,t) = u(\xi) e^{i\phi}, \quad \xi = x - vt, \tag{2}$$

where $u(\xi)$ and v represent amplitude and velocity of the soliton respectively while the phase component is defined as

$$\phi = -\kappa x + \omega t + \theta, \tag{3}$$

where κ is the soliton frequency, ω is the wave number and θ is the soliton phase constant. Substituting (2) into (1) and separating it into two parts real and imaginary parts yields

$$(a_1 - a_2 v) u'' - (\omega + a_1 \kappa^2 - \omega a_2 \kappa + \alpha \kappa) u + (\kappa \sigma + b) u^3 - \kappa \lambda u^{2m+1} = 0, \tag{4}$$

$$(v a_2 \kappa + \omega a_2 - v - 2a_1 \kappa - \alpha) u' + \sigma u' u^2 + [\lambda (2m+1) + 2m\mu] u' u^{2m} = 0. \tag{5}$$

respectively. From the imaginary part we obtain

$$m = 1, \tag{6}$$

$$\sigma = (2m+1)\lambda + 2m\mu, \tag{7}$$

$$v = \frac{\omega a_2 - 2a_1 \kappa - \alpha}{1 - a_2 \kappa} \tag{8}$$

So, real part reduces

$$(a_1 - a_2 v) u'' - (\omega + a_1 \kappa^2 - \omega a_2 \kappa + \alpha \kappa) u + (\kappa \sigma + b - \kappa \lambda) u^3 = 0, \tag{9}$$

The initial hypothesis is

$$u(\xi) = A sn^p (B\xi, \ell) \tag{10}$$

where sn is the Jacobi elliptic function, ℓ is the modulus of Jacobi elliptic function and $0 < \ell < 1$. A and B represent amplitude and inverse width of the soliton. Eq. (9) reduces

$$(a_1 - a_2 v)(\rho - 1)\rho A B^2 sn^{\rho-2} (B\xi, \ell)$$

$$- (a_1 - a_2 v)\rho \left[\ell^2 (\rho - 1) + \ell + \rho\right] A B^2 sn^\rho (B\xi, \ell)$$

$$+ (a_1 - a_2 v)\ell \rho (\ell \rho + 1) A B^2 sn^{\rho+2} (B\xi, \ell)$$

$$- (\omega + a_1 \kappa^2 - \omega a_2 \kappa + \alpha \kappa) A sn^\rho (B\xi, \ell)$$

$$+ (\kappa \sigma + b - \kappa \lambda) A^3 sn^{3\rho} (B\xi, \ell) = 0. \tag{11}$$

From Eq. (11), matching the exponents $sn^{\rho+2} (B\xi, \ell)$ and $sn^{3\rho} (B\xi, \ell)$ yields

$$p + 2 = 3p \tag{12}$$

which gives

$$p = 1 \tag{13}$$

Equating coefficients of them and setting coefficients of $sn^{\rho+j}(B\xi,\ell)$, for $j = -2, 0$, to zero in (11) as these are linearly independent functions we obtain

$$A = \sqrt{\frac{\ell(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)}{(\kappa\sigma + b - \kappa\lambda)}}, \tag{14}$$

$$B = \sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{(a_2 v - a_1)(\ell + 1)}}, \tag{15}$$

So, the Jacobi elliptic function solution is obtained as follows:

$$q(x,t) = \sqrt{\frac{\ell(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)}{(\kappa\sigma + b - \kappa\lambda)}} sn\left[\sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{(a_2 v - a_1)(\ell + 1)}}\right.$$

$$\left.\left(x - \frac{\omega a_2 - 2a_1\kappa - \alpha}{1 - a_2\kappa}t\right),\ell\right].e^{i(-\kappa x + \omega t + \theta)}, \tag{16}$$

which requires the constraints

$$(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)(\kappa\sigma + b - \kappa\lambda) > 0, \tag{17}$$

$$(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)(a_2 v - a_1) > 0. \tag{18}$$

When the modulus $\ell \to 1$ in Eq. (16), dark optical soliton solution is obtained as:

$$q(x,t) = \sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{\kappa\sigma + b - \kappa\lambda}} \tanh\left[\sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{2(a_2 v - a_1)}}\right.$$

$$\left.\left(x - \frac{\omega a_2 - 2a_1\kappa - \alpha}{1 - a_2\kappa}t\right)\right].e^{i(-\kappa x + \omega t + \theta)}. \tag{19}$$

In order to construct exact solutions for Eq. (1), we use hypothesis in the form

$$u(\xi) = A cn^{\rho}(B\xi,\ell), \tag{20}$$

From (20), Eq. (9) reduces to

$$(a_1 - a_2 v)(1 - \ell^2)(\rho - 1)\rho AB^2 cn^{\rho-2}(B\xi,\ell) + (a_1 - a_2 v)\rho\left[\ell^2(2\rho - 1) + \ell - \rho\right]AB^2 cn^{\rho}(B\xi,\ell)$$
$$- (a_1 - a_2 v)\ell\rho(\ell\rho + 1)AB^2 cn^{\rho+2}(B\xi,\ell) - (\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)A cn^{\rho}(B\xi,\ell)$$
$$+ (\kappa\sigma + b - \kappa\lambda)A^3 cn^{3\rho}(B\xi,\ell) = 0. \tag{21}$$

Setting the exponents and coefficients of functions $cn^{\rho+2}(B\xi,\ell)$ and $cn^{3\rho}(B\xi,\ell)$ equal to one another, and then setting the coefficients functions of $cn^{\rho+j}(B\xi,\ell)$ to zero for $j = -2, 0$, we acquire the same value of which is in (13) and following equations

$$A = \sqrt{\frac{\ell(\ell + 1)(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)}{(\ell^2 + \ell - 1)(\kappa\sigma + b - \kappa\lambda)}} \tag{22}$$

$$B = \sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{(a_1 - a_2 v)(\ell^2 + \ell - 1)}} \tag{23}$$

So, the Jacobi elliptic function solution is obtained as

$$q(x,t) = \sqrt{\frac{\ell(\ell+1)(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)}{(\ell^2 + \ell - 1)(\kappa\sigma + b - \kappa\lambda)}} cn\left[\sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{(a_1 - a_2 v)(\ell^2 + \ell - 1)}}\right.$$

$$\left.\left(x - \frac{\omega a_2 - 2a_1\kappa - \alpha}{1 - a_2\kappa}t\right), \ell\right].e^{i(-\kappa x + \omega t + \theta)}. \tag{24}$$

In Eq. (24), when the modulus $\ell \to 1$, we obtain following dark optical soliton solution

$$q(x,t) = \sqrt{\frac{2(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)}{\kappa\sigma + b - \kappa\lambda}} \sec h\left[\sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{a_1 - a_2 v}}\right.$$

$$\left.\left(x - \frac{\omega a_2 - 2a_1\kappa - \alpha}{1 - a_2\kappa}t\right)\right].e^{i(-\kappa x + \omega t + \theta)}. \tag{25}$$

with conditions

$$(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)(\kappa\sigma + b - \kappa\lambda) > 0, \tag{26}$$

$$(\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)(a_1 - a_2 v) > 0. \tag{27}$$

Now, if we use the assumption

$$u(\xi) = Ans^\rho(B\xi, \ell), \tag{28}$$

From (28), Eq. (9) reduces to

$$(a_1 - a_2 v)(\rho - 1)\rho\ell^2 AB^2 ns^{\rho-2}(B\xi, \ell)$$
$$- (a_1 - a_2 v)\rho^2(1 + \ell^2)AB^2 ns^\rho(B\xi, \ell)$$
$$+ (a_1 - a_2 v)\rho(\rho + 1)AB^2 ns^{\rho+2}(B\xi, \ell)$$
$$- (\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa)Ans^\rho(B\xi, \ell)$$

$$+ (\kappa\sigma + b - \kappa\lambda)A^3 ns^{3\rho}(B\xi, \ell) = 0. \tag{29}$$

When the modulus $\ell \to 1$ in Eq. (29), we obtain following singular optical soliton solution

$$q(x,t) = \sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{\kappa\sigma + b - \kappa\lambda}} \coth\left[\sqrt{\frac{\omega + a_1\kappa^2 - \omega a_2\kappa + \alpha\kappa}{2(a_1 - a_2 v)}}\right.$$

$$\left.\left(x - \frac{\omega a_2 - 2a_1\kappa - \alpha}{1 - a_2\kappa}t\right)\right].e^{i(-\kappa x + \omega t + \theta)}. \tag{30}$$

with conditions (26) and (27).

3 Conclusion

This paper consider FLE with full nonlinearity and perturbation terms. Jacobi elliptic functions are used for the integration scheme here. Bright and dark and singular optical soliton solutions are obtained using three types Jacobi

elliptic functions. The existence criteria of these solutions are also indicated.These solutions provide recognise physical phenomena described by the equation.Due to the fact that bright and dark optical soliton solutions always help to address the soliton dynamics in long distance telecommunication system, the results of the paper are useful in the fiber optics communication technology. It can be obtained more solutions of the FLE using the other Jacobi elliptic functions. This technique is very useful and effective to get soliton solutions of nonlinear partial differential equations in mathematical physics.

Fuzzy logic can be a useful tool for the navigation process in the distributed MRS and more particularly in a dynamic and uncertain environment. It can be used to formulate the different information received and observed by robots in the form of linguistic terms similar to those used by human. From this point of view, we can design robots with reasoning abilities which are similar to those found in humans. This may be possible if we choose a set of rules that allow the use these linguistic variables to conclude other information in order to make a particular decision. For this, it is necessary to use an appropriate architecture, especially when the functionalities of the robots are effectively separated. The process of selection behaviours proposed shows a great flexibility to implement the different navigation approaches in multi-robot systems.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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