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Soliton solutions of nonlinear (2 + 1)-dimensional Biswas-Milovic equation via new approach of generalized Kudryashov scheme

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Abstract: This paper deals with the (2+1)-dimensional Biswas-Milovic equation (BME) for soliton propagation in optical fiber utilizing the new approach of the generalized Kudryashov method. We derive a plethora of traveling wave solutions of the investigated problem. Some 3-d representations for the given special parameters are given. Moreover, the obtained results might be helpful for the future studies not only for explaining the dynamic behavior of the considered problem but also effectiveness and reliability of the proposed new approach of the method.

Keywords: (2+1) dimensional Biswas-Milovic equation, generalized Kudryashov method, Riccati equation, optical soliton.

1 Introduction

In the past two decades, nonlinear partial differential equations (NLPDE) have gained great importance to explain many physical phenomena and have been the focus of many researchers. NLPDE are observed in various fields such as fluid mechanics, plasma physics, telecommunications, heat conduction, optics, optical physics, optical fibers, quantum mechanics, genetics, wave propagation etc. Therefore, many studies have been carried out in these areas and many methods have been developed for the solution of NLPDE such as exp-function method, modified extended tanh expansion method, extended rational sine-cosine/ sinh-cosh method, Kudryashov technique, G'/G-expansion method, modified variational iteration method, the Riccati-Bernoulli sub-ODE technique, the Jacobi elliptic function expansion method, Backlund transformation, Hirota bilinear method [1,2,3,4,5,6,7,8,9,10,11] and so on. Optic wave propagation and optical soliton solution within the NLPD equations have also become the special focus of attention for researchers and have been a field of intense study. The perturbed NLSE in Kerr media, nonlinearities related to nonlinear Schrodinger equation (NLSE), Lakshmanan-Porsezian-Daniel equation, Manakov model, Triki-Biswas equation, Kaup-Newell equation, complex Ginzburg-Landau equation (CGLE), Kundu-Mukherjee-Naskar model, Sasa-Satsuma model, Chen-Lee-Liu model, Kundu-Eckhaus equation, Biswas-Milovic equation can be given as an example of the problems that have been studied intensively in optics recently [12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27].

Biswas and Milovic introduced the Biswas-Milovic equation which is a generalized type of the NLSE [28].
The (2+1)-BME [26,27] is given,

$$i\frac{\partial S}{\partial t}-\frac{1}{2}\frac{\partial^2 S}{\partial x^2}+i\left(\beta-|S|^2\right)S=0\ ,\quad i=\sqrt{-1} \tag{1}$$

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$$S(x,y,t) = e^{i\theta} S(\xi) \;\;,\;\; \xi = x + y + at \;\;,\;\; \theta = ax + by + \omega t. \tag{2}$$

In eq.(1) and eq.(2) $S(x,y,t)$ is a complex, $S(\zeta)$ is a real valued function and x, y are the independent variables which represent spatial co-ordinates, t is the temporal variable. In addition θ is the phase component and a,b,ω,β are real values.

The rest of the paper is presented as follows: In Section 2, algorithm of the method and application to considered problem is given; In Section 3, we present numerical results and discussion and finally, we conclude the obtained results in Section 4.

2 Algorithm of the method and application to problem

Step 1: Consider the (2+1)-dimensional BME in eq.(1) and the wave transformation in eq.(2). Substituting eq.(2) into eq.(1), we get the following nonlinear ordinary differential equation (NLODE):

$$-2(S(\xi))^3 + \left(a^2 + 2\beta - 2\omega\right) S(\xi) - \frac{\mathrm{d}^2 S(\xi)}{\mathrm{d}\xi^2} = 0 \tag{3}$$

Step 2: According to generalized Kudryashov technique the solution of eq.(3) is offered in the following form.

$$S(\xi) = \frac{\sum\limits_{i=0}^{p} A_i T^i(\xi)}{\sum\limits_{j=0}^{q} B_j T^j(\xi)}, \;\; A_p, B_q \neq 0 \tag{4}$$

In eq.(4), $A_0,\ldots,A_p,B_0,\ldots,B_q$ are unknowns real parameters to be computed later (A_p,B_q should not be zero simultaneously). The p and q are the balancing constants which are computed using the homogeneous balance principle in eq.(3) as p=q+1. If we select q=1, produces p=2 then eq.(4) takes the form,

$$S(\xi) = \frac{A_0 + A_1 T(\xi) + A_2 T^2(\xi)}{B_0 + B_1 T(\xi)}, \;\; A_2, B_1 \neq 0 \tag{5}$$

where A_2, B_1 are should not be zero simultaneously.

Step 3: We take the auxiliary function in eq.(5) which satisfies the following first order Riccati differential equation.

$$\frac{dT(\xi)}{d\xi} = T'(\xi) = w - [T(\xi)]^2 = 0 \tag{6}$$

The form of the Riccati equation $T'(\xi) = w + T^2(\xi)$ is used by many researchers under the names tanh expansion method, extended tanh expansion method and lastly modified extended tanh function/expansion method (METFM/METEM) with the following solutions [3,4].

$$T(\xi) = \begin{cases} -\sqrt{-w}\tanh\left(\sqrt{-w}\xi\right), \\ -\sqrt{-w}\coth\left(\sqrt{-w}\xi\right) \end{cases} .w < 0..,..T(\xi) = \begin{cases} \sqrt{w}\tan\left(\sqrt{w}\xi\right), \\ -\sqrt{w}\cot\left(\sqrt{w}\xi\right) \end{cases} .w > 0..,..T(\xi) = \left\{ -\frac{1}{\xi}.,w = 0 \right. \tag{7}$$

Step 4: In this study, we use the Riccati equation is given in eq.(6) with the following solutions,

$$T(\xi)=\begin{cases}\sqrt{w}\tanh\left(\sqrt{w}\xi\right),\\ \sqrt{w}\coth\left(\sqrt{w}\xi\right)\end{cases} w>0\,,\; T(\xi)=\begin{cases}-\sqrt{-w}\tan\left(\sqrt{-w}\xi\right),\\ \sqrt{-w}\cot\left(\sqrt{-w}\xi\right)\end{cases} w<0\,,\; T(\xi)=\left\{\frac{1}{\xi}\,,w=0\right. \tag{8}$$

where w is a real parameter.

Step 5: Substituting eq.(5) and its necessary derivatives along with eq.(6) into eq.(3) produces a polynomial in powers of $T(\zeta)$. Summing the each coefficients of $T^i(\zeta)$ with the same power and taking to zero, we get a set of algebraic equations in the following form,

$$\begin{aligned}
&T^0(\xi): -2A_0{}^3+\left(\left(a^2+2\beta-2\omega\right)B_0{}^2-2w^2B_1{}^2\right)A_0-2w^2B_0\left(-A_1B_1+A_2B_0\right)=0\\
&T^1(\xi): A_1\left(a^2+2\beta-2\omega+2w\right)B_0{}^2+2B_1A_0\left(a^2+2\beta-2\omega-w\right)B_0-6A_0{}^2A_1=0\\
&T^2(\xi): A_0\left(a^2+2\beta-2\omega+2w\right)B_1{}^2+2B_1A_1\left(a^2+2\beta-2\omega-w\right)B_0+A_2\left(a^2+2\beta-2\omega+8w\right)B_0{}^2-6A_0{}^2A_2-6A_0A_1{}^2=0\\
&T^3(\xi): A_1\left(a^2+2\beta-2\omega\right)B_1{}^2+2\left(\left(a^2+2\beta-2\omega+3w\right)A_2+A_0\right)B_1B_0-2A_1{}^3+\left(-12A_0A_2-2B_0{}^2\right)A_1=0\\
&T^4(\xi): A_2\left(-6B_0{}^2+\left(a^2+2\beta-2\omega+2w\right)B_1{}^2-6A_0A_2-6A_1{}^2\right)=0\\
&T^5(\xi): -6A_1A_2{}^2-6A_2B_0B_1=0\\
&T^6(\xi): -2A_2{}^3-2A_2B_1{}^2=0
\end{aligned} \tag{9}$$

Step 6: Solving the algebraic system in eq.(9) by aid of a Computer Algebraic Software we obtain the following possible solution sets:

$$\begin{aligned}
&Set_{1,2}=\left\{A_0=A_0,A_1=A_1,A_2=0,B_0=\frac{\mp 2A_0}{\sqrt{2a^2+4\beta-4\omega}},B_1=\frac{\mp 2A_1}{\sqrt{2a^2+4\beta-4\omega}}\right\},\\
&Set_{3,4}=\left\{A_0=\frac{\mp B_0\sqrt{2a^2+4\beta-4\omega}}{2},A_1=\frac{\mp B_1\sqrt{2a^2+4\beta-4\omega}}{2},A_2=0,B_0=B_0,B_1=B_1\right\}
\end{aligned} \tag{10}$$

Considering the eqs.(5) and eq.(2) by using the solutions of eq.(6) given in eq.(8), we can construct the soliton solution of eq.(1) in the following form:

for $w<0$**;**

$$S_1(x,y,t)=\mathrm{e}^{i(ax+by+\omega t)}\frac{A_0-A_1\sqrt{-w}\tan\left(\sqrt{-w}\,(at+x+y)\right)-A_2w\left(\tan\left(\sqrt{-w}\,(at+x+y)\right)\right)^2}{B_0-B_1\sqrt{-w}\tan\left(\sqrt{-w}\,(at+x+y)\right)} \tag{11}$$

$$S_2(x,y,t)=\mathrm{e}^{i(ax+by+\omega t)}\frac{A_0+A_1\sqrt{-w}\cot\left(\sqrt{-w}\,(at+x+y)\right)-A_2w\left(\cot\left(\sqrt{-w}\,(at+x+y)\right)\right)^2}{B_0+B_1\sqrt{-w}\cot\left(\sqrt{-w}\,(at+x+y)\right)} \tag{12}$$

for $w>0$**;**

$$S_3(x,y,t)=\mathrm{e}^{i(ax+by+\omega t)}\frac{A_0+A_1\sqrt{w}\tanh\left(\sqrt{w}\,(at+x+y)\right)+A_2w(\tanh\left(\sqrt{w}\,(at+x+y)\right))^2}{B_0+B_1\sqrt{w}\tanh\left(\sqrt{w}\,(at+x+y)\right)} \tag{13}$$

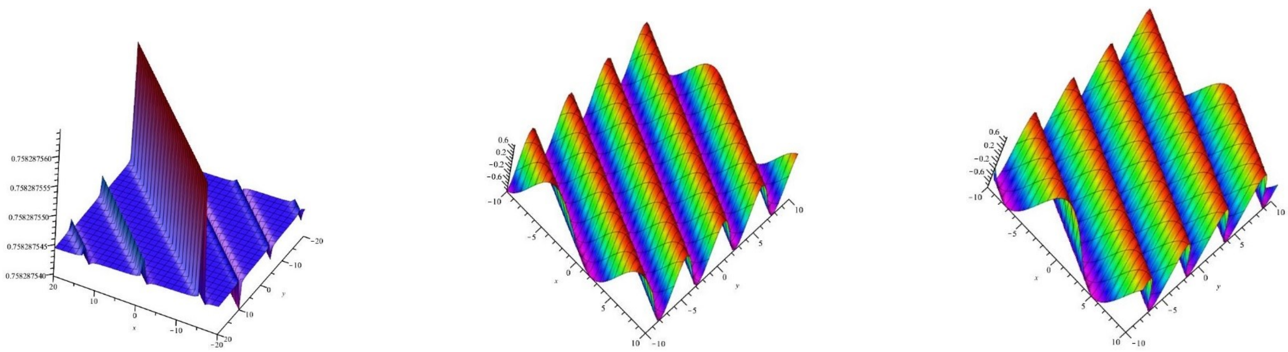
$$S_4(x,y,t)=\mathrm{e}^{i(ax+by+\omega t)}\frac{A_0+A_1\sqrt{w}\coth\left(\sqrt{w}\,(at+x+y)\right)+A_2w(\coth\left(\sqrt{w}\,(at+x+y)\right))^2}{B_0+B_1\sqrt{w}\coth\left(\sqrt{w}\,(at+x+y)\right)} \tag{14}$$

for $w = 0$**;**

$$S_5(x,y,t) = \mathrm{e}^{i(ax+by+\omega t)} \frac{A_0 + \frac{A_1}{at+x+y} + \frac{A_2}{(at+x+y)^2}}{B_0 + \frac{B_1}{at+x+y}} \tag{15}$$

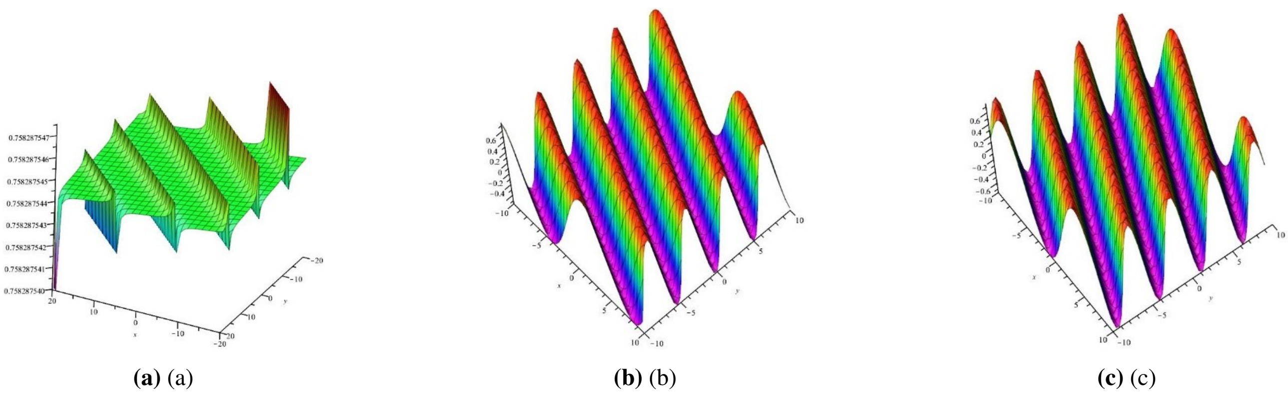
Selecting the any set given in eq.(10), then inserting into eqs.(11-15) by considering with suitable selected parameter values, we get the different graphical representations given by Fig.1-Fig.5.

3 Results and graphical representation



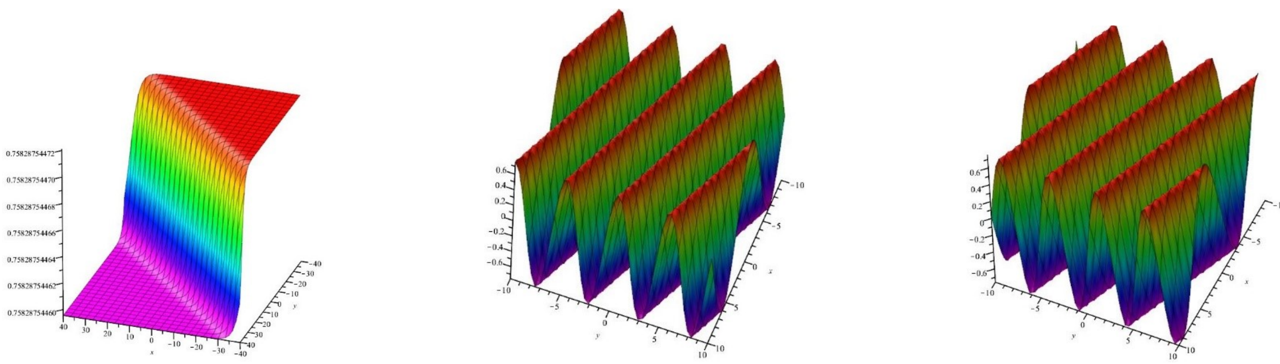
(a) (a) **(b)** (b) **(c)** (c)

Fig. 1: The 3D (a) modulus, (b) real, (c) imaginary surface of $S_1(x,y,t)$ in eq.(11) by selecting set Set_3 in eq.(10) for the parameters $w = -0.05$, a=0.5, b=1.2, ω=0.75, A_0=0.5, A_1=1, β=1.2, t=1.



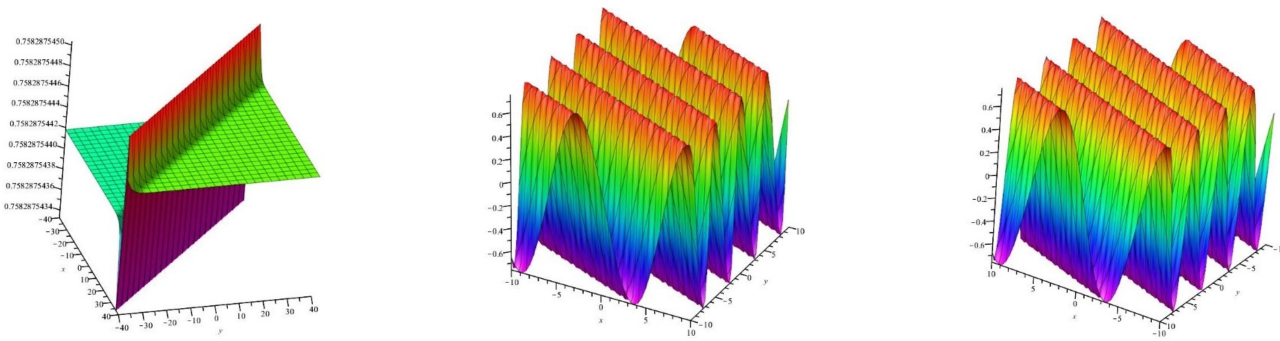
(a) (a) **(b)** (b) **(c)** (c)

Fig. 2: The 3D (a) modulus, (b) real, (c) imaginary surface of $S_2(x,y,t)$ in eq.(12) by selecting set Set_4 in eq.(10) for the parameters $w = -0.05$, a=0.5, b=1.2, ω=0.75, B_0=0.5, B_1=1, β=1.2, t=2.



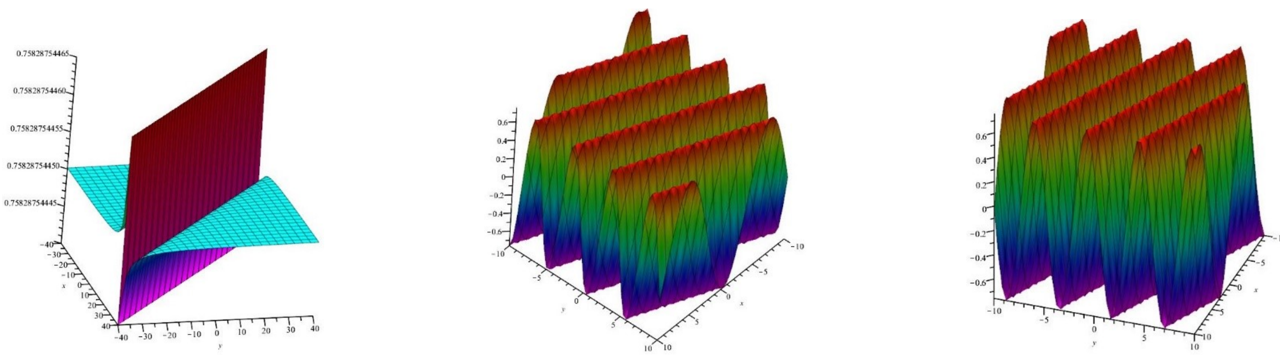
(a) (a) **(b)** (b) **(c)** (c)

Fig. 3: The 3D (a) modulus, (b) real, (c) imaginary surface of $S_3(x,y,t)$ in eq.(13) by selecting set Set_1 in eq.(10) for the parameters $w = 0.05, a$=0.5, b=1.2, ω=0.75,A_0=0.5,A_1=1,β=1.2,t=1.



(a) (a) **(b)** (b) **(c)** (c)

Fig. 4: The 3D (a) modulus, (b) real, (c) imaginary surface of $S_4(x,y,t)$ in eq.(14) by selecting set Set_3 in eq.(10) for the parameters $w = 0.05, a$=0.5, b=1.2, ω=0.75,B_0=0.5,B_1=1,β=1.2,t=1.



(a) (a) **(b)** (b) **(c)** (c)

Fig. 5: The 3D (a) modulus, (b) real, (c) imaginary surface of $S_5(x,y,t)$ in eq.(15) by selecting set Set_5 in eq.(10) for the parameters $w = 0, a$=0.5, b=1.2, ω=0.75,B_0=0.5,B_1=1,β=1.2,t=1.

4 Conclusion

Modeling of optical solitons and obtaining their solutions are of great importance. The Biswas-Milovic equation is not only a generalized version of the nonlinear Schrodinger equation, but also has an special importance as a realistic model. In this study, the (2+1)-dimensional Biswas-Milovic equation which has not been studied much, is investigated, soliton solutions and graphics that have not been given in the literature before are presented such as periodic, singular periodic, kink, singular kink, periodic singular kink. Moreover, unlike its common usage, the Riccati equation was taken as $T'(\xi) = w - T^2(\xi)$ and combining with the generalized Kudryashov technique the specified solutions were obtained.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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