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Optical soliton perturbation of the quintic Gerdjikov-Ivanov model

Muslum Ozisik

Yildiz Technical University, Istanbul

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Abstract: This aim of this paper is revealing the optical soliton solutions of the perturbed quintic Gerdjikov-Ivanov equation, which models optical soliton transmission in the photonic crystal fibers and optical fibers. With the aid of the new Kudryashov scheme, we successful achieved the bright and dark solitons and we simulated their graphical presentation.

Keywords: Perturbation; Self-steepening; Photonic crystal; DNLSIII equation.

1 Introduction

The last quarter of a century is full of studies and newly developed models of nonlinear optics due to the importance of optical fibers in connection with issues such as internet, communication, data transmission and intercontinental data transfer. One of these model is Gerdjikov-Ivanov equation.

$$iq_{1t}+q_{1xx}+i\varepsilon_1 q_1^2 q_x^* + \frac{\varepsilon_1^2}{g}\left|q_1\right|^4 q_1 = 0. \tag{1}$$

Equation (1) presented by Gerdjikov and Ivanov in 1983 [1]. Later it is called the Gerdjikov-Ivanov equation. Equation (1) has a great importance for ultrashort solitons in nonlinear metamaterials for nonlinear optics, plasma-physics and hydrodynamic wave packets. The following form is very familiar structure of eq. (1):

$$iq_t + aq_{xx} + b\left|q\right|^4 q + icq^2 q_x^* = 0, \tag{2}$$

where $q = q(x,t)$ is a complex function which represents the soliton profile. The first term is temporal evolution, the second group velocity dispersion (GVD) or namely chromatic dispersion (CD), the third one is the quintic nonlinearity and the last one is the self-steepening for short-pulses. Moreover, a, b, and c are real values.

If a=1, c=-1, b=1/2, eq. (2) converts into regular Gerdjikov-Ivanov equation:

$$iq_t + q_{xx} + \frac{1}{2}\left|q\right|^4 q - iq^2 q_x^* = 0, \tag{3}$$

in which also called the derivative type nonlinear Schrödinger equation (DNLSIII) [2,3]. The other form are as follows [3,4]:

$$iq_t + q_{xx} = i\left(q^2 q\right)_x, \tag{4}$$

* Corresponding author e-mail: ozisik@yildiz.edu.tr

which is the DNLSI type equation or Kaup-Newell equation. The last one is the DNLSII type,

$$iq_t + q_{xx} + i\left|q^2\right| q_x = 0, \tag{5}$$

which is also known as Chen-Lee-Liu model.

Due to its importance, the GI equation has been the subject of studies by many researchers [5-11].

The perturbed Gerdjikov-Ivanov having spatio-temporal dispersion is reads [11]:

$$ig_t + \alpha g_{xx} + \beta g_{xt} + \lambda |g|^4 g + i\mu g^2 g_x^* = i\left(\tau g_x + \eta\left(|g|^2 g\right)_x + \kappa\left(|g|^2\right)_x g\right), \tag{6}$$

where $g = g(x,t)$ stands for soliton profile. The terms for which, $\alpha, \beta, \lambda, \mu, \tau, \eta, \kappa$ are the coefficients GVD, STD, quintic nonlinearity, self-steepening, inter-modal dispersion, self-steepening, self-frequency shift, respectively.

The designation of the manuscript is as follows: The wave transformation and obtaining the NODE form is given in Phase 2. Presentation and implementation of the proposed method (the new Kudryashov method-nKM) is given in Phase 3. Phase 4 is devoted for graphical simulations. Phase 5 is the Conclusion.

2 Gaining the NODE form of eq. (6)

Let we consider the following form:

$$g(x,t) = G(\xi)\exp(i\Omega(x,t)), \Omega(x,t) = kx - \omega t + \varphi_0, \xi = x - vt \tag{7}$$

where $G(\xi)$ is the amplitude of the wave, $\xi$is the new variable, $\Omega(x,t)$ is the phase component, v is velocity, ω is frequency, $k$is wave number and $\varphi_0$is phase constant. Implementation of eq. (7) in eq. (6), gives imaginary part equation as:

$$\left((-3\eta - 2\kappa + \mu) G(\xi)^2 - \beta vk + 2\alpha k - \beta\omega - \tau - v\right)\left(\frac{dG(\xi)}{d\xi}\right) = 0, \tag{8}$$

which allows us to get the following constraint:

$$v = \frac{2\alpha k - \beta\omega - \tau}{\beta k + 1},\ \kappa = \frac{\mu - 3\eta}{2}, \tag{9}$$

where v is the velocity.

The real part of the equation gives the following form:

$$\left(\beta k\omega + k\tau + \omega - \alpha k^2\right) G(\xi) + (\eta k + k\mu) G(\xi)^3 + \lambda G(\xi)^5 + (\alpha - \beta v)\frac{d^2G(\xi)}{d\xi^2} = 0. \tag{10}$$

In eq. (10), the application of the balance rule is the terms $G''(\xi), G^5(\xi)$, serves N=1/2, N represents the balancing constant. In order to get the N as a positive integer, we need to define the following algebraic relation:

$$G(\xi) = \sqrt{M(\xi)}. \tag{11}$$

Implementing the eq. (11) in eq. (10), it is achieved as:

$$4\lambda M^4 + 4k(\eta + \mu) M^3 + 4\left((\beta\omega + \tau)k - \alpha k^2 + \omega\right) M^2 + (\beta v - \alpha)\left(M'\right)^2 - 2(\beta v - \alpha) MM'' = 0, \tag{12}$$

where $M = M(\xi)$. The balance of the terms MM'',M^4under the homogeneous balance principle, gives the balance number as N=1.

3 The basic algorithm of the nKM [12, 13] and its application

In this section, analytical optical soliton solutions of eq. (6) are obtained by nKM algorithm. First, the general features of the nKM were introduced, and then the method was successfully applied. One of the most important reasons for choosing the nKM is that the study is based on obtaining certain known optical soliton solutions (dark, bright). At this point, the nKM is a method that can be applied easily without requiring much processing and difficulty, and can effectively achieve dark and bright closed-form solutions.

Let consider eq. (13) as a solution of eq. (12):

$$M(\xi) = A_0 + \sum_{i=1}^{N} A_i R^i(\xi), A_N \neq 0, \tag{13}$$

where A_i are coefficients. Since N=1, eq. (13) takes the following form:

$$M(\xi) = A_0 + A_1 R(\xi), \tag{14}$$

in which $R(\xi)$ satisfies the eq. (15) [12, 13]:

$$\left(\frac{d^2R(\xi)}{dR(\xi)^2}\right)^2 - \delta^2 R^2(\xi)\left(1 - \chi R^2(\xi)\right) = 0, \tag{15}$$

which has the solution as:

$$R(\xi) = \frac{4L}{4L^2 e^{\delta\xi} + \chi e^{-\delta\xi}}. \tag{16}$$

χ, δ, L are arbitrary constants.

Inserting the eqs. (14), (15) into eq. (12), we reach a polynomial of $R(\xi)$ which serves the following algebraic equations system:

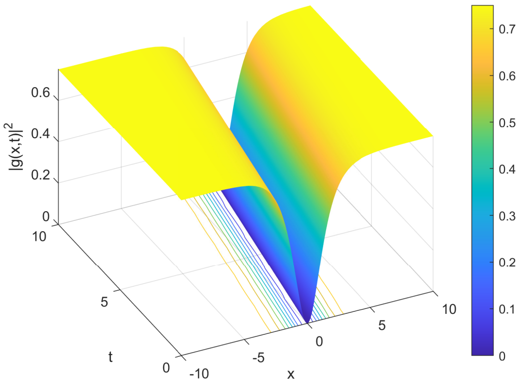
$$\begin{aligned}
&R^0(\xi): \omega(\beta k + 1) + kA_0(\eta + \mu) + \lambda A_0^2 - \alpha k^2 + k\tau = 0,\\
&R^1(\xi): \Omega + 8\lambda A_0^2 + 4\omega(1 + \beta k) + 6kA_0(\eta + \mu) + 4k(\tau - \alpha k) + \alpha\delta^2 = 0,\\
&R^2(\xi): \Omega + 24\lambda A_0^2 + 4\omega(\beta k + 1) + 12kA_0(\eta + \mu) + 4k(\tau - \alpha k) + \alpha\delta^2 = 0,\\
&R^3(\xi): -A_0\chi\Omega + 4\lambda A_0 A_1^2 + kA_1^2(\eta + \mu) - A_0\alpha\delta^2\chi = 0,\\
&R^4(\xi): -3\chi\Omega + 4\lambda A_1^2 - 3\alpha\delta^2\chi = 0,
\end{aligned} \tag{17}$$

where $\Omega = \frac{\beta\delta^2(\beta\omega + \tau - 2\alpha k)}{\beta k + 1}$.

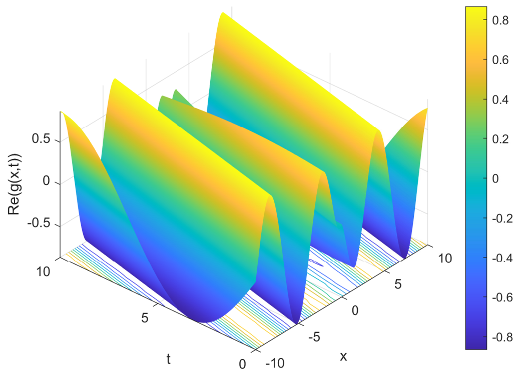
The solution of eq. (17) with appropriate auxiliary algebraic calculation programs allows us to reach the following solution sets:

$$\begin{aligned}
&Gset^{1,2} = \left\{\delta = \frac{6k(\beta k + 1)\Upsilon}{\Lambda}, \omega = \frac{\Theta}{64(\beta k + 1)\lambda}, A_0 = -\frac{3k\Upsilon}{8\lambda}, A_1 = \mp\frac{3k\sqrt{\chi}\Upsilon}{8\lambda}\right\},\\
&Gset^{3,4} = \left\{\delta = -\frac{6k(\beta k + 1)\Upsilon}{\Lambda}, \omega = \frac{\Theta}{64(\beta k + 1)\lambda}, A_0 = -\frac{3k\Upsilon}{8\lambda}, A_1 = \mp\frac{3k\sqrt{\chi}\Upsilon}{8\lambda}\right\},
\end{aligned} \tag{18}$$

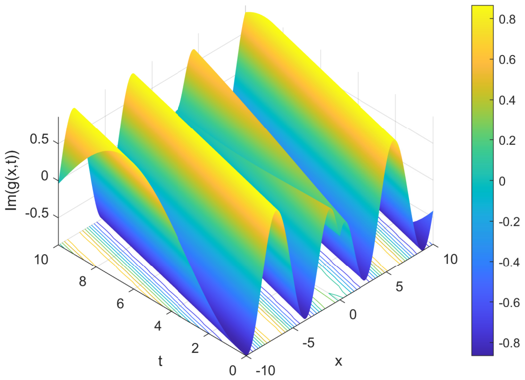
where $\Lambda = \sqrt{(192\beta\tau + 192\alpha)\lambda + 45k^2\Upsilon^2\beta^2}$, $\Theta = k\left(64\lambda(\alpha k - \tau) + 15k\Upsilon^2\right)$, $\Upsilon = (\eta + \mu)$.



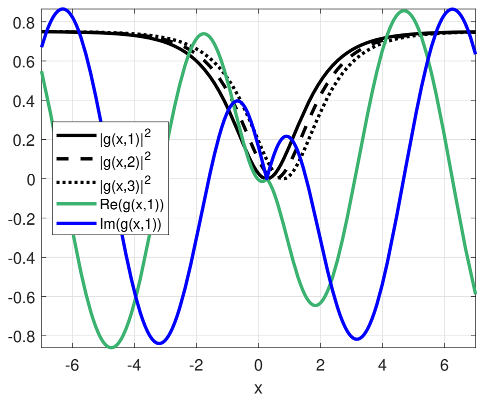
(a) $|\mathrm{g}(\mathrm{x},\mathrm{t})|^2$in 3D.



(b) $Re\,(\mathrm{g}(\mathrm{x},\mathrm{t}))$



(c) $Im\,(\mathrm{g}(\mathrm{x},\mathrm{t}))$.



(d) $|\mathrm{g}(\mathrm{x},\mathrm{t})|\,,\mathrm{Im}(\mathrm{g}(\mathrm{x},1)),\,\mathrm{Re}(\mathrm{g}(\mathrm{x},1))$

Fig. 1: Dark soliton view of eq. (19), and $Gset^{1,3}$with the parameters as $L=1,\,\chi=4,\,\eta=\mu=\lambda=\tau=\alpha=\beta=k=1,\;\varphi_0=0.5.$

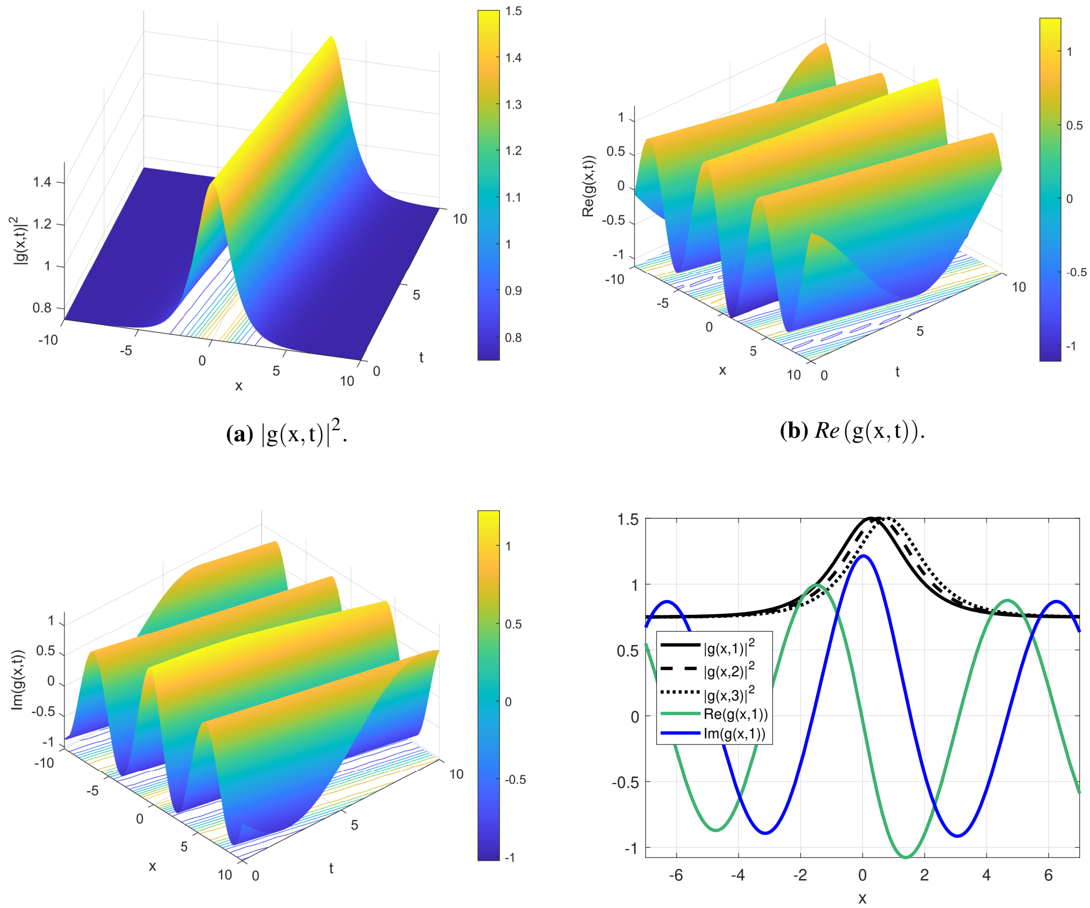
Combination of eq. (14) with eqs. (7), (11), (16) gives:

$$g(x,t)=\left(A_0+\frac{4A_1L}{4L^2\mathrm{e}^{\delta\left(x-\frac{(2\alpha k-\beta\omega-\tau)t}{\beta k+1}\right)}+\chi\mathrm{e}^{-\delta\left(x-\frac{(2\alpha k-\beta\omega-\tau)t}{\beta k+1}\right)}}\right)^{1/2}\mathrm{e}^{i(kx-\omega t+\phi_0)},\tag{19}$$

where $\delta,\,\omega,A_0,\,A_1$are presented in eq. (18).

4 Result and discussion

This phase covers the graphical representation of the eq. (19). Figure 1a is the 3d representation of the solution, reflecting the dark soliton representation. Figure 1b and fig. 1c, on the other hand, describe the periodic wave motion of the imaginary and real components eq. (19). Figure 1d is dedicated to 2d reflections. The expressions from solid black to dotted black lines sign the dark soliton shifting to the right with time. Similarly, fig. 2a depicts the bright soliton representation of $|\mathrm{g}(\mathrm{x},\mathrm{t})|^2$. Figure 2b and fig. 2c simulate the periodic wave motion of the $Re\,(\mathrm{g}(\mathrm{x},\mathrm{t}))$and $Im\,(\mathrm{g}(\mathrm{x},\mathrm{t}))$ .Figure 2d reflects the traveling wave view of the bright soliton when t=1,2,3.



(a) $|\mathrm{g}(\mathrm{x},\mathrm{t})|^2$. **(b)** $Re\,(\mathrm{g}(\mathrm{x},\mathrm{t}))$.

(c) $Im\,(\mathrm{g}(\mathrm{x},\mathrm{t}))$. **(d)** $|\mathrm{g}(\mathrm{x},\mathrm{t})|\,,\mathrm{Im}(\mathrm{g}(\mathrm{x},1)),\mathrm{Re}(\mathrm{g}(\mathrm{x},1))$.

Fig. 2: Bright soliton face of eq. (19), and $Gset^{2,4}$with the parameters as $L=1,\ \chi=4,\ \eta=\mu=\lambda=\tau=\alpha=\beta=k=1,\ \varphi_0=0.5$.

5 Conclusion

In this study, we successfully investigated the perturbed quintic Gerdjikov-Ivanov equation which equation, which identifies the optical soliton propagation in the photonic crystal and optical fibers. By implementing the new Kudryashov technique we derived the dark and bright solitons and we presented their graphical simulations. Today, the slice occupied by both communication and the internet in nonlinear optics is increasing, and the technological developments in these sectors are increasing in parallel with the researches. At this point, the importance of investigations in nonlinear optics comes to the fore once again. At this point, it is aimed that this study will contribute to the studies in this field.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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