On optical solitons of the (1+1)-dimensional Biswas-Milovic equation having Kerr law and parabolic-law with weak non-local nonlinearity in the presence of spatio-temporal dispersion

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Abstract: In this study, an effort has been made to obtain optical soliton solutions of the (1+1)-dimensional Biswas-Milovic equation which was introduced by Biswas and Milovic in 2010, having Kerr law and parabolic-law with weak non-local nonlinearity in the presence of spatio-temporal dispersion, which is one of the important models for nonlinear optics. Although, the model is an equation that has been studied by many researchers, the fact that the form to be examined has not been studied before. As a general algorithm, first converting the model to nonlinear ordinary differential form with a complex wave transformation, then obtaining candidate optical soliton solutions by utilizing the new Kudryashov technique, determining the ones that satisfy the main equation from these solutions as the exact solution, and in order to better understand the obtained solutions by making graphical presentation and providing the necessary comments constitute the main framework of the article.

Keywords: Self-phase modulation; Generalization parameter; Pulse propagation; Complex transform.

1 Introduction

Today, the internet is one of the most indispensable elements in the lives of individuals and societies. When it comes to the Internet, naturally, social media, communication, data sharing/transmission, especially very large volumes of data transmission are the sub-headings associated with the Internet. Especially in the last 20 years, there have been huge and unpredictable developments in this field. All these situations brought the importance of fiber optic communication to the fore and studies in this field showed an even faster acceleration. Of course, some analytical, semi-analytical and soliton solutions developed for the solution of nonlinear partial differential equations (NLPDEs) in the early 2000s had a great impact on the basis of such advances. Advances in the software industry, developed symbolic algebraic computations software have played an important role in the development of many optical models [1-9] apart from many solution methods [10-15].

In this work, we consider (1+1)-dimensional Biswas-Milovic (BM) [16] equation consisting spatio-temporal [17] dispersion:

\[ i(M^n)t + \beta (M^n)_{xx} + \alpha (M^n)_{tt} + c |M|^2 M^n = 0, \]  

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and with parabolic law with nonlocal nonlinearity as:

\[ i(M^n)_t + \beta (M^n)_{xx} + \alpha (M^n)_x + \left( c_1 |M|^2 + c_2 |M|^4 + c_3 \left( |M|^2 \right)_x \right) M^n = 0, \tag{2} \]

where \( M = M(x,t) \) is the (1+1)-dimensional a complex valued function, the temporal evolution, group velocity dispersion, spatio-temporal dispersino and self-phase modulation terms are represented by from the first to the fourth terms, respectively. \( \alpha, \beta, c_i \) are real values and they are the coefficient to the relevant terms. The parameter \( n \) is also for generalization parameter BM equation to NLS equation and \( n \geq 1 \). The BM equation equation was presented to the literature as a generalization of the NLSE equation in 2010 [16] and has a very wide application area and therefore it has been in the working range of many researchers [18-29].

The paper are designed as follows: In section 2, NODE forms of eq.(1) and eq.(2) are derived. Section 3, introduces the utilized method which is the new Kudryashov and and its implementation. Section 4, covers the the results and some graphical presentations. Section is the final, Conclusion.

### 2 NODE representations of the eq.(1) and eq.(2)

The following transformation is utilized to gain the NODE structure of the eq.(1) and eq.(2):

\[ M(x,t) = M(\xi) e^{i\theta}, \quad \theta = x - vt, \quad \xi = -\kappa x + \omega t + \phi_0, \tag{3} \]

where \( M(\xi), \theta, \kappa, \omega, v, \phi_0 \) stand for the amplitude in real, phase component, frequency, wave number, velocity, phase constant, respectively. They are all real nonzero values to be determined.

#### 2.1 NODE representation of eq. (1)

Inserting the eq. (3) into eq. (1) we gain the following nonlinear ordinary equations which come from real and imaginary parts, respectively:

\[ n \left( (\kappa v + \omega) \alpha - 2\beta \kappa n - v \right) M^{n+1} M' = 0, \tag{4} \]

\[ M^4 c - n \left( (\beta \kappa^2 - \alpha \kappa \omega) n + \omega \right) M^2 - n (\alpha v - \beta) MM'' - n (n (\alpha v - \beta) (n - 1) (M')^2 = 0, \tag{5} \]

where \( M = M(\xi) \) and superscripts denote the normal derivative with respect to new variable \( \xi \). Equation (4) permits us to write:

\[ v = \frac{n (2\beta \kappa - \alpha \omega)}{\kappa \alpha n - 1}, \tag{6} \]

which is the velocity. Therefore, eq. (5) is the NODE representation of eq. (1).

#### 2.2 NODE representation of eq.(2)

Injecting the eq. (3) into eq. (2), we derive the following equations:

\[ n \left( (\kappa v + \omega) \alpha - 2\beta \kappa n - v \right) M^{n+1} M' = 0, \tag{7} \]

\[ (2c_3 M^2 - n (\alpha v - \beta) (n - 1)) (M')^2 + M^6 c_2 + M^4 c_1 - n \left( (\kappa^2 \beta - \alpha \kappa \omega) n + \omega \right) M^2 - n (\alpha v - \beta) - 2c_3 M^2) MM'' = 0, \tag{8} \]

where \( M = M(\xi) \) and superscripts denote the normal derivative with respect to \( \xi \) as stated before. Equation (7) gives the following constraint as eq. (6):

\[ v = \frac{n (2\beta \kappa - \alpha \omega)}{\kappa \alpha n - 1}, \tag{9} \]
Similarly, eq. (8) is the NODE representation of eq. (2).

3 The new Kudryashov integration algorithm and its utilization

In order to utilize the proposed nKM [11,12], first of all, the following form is considered as solution of eqs. (5), (8), respectively:

\[ M(\xi) = \sum_{i=0}^{p} A_i K^i(\xi), \]

(10)

with the restriction \( A_p \neq 0 \). In eq.(10), \( A_i \) are real values to be determined, \( p \) represents the positive integer balance number and \( K(\xi) \) is the solution of the following formula:

\[ \frac{dK(\xi)}{d\xi} = \sqrt{\delta^2 K^2(\xi)(1 - \lambda K^2(\xi))}, \]

(11)

where \( \delta, \lambda \) are arbitrary nonzero real numbers. Equation (11) gives the following well-known solution:

\[ K(\xi) = \frac{4L}{4L^2 e^{\delta^2} + \lambda e^{-\delta^2}}, \]

(12)

where, the parameters \( \delta, \lambda \), and \( L \) are arbitrary real constants. Equation (12) has the capability to generate bright and singular solution for \( \lambda = \mp 4L^2 \), respectively.

3.1 Utilization of the method to eq. (4)

In eq. (4), balancing the highest order linear and nonlinear terms \( M''', M^4 \) with the aid of homogeneous balance rule, we obtain \( p = 1 \). In the line of this calculation, eq.(10) is given as:

\[ M(\xi) = A_0 + A_1 K(\xi). \]

(13)

Inserting eq. (13) into eq. (4), by assuming the eq. (11), we have a polynomial form of \( K(\xi) \) which leads to following system:

\begin{align*}
\text{Coef. } K^0 & : n\omega (\alpha \kappa n - 1) - \beta \kappa^2 n^2 + cA_0^2 = 0, \\
\text{Coef. } K^1 & : (\alpha \omega - \beta \kappa ) (2\alpha \kappa^2 n^3 + \alpha n^2 \delta^2) + 4n\alpha \kappa (cA_0^2 - n\omega) + n\beta (2n\kappa^2 - \delta^2) - 4cA_0^2 + 2n\omega = 0, \\
\text{Coef. } K^2 & : -\alpha (\delta^2 + \kappa^2 ) (\beta \kappa - \alpha \omega)n^3 + (\beta \kappa^2 - \delta^2 - 2\alpha \kappa \omega)n^2 + (6\alpha \kappa A_0^2 + \omega)n - 6cA_0^2 = 0, \\
\text{Coef. } K^3 & : \delta^2 \lambda n (\alpha^2 n\omega - \beta - \alpha n \kappa \beta) + 2cA_0^2 (1 - \alpha \kappa n) = 0, \\
\text{Coef. } K^4 & : \alpha \delta^2 \lambda (\beta \kappa - \alpha \omega)n^3 + \delta^2 \lambda (\alpha \beta \kappa - \alpha^2 \omega + \beta)n^2 + (\alpha \kappa A_0^2 + \delta^2 \lambda)n - cA_0^2 = 0.
\end{align*}

(14)

Solution of the above system gives the following sets:

\[ \text{Set}_1^\delta : \delta = \delta, \omega = \frac{\alpha \kappa (\delta^2 + \kappa^2) n + \delta^2 - \kappa^2 - \beta \kappa (\delta^2 + \kappa^2)n^2 - 2\kappa \alpha n}{1 + \alpha^2 (\delta^2 + \kappa^2)n^2 - 2\kappa \alpha n}, A_0 = 0, A_1 = \mp \sqrt{\frac{4\lambda n(n+1)(1+\alpha^2(\delta^2 + \kappa^2)n^2 - 2\kappa \alpha n)\beta \delta}{c(1+\alpha^2(\delta^2 + \kappa^2)n^2 - 2\kappa \alpha n)}}. \]

(15)

Evaluation of eq. (15) with eqs. (3), (12), gives the following solution:

\[ M(x,t) = \frac{4L A_1}{4L^2 e^{\delta\left(x - \frac{n - \alpha \kappa - 2\kappa \alpha}{\kappa \alpha + 1}\right)} + \lambda e^{-\delta\left(x - \frac{n - \alpha \kappa - 2\kappa \alpha}{\kappa \alpha + 1}\right)}} e^{i(\kappa x + \omega t + \phi_0)}, \]

(16)

where \( A_1, \omega \) are presented in eq. (15).
3.2 Utilization of the method to eq. (8)

In eq. (8), balancing the highest order linear and nonlinear terms $M^3M'$, $M^6$ with the aid of homogeneous balance rule, we obtain $p = 1$. so, eq. (10) takes the following form:

$$M(\xi) = A_0 + A_1 K(\xi).$$

(17)

Inserting eq. (17) into eq. (8), by assuming the eq. (11), we gather a polynomial form of $K(\xi)$ which permits us to write the following system:

Coef. $K^0$ : $(\beta k^2 - \alpha k \omega) n^2 + n \omega - A_0^2 c_2 - A_0^3 c_1 = 0$,

Coef. $K^1$ : $2(-\alpha k \omega + \beta k^2) n^2 + ((\alpha \omega - \beta) \delta^2 + 2 \omega) n - 3A_0^4 c_2 - (\delta^2 c_3 + 2c_1) A_0^2 = 0$,

Coef. $K^2$ : $(\alpha \omega - \beta) \delta^2 + \beta k^2 - \alpha k \omega) n^2 + n \omega - A_0^6 c_4^2 A_0^2 + (15c_2 A_0^4 + 6c_1) = 0$,

Coef. $K^3$ : $(5 \delta c_3 + 10c_2 A_0^2 + 2c_1) A_1^2 + \lambda A_1 (-3c_2 A_0^2 + n(\alpha \omega - \beta)) \delta^2 = 0$,

Coef. $K^4$ : $((\alpha \omega - \beta) n^2 + n(\alpha \omega - \beta) - 14A_0^2 A_0^2 \lambda + 4c_3 A_0^4) \delta^2 + (15c_2 A_0^4 + c_1) A_1^2 = 0$,

Coef. $K^5$ : $-8\delta^2 \lambda c_3 + 3A_0^2 c_1 = 0$,

Coef. $K^6$ : $-6\delta^3 A_0^2 + 3A_0^2 c_1 = 0$.

Solution of the above system gives the following sets:

Set\n
$$\xi_\pm : \left\{ \begin{array}{l} \delta = \sqrt{-6n(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)} \cdot A_0 = 0, A_1 = \mp \sqrt{\frac{\lambda c_2 c_3 (\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)}{2c_2 c_3}} \\ \omega = -\frac{n(2c_1(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta))}{2c_2 c_3} \end{array} \right. \right\},$$

Set\n
$$\xi_\pm : \left\{ \begin{array}{l} \delta = \sqrt{-6n(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)} \cdot A_0 = 0, A_1 = \mp \sqrt{\frac{\lambda c_2 c_3 (\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)}{2c_2 c_3}} \\ \omega = -\frac{n(2c_1(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta)(\alpha \omega - \beta))}{2c_2 c_3} \end{array} \right. \right\},$$

(19)

In eqs. (18), (19), $v = \frac{n(\beta k - \alpha \omega)}{\kappa \alpha - \lambda}$ as given by eq. (9).

Evaluation of eq. (17) with eqs. (3), (12), gives the following solution:

$$M(x,t) = \frac{4A_1 L}{4L^2 e^{\delta(x-\nu)} + \lambda e^{-\delta(x-\nu)}} e^{(\kappa x + \omega t + \phi_0)},$$

(20)

where $A_1$, $\phi_0$ are presented in eq. (19) and $v$ is given by eq. (9).

4 Result and discussion

In this section, we demonstrate the some graphical simulations of the obtained functions which is given by eq. (16) and eq. (20). In fig.1, we portray the $M(x,t)$ in eq. (16) with the solution set $Set_\xi^+$ given in eq. (15). After the combination of eq. (16) and eq. (15), we selected the parameter values as $\delta = 1, L = 1, \lambda = 4, \kappa = 0.25, \beta = 2.5, \alpha = 1, c = 0.75, \phi_0 = 0.5, n = 2, t = 1, 2, 3$. Fig.1a-1c, represent the 3D of $|M(x,t)|$, $Re(M(x,t))$, $Im(M(x,t))$, fig.1d-1e belong to contour of $|M(x,t)|$, $Re(M(x,t))$ and fig.1e depicts the 2D of $|M(x,t)|$, $Re(M(x,t))$, $Im(M(x,t))$ when $t_\nu = 1, 2, 3$, together. Similarly, fig.2 simulated for the $M(x,t)$ in eq. (20) with the solution set $Set_\xi^+$ given in eq. (19). After the combination of eq. (20) and eq. (19), we selected the parameter values as $L = 1, \lambda = 4, \kappa = 0.25, \beta = 2.5, \alpha = 1, c_1 = c_2 = 1, c_3 = 0.75, \phi_0 = 0.5, n = 2, t = 1, 2, 3$. Fig.1a-1c, represent the 3D of $|M(x,t)|$, $Re(M(x,t))$, $Im(M(x,t))$, fig.1d-1e belong to contour of $|M(x,t)|$, $Re(M(x,t))$ and fig.1e depicts the 2D of $|M(x,t)|$, $Re(M(x,t))$, $Im(M(x,t))$ when $t_\nu = 1, 2, 3$, together.
Fig. 1: Bright soliton depiction of eq. (16) which belongs to BM equation which is consisting spatio-temporal dispersion with Kerr law nonlinearity given by eq. (1).
Fig. 2: Bright soliton depiction of eq. (20) which belongs to BM equation which is consisting spatio-temporal dispersion having parabolic law with nonlocal nonlinearity given by eq. (2).
5 Conclusion

This study was carried out to examine the existence of optical soliton solutions in the presence of spatio-temporal dispersion and chromatic dispersion terms of Kerr and parabolic law with nonlocal nonlinearity forms of the (1+1)-dimensional Biswas-Milovic equation, which was introduced to the literature in 2010 as a general form of the NLS equation. As a method, the new Kudryashov technique with an recent and effective integration algorithm was applied and bright optical soliton solutions of both forms were obtained. 3D, contour and 2D graphic images were drawn to reflect the different physical properties of the solutions obtained. In addition to the fact that the study will contribute to the range of those working in this field, it is also possible to study the forms of the studied model with fractional and perturbation terms.

Competing interests

The authors declare that they have no competing interests.

Authors’ contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

References


