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On optical solitons of the (1+1)-dimensional Biswas-Milovic equation having Kerr law and parabolic-law with weak non-local nonlinearity in the presence of spatio-temporal dispersion

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Abstract: In this study, an effort has been made to obtain optical soliton solutions of the (1+1)-dimensional Biswas-Milovic equation which was introduced by Biswas and Milovic in 2010, having Kerr law and parabolic-law with weak non-local nonlinearity in the presence of spatio-temporal dispersion, which is one of the important models for nonlinear optics. Although, the model is an equation that has been studied by many researchers, the fact that the form to be examined has not been studied before. As a general algorithm, first converting the model to nonlinear ordinary differential form with a complex wave transformation, then obtaining candidate optical soliton solutions by utilizing the new Kudryashov technique, determining the ones that satisfy the main equation from these solutions as the exact solution, and in order to better understand the obtained solutions by making graphical presentation and providing the necessary comments constitute the main framework of the article.

Keywords: Self-phase modulation; Generalization parameter; Pulse propagation; Complex transform.

1 Introduction

Today, the internet is one of the most indispensable elements in the lives of individuals and societies. When it comes to the Internet, naturally, social media, communication, data sharing/transmission, especially very large volumes of data transmission are the sub-headings associated with the Internet. Especially in the last 20 years, there have been huge and unpredictable developments in this field. All these situations brought the importance of fiber optic communication to the fore more and studies in this field showed an even faster acceleration. Of course, some analytical, semi-analytical and soliton solutions developed for the solution of nonlinear partial differential equations (NLPDEs) in the early 2000s had a great impact on the basis of such advances. Advances in the software industry, developed symbolic algebraic computations software have played an important role in the development of many optical models [1-9] apart from many solution methods [10-15].

In this work, we consider (1+1)-dimensional Biswas-Milovic (BM) [16] equation consisting spatio-temporal [17] dispersion:
with Kerr law nonlinearity as:

$$i(M^n)_t+\beta\,(M^n)_{xx}+\alpha\,(M^n)_{xt}+c\,|M|^2M^n=0, \tag{1}$$

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and with parabolic law with nonlocal nonlinearity as:

$$i(M^n)_t + \beta\,(M^n)_{xx} + \alpha\,(M^n)_{xt} + \left(c_1\,|M|^2 + c_2\,|M|^4 + c_3\left(|M|^2\right)_{xx}\right)M^n = 0, \tag{2}$$

where $M = M(x,t)$ is the (1+1)-dimensional a complex valued function, the temporal evolution, group velocity dispersion, spatio-temporal dispersino and self-phase modulation terms are represented by from the first to the fourth terms, respectively. α, β, c_i are real values and they are the coefficient to the relevant terms. The parameter n is also for generalization parameter BM equation to NLS equation and $n \geq 1$. The BM equation equation was presented to the literature as a generalization of the NLSE equation in 2010 [16] and has a very wide application area and therefore it has been in the working range of many researchers [18-29].

The paper are designed as follows: In section 2, NODE forms of eq.(1) and eq.(2) are derived. Section 3, introduces the utilized method which is the new Kudryashov and and its implementation. Section 4, covers the the results and some graphical presentations. Section is the final, Conclusion.

2 NODE representations of the eq.(1) and eq.(2)

The following transformation is utilized to gain the NODE structure of the eq.(1) and eq.(2):

$$M(x,t) = M(\xi)\,e^{i\theta},\ \theta = x - vt,\ \xi = -\kappa x + \omega t + \varphi_0, \tag{3}$$

where $M(\xi)$, θ, κ, ω, v, φ_0 stand for the amplitude in real, phase component, frequency, wave number, velocity, phase constant, respectively. They are all real nonzero values to be determined.

2.1 NODE representation of eq. (1)

Inserting the eq. (3) into eq. (1) we gain the following nonlinear ordinary equations which come from real and imaginary parts, respectively:

$$n\,(((\kappa v + \omega)\,\alpha - 2\beta\kappa)\,n - v)\,M^{n+1}M' = 0, \tag{4}$$

$$M^4 c - n\left(\left(\beta\kappa^2 - \alpha\kappa\omega\right)n + \omega\right)M^2 - n\,(\alpha v - \beta)\,MM'' - n\,(\alpha v - \beta)\,(n-1)\,\left(M'\right)^2 = 0, \tag{5}$$

where $M = M(\xi)$ and superscripts denote the normal derivative with respect to new variable ξ. Equation (4) permits us to write:

$$v = \frac{n\,(2\beta\kappa - \alpha\omega)}{\kappa\alpha n - 1}, \tag{6}$$

which is the velocity. Therefore, eq. (5) is the NODE representation of eq. (1).

2.2 NODE representation of eq.(2)

Injecting the eq. (3) into eq. (2), we derive the following equations:

$$n\,(((\kappa v + \omega)\,\alpha - 2\beta\kappa)\,n - v)\,M^{n+1}M' = 0, \tag{7}$$

$$\begin{aligned}&\left(2c_3M^2 - n\,(\alpha v - \beta)\,(n-1)\right)(M')^2 + M^6c_2 + M^4c_1\\ &-n\left(\left(\kappa^2\beta - \kappa\alpha\omega\right)n + \omega\right)M^2 - \left(n\,(\alpha v - \beta) - 2c_3M^2\right)MM'' = 0,\end{aligned} \tag{8}$$

where $M = M(\xi)$ and superscripts denote the normal derivative with respect toξ as stated before. Equation (7) gives the following constraint as eq. (6):

$$v = \frac{n\,(2\beta\kappa - \alpha\omega)}{\kappa\alpha n - 1}. \tag{9}$$

Similarly, eq. (8) is the NODE representation of eq. (2).

3 The new Kudryashov integration algorithm and its utilization

In order to utilize the proposed nKM [11,12], first of all, the following form is considered as solution of eqs. (5), (8), respectively:

$$M(\xi)=\sum_{i=0}^{p}A_iK^i(\xi), \tag{10}$$

with the restriction $A_p\neq 0$. In eq.(10), A_i are real values to be determined, p represents the positive integer balance number and $K(\xi)$ is the solution of the following formula:

$$\frac{dK(\xi)}{d\xi}=\sqrt{\delta^2K^2(\xi)\left(1-\lambda K^2(\xi)\right)}, \tag{11}$$

where δ,λ are arbitrary nonzero real numbers. Equation (11) gives the following well-known solution:

$$K(\xi)=\frac{4L}{4L^2e^{\delta\xi}+\lambda e^{-\delta\xi}}, \tag{12}$$

where, the parameters δ,λ and L are arbitrary real constants. Equation (12) has the capable to generate bright and singular solution for $\lambda=\mp 4L^2$,respectively.

3.1 *Utilization of the method to eq. (4)*

In eq. (4), balancing the highest order linear and nonlinear terms MM'', M^4 with the aid of homogeneous balance rule, we obtain $p=1$. In the line of this calculation, eq.(10) is given as:

$$M(\xi)=A_0+A_1K(\xi). \tag{13}$$

Inserting eq. (13) into eq. (4), by assuming the eq. (11), we have a polynomial form of $K(\xi)$ which leads to following system:

$$\begin{aligned}
&Coef.K^0: n\omega\left(\alpha\kappa n-1\right)-\beta\kappa^2n^2+cA_0^2=0,\\
&Coef.K^1: \left(\alpha\omega-\beta\kappa\right)\left(2\alpha\kappa^2n^3+\alpha n^2\delta^2\right)+4n\alpha\kappa\left(cA_0^2-n\omega\right)+n\beta\left(2n\kappa^2-\delta^2\right)-4cA_0^2+2n\omega=0,\\
&Coef.K^2: -\alpha\left(\delta^2+\kappa^2\right)\left(\beta\kappa-\alpha\omega\right)n^3+\left(\beta\kappa^2-\beta\delta^2-2\alpha\kappa\omega\right)n^2+\left(6c\alpha\kappa A_0^2+\omega\right)n-6cA_0^2=0,\\
&Coef.K^3: \delta^2\lambda n\left(\alpha^2n\omega-\beta-\alpha n\kappa\beta\right)+2cA_1^2\left(1-\alpha\kappa n\right)=0,\\
&Coef.K^4: \alpha\delta^2\lambda\left(\beta\kappa-\alpha\omega\right)n^3+\delta^2\lambda\left(\alpha\beta\kappa-\alpha^2\omega+\beta\right)n^2+\left(\alpha c\kappa A_1^2+\beta\delta^2\lambda\right)n-cA_1^2=0.
\end{aligned} \tag{14}$$

Solution of the above system gives the following sets:

$$Set_1^{\mp}:\left\{\delta=\delta,\omega=\frac{\left(\alpha\kappa\left(\delta^2+\kappa^2\right)n+\delta^2-\kappa^2\right)\beta n}{1+\alpha^2\left(\delta^2+\kappa^2\right)n^2-2\kappa\alpha n},A_0=0,A_1=\mp\frac{\sqrt{\lambda n\left(n+1\right)c\left(1+\alpha^2\left(\delta^2+\kappa^2\right)n^2-2\kappa\alpha n\right)\beta\delta}}{c\left(1+\alpha^2\left(\delta^2+\kappa^2\right)n^2-2\kappa\alpha n\right)}\right\}, \tag{15}$$

Evaluation of eq. (15) with eqs. (3), (12), gives the following solution:

$$M(x,t)=\frac{4LA_1}{4L^2\mathrm{e}^{\delta\left(x-\frac{n(-\alpha\omega+2\beta\kappa)}{\kappa\alpha n-1}t\right)}+\lambda\mathrm{e}^{-\delta\left(x-\frac{n(-\alpha\omega+2\beta\kappa)}{\kappa\alpha n-1}t\right)}}\mathrm{e}^{i(-\kappa x+\omega t+\phi_0)}, \tag{16}$$

where A_1,ω are presented in eq. (15).

3.2 Utilization of the method to eq. (8)

In eq. (8), balancing the highest order linear and nonlinear terms M^3M'', M^6 with the aid of homogeneous balance rule, we obtain $p=1$. so, eq. (10) takes the following form:

$$M(\xi)=A_0+A_1K(\xi). \tag{17}$$

Inserting eq. (17) into eq. (8), by assuming the eq. (11), we gather a polynomial form of $K(\xi)$ which permits us to write the following system:

$$\begin{aligned}
&Coef.K^0:\left(\beta\kappa^2-\alpha\kappa\omega\right)n^2+n\omega-A_0^4c_2-A_0^2c_1=0,\\
&Coef.K^1:2\left(-\alpha\kappa\omega+\beta\kappa^2\right)n^2+\left(\left(v\alpha-\beta\right)\delta^2+2\omega\right)n-3A_0^4c_2-\left(\delta^2c_3+2c_1\right)A_0^2=0,\\
&Coef.K^2:\left(\left(v\alpha-\beta\right)\delta^2+\beta\kappa^2-\alpha\kappa\omega\right)n^2+n\omega-A_0^2A_1^2\left(15c_2A_0^2+8\delta^2c_3+6c_1\right)=0,\\
&Coef.K^3:\left(5\delta^2c_3+10c_2A_0^2+2c_1\right)A_1^2+\lambda A_1\left(-2c_3A_0^2+n\left(v\alpha-\beta\right)\right)\delta^2=0,\\
&Coef.K^4:\left(\left(\left(v\alpha-\beta\right)n^2+n\left(v\alpha-\beta\right)-14c_3A_0^2\right)\lambda+4c_3A_1^2\right)\delta^2+\left(15c_2A_0^2+c_1\right)A_1^2=0,\\
&Coef.K^5:-8\delta^2\lambda c_3+3A_1^2c_2=0,\\
&Coef.K^6:-6\delta^2\lambda c_3+A_1^2c_2=0.
\end{aligned} \tag{18}$$

Solution of the above system gives the following sets:

$$\begin{aligned}
&Set_2^{\mp}:\left\{\begin{array}{l}\left\{\delta=\frac{\sqrt{-6n(v\alpha-\beta)(n+1)c_2-36c_1c_3}}{12c_3},A_0=0,A_1=\mp\frac{\sqrt{-\lambda c_2c_3(n(v\alpha-\beta)(n+1)c_2+6c_1c_3)}}{2c_2c_3}\right\},\\ \omega=-\frac{n\left(c_2(v\alpha-\beta)^2n^2+c_2(v\alpha-\beta)^2n+6c_3\left(v\alpha c_1-4\left(\kappa^2c_3+\frac{c_1}{4}\right)\beta\right)\right)}{24c_3^2(\alpha\kappa n-1)}\end{array}\right\},\\
&Set_3^{\mp}:\left\{\begin{array}{l}\left\{\delta=-\frac{\sqrt{-6n(v\alpha-\beta)(n+1)c_2-36c_1c_3}}{12c_3},A_0=0,A_1=\mp\frac{\sqrt{-\lambda c_2c_3(n(v\alpha-\beta)(n+1)c_2+6c_1c_3)}}{2c_2c_3}\right\},\\ \omega=-\frac{n\left(c_2(v\alpha-\beta)^2n^2+c_2(v\alpha-\beta)^2n+6c_3\left(v\alpha c_1-4\left(\kappa^2c_3+\frac{c_1}{4}\right)\beta\right)\right)}{24c_3^2(\alpha\kappa n-1)}\end{array}\right\},
\end{aligned} \tag{19}$$

In eqs. (18), (19), $v=\frac{n(2\beta\kappa-\alpha\omega)}{\kappa\alpha n-1}$ as given by eq. (9).

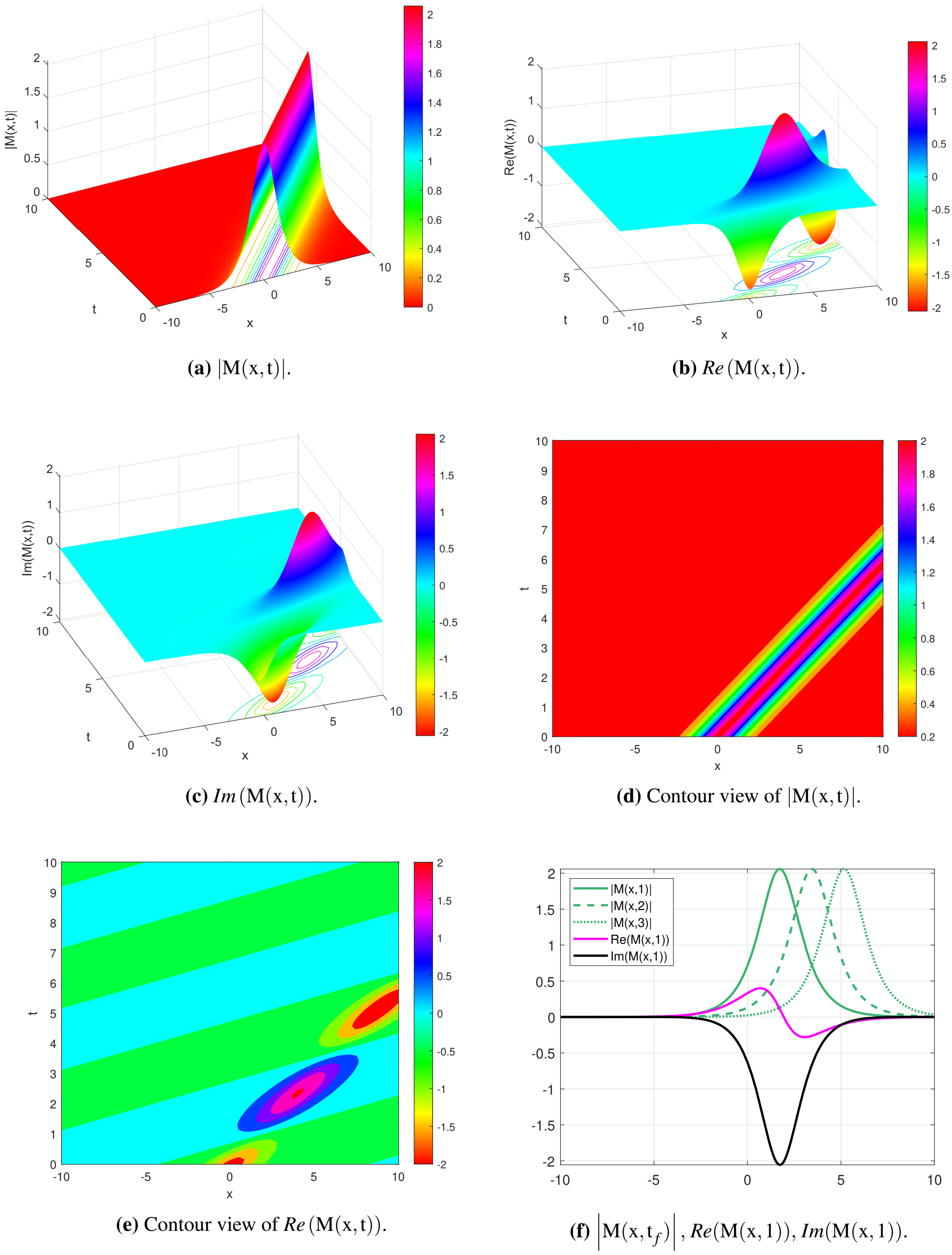
Evaluation of eq. (17) with eqs. (3), (12), gives the following solution:

$$M(x,t)=\frac{4A_1L}{4L^2e^{\delta(x-vt)}+\lambda e^{-\delta(x-vt)}}e^{i(-\kappa x+\omega t+\phi_0)}, \tag{20}$$

where $A_1,\omega$are presented in eq. (19) and v is given by eq. (9).

4 Result and discussion

In this section, we demonstrate the some graphical simulations of the obtained functions which is given by eq. (16) and eq. (20). In fig.1, we portrait the M(x,t)in eq. (16) with the solution set Set_1^+given in eq. (15). After the combination of eq. (16) and eq. (15), we selected the parameter values as $\delta=1,L=1,\ \lambda=4,\ \kappa=0.25,\ \beta=2.25,\ \alpha=1,\ c=0.75,\phi_0=0.5,\ n=2,\ t=1,2,3$. Fig.1a-1c, represent the 3D of $|\mathrm{M(x,t)}|$, $Re(\mathrm{M(x,t)})$, $Im(\mathrm{M(x,t)})$,fig.1d-1e belong to contour of $|\mathrm{M(x,t)}|$, $Re(\mathrm{M(x,t)})$and fig.1e depicts the 2D of $\left|\mathrm{M(x,t_{\mathit{f}})}\right|$, $Re(\mathrm{M(x,1)})$, $Im(\mathrm{M(x,1)})$ when $t_f=1,2,3$,together. Similarly, fig.2 simulated for the M(x,t)in eq. (20) with the solution set Set_2^+given in eq. (19). After the combination of eq. (20) and eq. (19), we selected the parameter values as $L=1,\ \lambda=4,\ \kappa=0.25,\ \beta=2.25,\ \alpha=1,\ c_1=c_2=1,\ c_3=0.75,\ \phi_0=0.5,\ n=2,\ t=1,2,3$. Fig.1a-1c, represent the 3D of $|\mathrm{M(x,t)}|$, $Re(\mathrm{M(x,t)})$, $Im(\mathrm{M(x,t)})$,fig.1d-1e belong to contour of $|\mathrm{M(x,t)}|$, $Re(\mathrm{M(x,t)})$and fig.1e depicts the 2D of $\left|\mathrm{M(x,t_{\mathit{f}})}\right|$, $Re(\mathrm{M(x,1)})$, $Im(\mathrm{M(x,1)})$ when $t_f=1,2,3$,together.



(a) $|\mathrm{M}(\mathrm{x},\mathrm{t})|$.

(b) $Re\,(\mathrm{M}(\mathrm{x},\mathrm{t}))$.

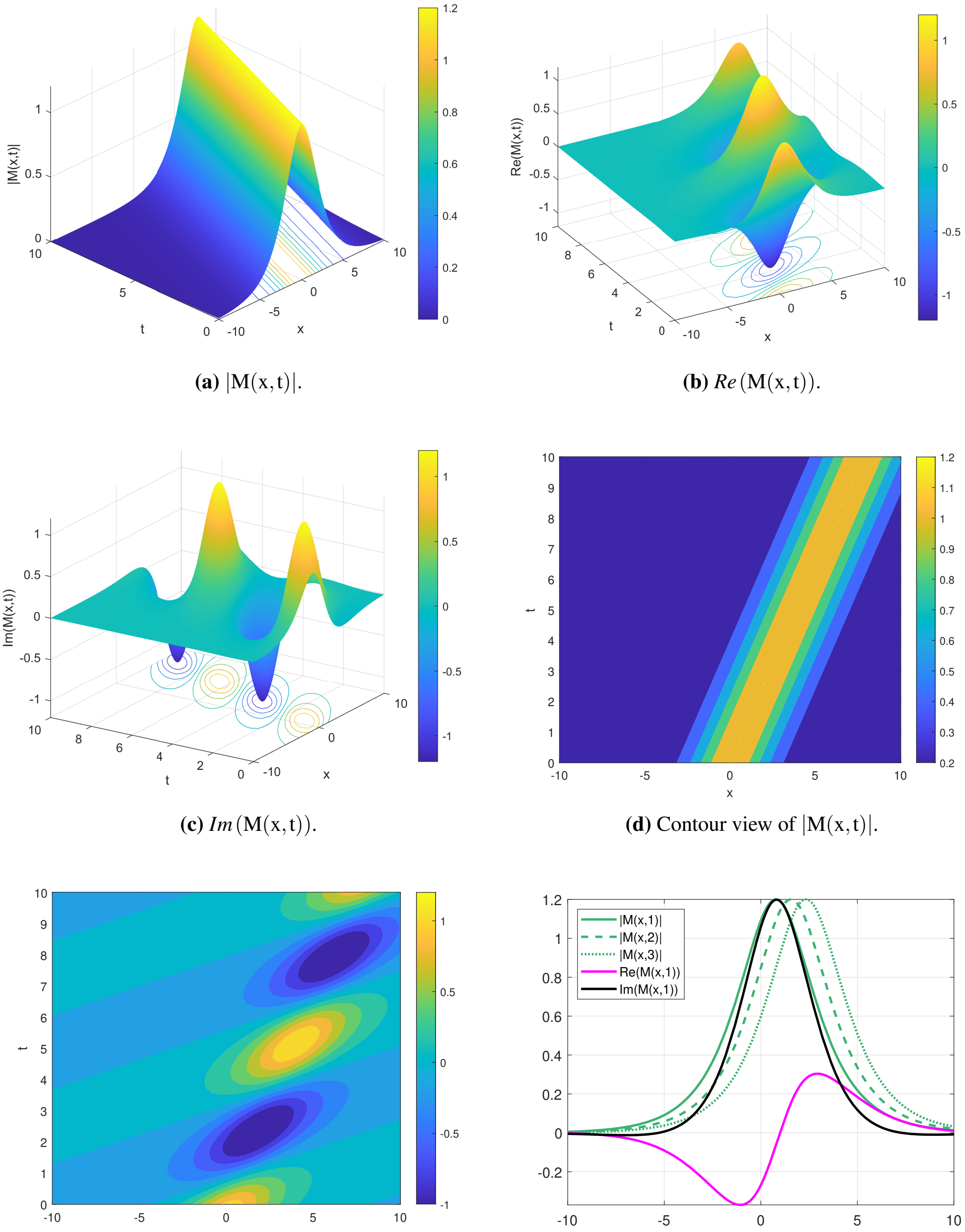
(c) $Im\,(\mathrm{M}(\mathrm{x},\mathrm{t}))$.

(d) Contour view of $|\mathrm{M}(\mathrm{x},\mathrm{t})|$.

(e) Contour view of $Re\,(\mathrm{M}(\mathrm{x},\mathrm{t}))$.

(f) $\left|\mathrm{M}(\mathrm{x},\mathrm{t}_f)\right|$, $Re(\mathrm{M}(\mathrm{x},1))$, $Im(\mathrm{M}(\mathrm{x},1))$.

Fig. 1: Bright soliton depiction of eq. (16) which belongs to BM equation which is consisting spatio-temporal dispersion with Kerr law nonlinearity given by eq. (1).



(a) $|\mathrm{M}(\mathrm{x},\mathrm{t})|$.

(b) $Re\,(\mathrm{M}(\mathrm{x},\mathrm{t}))$.

(c) $Im\,(\mathrm{M}(\mathrm{x},\mathrm{t}))$.

(d) Contour view of $|\mathrm{M}(\mathrm{x},\mathrm{t})|$.

(e) Contour view of $Re\,(\mathrm{M}(\mathrm{x},\mathrm{t}))$.

(f) $\left|\mathrm{M}(\mathrm{x},\mathrm{t}_f)\right|, Re(\mathrm{M}(\mathrm{x},1)), Im(\mathrm{M}(\mathrm{x},1))$.

Fig. 2: Bright soliton depiction of eq. (20) which belongs to BM equation which is consisting spatio-temporal dispersion having parabolic law with nonlocal nonlinearity given by eq. (2).

5 Conclusion

This study was carried out to examine the existence of optical soliton solutions in the presence of spatio-temporal dispersion and chromatic dispersion terms of Kerr and parabolic law with nonlocal nonlinearity forms of the (1+1)-dimensional Biswas-Milovic equation, which was introduced to the literature in 2010 as a general form of the NLS equation. As a method, the new Kudryashov technique with an recent and effective integration algorithm was applied and bright optical soliton solutions of both forms were obtained. 3D, contour and 2D graphic images were drawn to reflect the different physical properties of the solutions obtained. In addition to the fact that the study will contribute to the range of those working in this field, it is also possible to study the forms of the studied model with fractional and perturbation terms.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

References

[1] Adem, A., Ntsime Basetsana, Biswas A., Khan Sabrin, Alzahrani A.K., Belić, Milivoj. (2021). Stationary optical solitons with nonlinear chromatic dispersion for Lakshmanan-Porsezian-Daniel model having Kerr law of nonlinear refractive index. Ukrainian Journal of Physical Optics. 22. 83-86. 10.3116/16091833/22/2/83/2021.

[2] Yildirim Yakup, Biswas A., Guggilla P., Khan Sabrin, Alshehri H.M., Belić, Milivoj. (2021). Optical solitons in fibre Bragg gratings with third- and fourth-order dispersive reflectivities. Ukrainian Journal of Physical Optics. 22. 239-254. 10.3116/16091833/22/4/239/2021.

[3] Arnous Ahmed, Nofal Taher, Biswas Anjan, Khan Salam, Moraru Luminita. (2022). Quiescent Optical Solitons with Kudryashov's Generalized Quintuple-Power and Nonlocal Nonlinearity Having Nonlinear Chromatic Dispersion. Universe. 8. 501. 10.3390/universe8100501.

[4] Nikolay A. Kudryashov, Optical solitons of the Schrödinger–Hirota equation of the fourth order, Optik, Volume 274, 2023, 170587, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2023.170587.

[5] Nikolay A. Kudryashov, Dispersive optical solitons of the generalized Schrödinger–Hirota model, Optik, Volume 272, 2023, 170365, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.170365.

[6] Kudryashov, N.A. Embedded Solitons of the Generalized Nonlinear Schrödinger Equation with High Dispersion. Regul. Chaot. Dyn. 27, 680–696 (2022). https://doi.org/10.1134/S1560354722060065.

[7] Nikolay A. Kudryashov, Solitary waves of the generalized Radhakrishnan-Kundu-Lakshmanan equation with four powers of nonlinearity, Physics Letters A, Volume 448, 2022, 128327, ISSN 0375-9601, https://doi.org/10.1016/j.physleta.2022.128327.

[8] Nikolay A. Kudryashov, Exact solutions of the complex Ginzburg–Landau equation with law of four powers of nonlinearity, Optik, Volume 265, 2022, 169548, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.169548.

[9] Nikolay A. Kudryashov, Governed optical solitons of the generalized Schrödinger equation with dual-power law of refractive index, Optik, Volume 266, 2022, 169619, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.169619.

[10] Nikolay A. Kudryashov, Method for finding optical solitons of generalized nonlinear Schrödinger equations, Optik, Volume 261, 2022, 169163, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.169163.

[11] Nikolay A. Kudryashov, Method for finding highly dispersive optical solitons of nonlinear differential equations, Optik, Volume 206, 2020, 163550, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2019.163550.

[12] Muslum Ozisik, Aydin Secer, Mustafa Bayram, Huseyin Aydin, An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices, Optik, Volume 265, 2022, 169499, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.169499.

[13] Gonzalez-Gaxiola Oswaldo, Biswas Anjan, Moraru Luminita, Alghamdi Abdulah. (2023). Solitons in Neurosciences by the Laplace–Adomian Decomposition Scheme. Mathematics. 11. 1080. 10.3390/math11051080.

[14] Naowarat S., Saifullah S., Ahmad S., De la Sen M. Periodic, Singular and Dark Solitons of a Generalized Geophysical KdV Equation by Using the Tanh-Coth Method. Symmetry. 2023; 15(1):135. https://doi.org/10.3390/sym15010135.

[15] Ahmad, S., Hameed, A., Ahmad, S. et al. Stability analysis and some exact solutions of a particular equation from a family of a nonlinear Schrödinger equation with unrestricted dispersion and polynomial nonlinearity. Opt Quant Electron 55, 666 (2023). https://doi.org/10.1007/s11082-023-04932-4.

[16] A. Biswas, D. Milovic, Bright and dark solitons of the generalized nonlinear SchrÂ¨odinger's equation, Communications in Nonlinear Science and Numerical Simulation, 15(6), (2010), 1473-1484, https://doi.org/10.1016/j.cnsns.2009.06.017.

[17] Tahir, M., Awan, A.U. Analytical solitons with the Biswas-Milovic equation in the presence of spatio-temporal dispersion in non-Kerr law media. Eur. Phys. J. Plus 134, 464 (2019). https://doi.org/10.1140/epjp/i2019-12887-3.

[18] Muslum Ozisik, Aydin Secer, Mustafa Bayram, Discovering optical soliton solutions in the Biswas-Milovic equation through five innovative approaches, May 2023, Optik - International Journal for Light and Electron Optics 286(**??**):170986, doi: 10.1016/j.ijleo.2023.170986.

[19] Kudryashov, Nikolay. First integrals and general solutions of the Biswas-Milovic equation. Optik. 210. 164490. (2020) 10.1016/j.ijleo.2020.164490.

[20] Elsayed M.E. Zayed, Reham M.A. Shohib, Mohamed E.M. Alngar, Taher A. Nofal, Khaled A. Gepreel, Yakup Yıldırım, Cubic–quartic optical solitons of perturbed Biswas–Milovic equation having Kudryashov's nonlinear form and two generalized non-local laws, Optik, Volume 259, 2022, 168919, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.168919.

[21] Elsayed M.E. Zayed, Khaled A. Gepreel, Reham M.A. Shohib, Mohamed E.M. Alngar, Yakup Yıldırım, Optical solitons for the perturbed Biswas-Milovic equation with Kudryashov's law of refractive index by the unified auxiliary equation method, Optik, Volume 230, 2021, 166286, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2021.166286.

[22] Elsayed M.E. Zayed, Reham M.A. Shohib, Khaled A. Gepreel, Mahmoud M. El-Horbaty, Mohamed E.M. Alngar, Cubic–quartic optical soliton perturbation Biswas–Milovic equation with Kudryashov's law of refractive index using two integration methods, Optik, Volume 239, 2021, 166871, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2021.166871.

[23] Muslum Ozisik, Novel (2+1) and (3+1) forms of the Biswas–Milovic equation and optical soliton solutions via two efficient techniques, Optik, Volume 269, 2022, 169798, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.169798.

[24] Nivan M. Elsonbaty, Niveen M. Badra, Hamdy M. Ahmed, Ahmed M. Elsherbeny, Derivation of new optical solitons for Biswas-Milovic equation with dual-power law nonlinearity using improved modified extended tanh-function method, Alexandria Engineering Journal, Volume 67, 2023, Pages 537-546, ISSN 1110-0168, https://doi.org/10.1016/j.aej.2022.12.068.

[25] Selvi Altun, Muslum Ozisik, Aydin Secer, Mustafa Bayram, Optical solitons for Biswas–Milovic equation using the new Kudryashov's scheme, Optik, Volume 270, 2022, 170045, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2022.170045.

[26] Nauman Raza, Muhammad Abdullah, Asma Rashid Butt, Analytical soliton solutions of Biswas–Milovic equation in Kerr and non-Kerr law media, Optik, Volume 157, 2018, Pages 993-1002, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2017.11.043.

[27] Pinar Albayrak, Optical solitons of Biswas–Milovic model having spatio-temporal dispersion and parabolic law via a couple of Kudryashov's schemes, Optik, Volume 279, 2023, 170761, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2023.170761.

[28] Mustafa Inc, Aliyu Isa Aliyu, Abdullahi Yusuf, Dumitru Baleanu, Optical solitons for Biswas-Milovic Model in nonlinear optics by Sine-Gordon equation method, Optik, Volume 157, 2018, Pages 267-274, ISSN 0030-4026, https://doi.org/10.1016/j.ijleo.2017.11.061.

[29] Lanre Akinyemi, Mohammad Mirzazadeh, Seyed Amin Badri, Kamyar Hosseini. Dynamical solitons for the perturbated Biswas–Milovic equation with Kudryashov's law of refractive index using the first integral method, Journal of Modern Optics, 69:3, 172-182, (2022) doi: 10.1080/09500340.2021.2012286.