# Online International Symposium on Applied Mathematics and Engineering ($\mathcal{ISAME}22$)

January 21-23, 2022
Istanbul-Turkey

# Abstracts Book

Editors
Prof. Dr. Mustafa Bayram
Prof. Dr. Aydın Seçer

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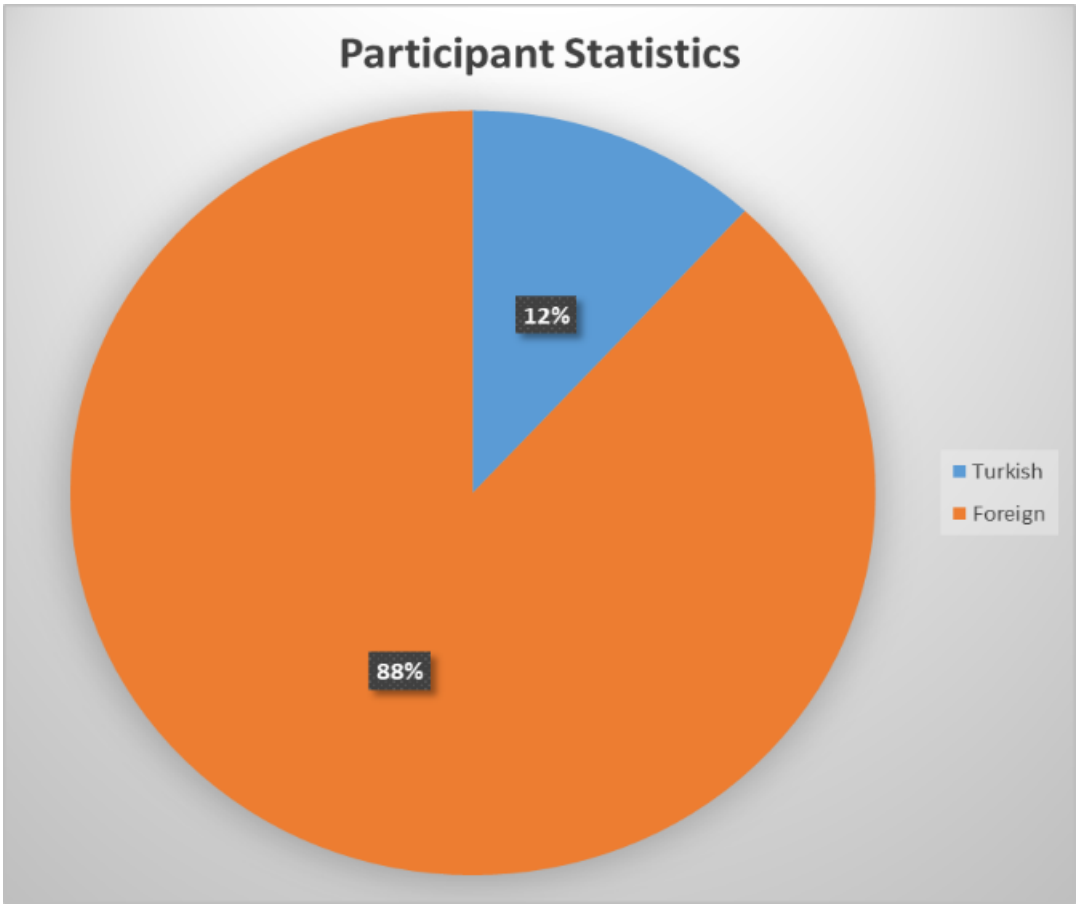
# Participant Statistics

237 participants from 35 different countries attended the conference, 28 of them from Turkey and the others from abroad, so 88% participants are foreigners and 12% participants are Turkish.



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# MESSAGE FROM CHAIRMAN

The "Online International Symposium on Applied Mathematics and Engineering, 2022" organized by Biruni University will be held on 21-23 January 2022 in Istanbul, Turkey. Due to the Covid-19 Pandemic, we could not meet face to face. For this reason, we decided to make it online by technology. The aim of this symposium is to bring the Mathematics & Engineering Sciences community working in the new trends of applications of Mathematics together in a wonderful city of the world, Istanbul.

There have been quite a big number of applications from different part of the world and as you know when the number increase task of the organizing committee will increase. Thus it was a very difficult task to select and classify the abstracts for all the participants. We tried to do our best to accommodate many speakers in order to have a better and enjoyable research session which will provide more interactions, exchanges among the participants.

Besides the scientific program, we had some social activities (excursion boat trip, city tour, etc.) where we could continue some informal discussions that would serve the purpose of our meeting in such a short time. We had to cancel due to the pandemic. As we can see from the list of participants, many speeches by young researchers will also serve the purpose of this symposium.

The talks will cover a wide range of mathematics and its applications such as analysis, algebra, statistics, computer mathematics, discrete mathematics, geometry, engineering, etc. as well as their use in modeling. We believe that this richness will provide the basis for interdisciplinary collaborations.

We also would very much thank to all presenters and participants for their interests in the symposium and believe and hope that each of them will get the maximum benefit in terms of networking and interaction from this meeting.

We would like to thank Dumitru Baleanu, Aydin Secer, Tuğçem Partal, Neslihan Ozdemir, Melih Cinar, Handenur Esen and Ismail Onder all our colleagues who worked for the organization of the symposium.

Finally, we also would to thank to chairman of the board of trustees of Biruni University and Prof. Dr. Adnan Yüksel the Rector of Biruni University which is Host University.

Further we thank to all the plenary speakers that kindly accepted our invitation and spend their precious time by sharing their ideas during the symposium. We also thank to all members of organizing committee.

We apologize for any shortcomings or might not be mentioned unintentionally or may have been forgotten to be mentioned explicitly here. We really hope their kind understanding, we thank all and each individual that have put their effort to make this occasion possible.

We welcome each and every one of you again to this symposium; we wish a enjoyable and productive symposium and hope to meet again in future occasions.

Sincerely Yours,
Prof. Dr. Mustafa Bayram,
Conference Chair

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.

# ORAL PRESENTATIONS

# Improving Prediction and classification of Water Quality Indices using Hybrid Machine learning Algorithms with features selection analysis

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**Abstract:** The assessment of surface water quality is a major environmental concern and one of the most important tasks in ensuring safe drinking water sources. The Water Quality Index (WQI) describes a number of water quality variables at a certain location environment and time. WQI computation takes time and is frequently affected by errors when subindex calculations are performed. Thus, it is highly necessary to provide an accurate WQI prediction model. Different input combinations were developed using the best dataset, and the work strategy was to demonstrate water quality variation where all inputs have been reduced using features selection analysis like as: principal component analysis (PCA) and self-organizing feature map (SOFM). Two machine learning methods have been applied in the current research: ANN and SVM models to investigate and try to emulate WQI's relationship with water quality variables in Tilesdit dam in Bouira (Algeria). Moreover, a comprehensive analysis has been performed for the performance assessment and sensitivity analysis of the variables. The models were appraised using several performance metrics. With high performance accuracy in two used models, the results achieved are promising. The proposed approach also provides an efficient alternative to calculate and predict the WQI by including long computing methods, transformations, the use of various subindex formulas for every value of the water quality component variables and time consumption.

**Keywords:** Water quality assessment, water quality index (WQI), features selection, PCA, SOFM, Machine learning, ANN, SVM, Algeria.

**Mathematics Subject Classification:** MSC2020 database: 68Uxx

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# Finite time blow-up of solutions for certain fifth order evolution equation

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**Abstract:** This paper studies the Cauchy problem of the fifth -order Boussinesq equation with a damping term. The local existence of weak solution of our problem is studied, then under suitable conditions on the initial, the global existence and finite time blow-up of weak solution, also are obtained by using Concavity method.

**Keywords:** Generalized Boussinesq equation, blow-up, weak solution, smooth solution, viscoelastic damping, local and global solution.

**Mathematics Subject Classification:** 47D09, 34G10, 34G20

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# On the estimate solution of fractional stochastic differential equation

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**Abstract:** This talk is devoted to studying a system which is governed by a nonlinear backward stochastic differential equation driven by fractional Brownian motion, in the case where the set of the control domain is convex. We give some results about the estimate of the solutions and the linearization of the equation of state and which will help us next.

**Keywords:** stochastic differential equation, fractional Brownian motion, control domain

**Mathematics Subject Classification:** 35F21, 60G22, 60H07

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# On some fixed point results for multivalued contractions with an application

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**Abstract:** In this paper, we prove the existence of multivalued fixed point, by combining the contractions of Geraghty type with $\theta$-contraction and $\alpha,\eta$-admissible concepts. Some consequences are given in metric spaces endowed with partial order or with graph, also we provide an example and an application to the existence of solutions of a boundary value problem for fractional differential inclusions to demonstrate the usability of our outcomes.

**Keywords:** $\theta$-contraction, $\alpha,\eta$-admissible, fractional differential inclusion.

**Mathematics Subject Classification:**54H25, 47H10.

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# Making hard fractional problems easy: on series and conjugation

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**Abstract:** Many different fractional derivative and integral operators are being introduced nowadays, but mostly they have one of the same few basic structures: for example, convolution integrals with a special function in the kernel, or operators with some weight function multiplied and divided. By understanding the structure of these fractional operators and how they connect with the classical Riemann-Liouville operators, we show that many results about new operators can be proved easily by harnessing their connection with classical operators. We illustrate this work by providing many examples of such easy proofs using connections such as conjugation relations and series formulae.

**Keywords:** fractional calculus; general analytic kernels; fractional calculus with respect to functions; weighted fractional calculus; conjugation relations; series formulae.

**Mathematics Subject Classification:** 26A33, 47A05, 34A08

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# On the linear moment problems and nonhomogeneous lineair recursive sequences

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**Abstract:** In view of its fundamental role in various fields of mathematics and applied science, the linear moment problem has been extensively studied in the literature. Especially, it has been shown that this problem is useful for some topics in physics, such that quantum dynamical systems. Furthermore, the linear moment problem is also related to the Lanczos numerical method. Recently, the linear moment problem has been investigated in the literature, by various methods. We aim to explore the linear moment problem for the real sequences defined by the nonhomogeneous linear recursive relation. Various properties are provided, especially, those related to the Hankel matrices. Some considerations in connection with the K-moment problem, for the nonhomogeneous recursive, are discussed.

**Keywords:** Linear moment problem, $K-$moment problem, Hankel matrix, nonhomogeneous linear recursive sequences

**Mathematics Subject Classification:** 30E05, 39A10

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# Social Housing in Albania: Using Shiny to enhance visualization and interpretation of study key findings

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**Abstract:** The problem that everybody unfamiliar with data analysis, come across from time to time, is to read statistical research and understand key findings. In this study, an intuitive approach is offered on how to fill this gap. It has been discussed what the advantages of using Shiny platform dashboards are when they are used to visualize data and study findings. An example of this dashboard is presented in this study, visualizing a previously written by author, statistical research about Social Housing in Albania.

**Keywords:**R Language, Shiny, Dashboard, Social Housing, Visualization.

**Mathematics Subject Classification:** 97K80

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# Solutions for Neumann Problem Involving Multiple Critical Exponents

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**Abstract:** We consider the solvability of the Neumann problem for an elliptic system of two equations with weights involving two critical Sobolev exponents on a bounded domain in RN . By using variational methods, we investigate the effect of the shape of the graph of the weight functionsand the geometry of the boundary on the existence of solutions.

**Keywords:** Critical Sobolev exponents, Palais-Smale, Neumann Problem

**Mathematics Subject Classification:**

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# The Effect of Bamboo and Hemp Natural Fibers on the Elastic Behavior of Composite Materials Based on PMMA Polymer Matrix

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**Abstract:** The exploitation by the industrials of vegetable fibers in the field of composite materials has made it possible to reduce the dependence on oil and this as a result of their mechanical properties, their thermal resistance and their biodegradability. In this work, we have investigated by a genetic simulation on two composite materials based on different natural reinforcements (Bamboo and Hemp) to see the influence of its fibers on the elastic behavior of composite materials. The results of our numerical simulation showed that the interface of Bamboo / PMMA is more resistant than Hemp/ PMMA and that shear damage of Bamboo / PMMA is lower than that of Hemp/ PMMA of 16,3%. Our results are in good agreement with the results found by Rao KMM where he showed by experimental tests that Bamboo fiber is the most resistant fiber.

**Keywords:** Shear damage, Bamboo, Hemp, PMMA, Interface, fibers.

# Mathematical modelling and numerical simulations of a compressible two-phase flow in the anaerobic biodegradation for biogas production

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**Abstract:** Waste management and renewable energy generation are two key issues in nowadays society. A major research field arising in recent years focuses on combining the two aforementioned topics by developing new techniques to handle waste and to use it in the energy production. The anaerobic digestion process is a natural biological process of decomposition of organic matter by microorganisms (bacteria) activated in the absence of oxygen. It consists of complex chemical reactions. In the long term, the organic matter is transformed into "biogas" which is a mixture of methane and carbon dioxide [1,3]. The model given in [2] describes both the biological activity and the leachate single phase flow, represented by a system of reaction-diffusion coupled with Darcy flow equations, during the anaerobic digestion process in two-step for biogas production considering a landfill as a reactive porous medium. However, in order to have a more accurate consideration of the mechanical aspects of the complex interaction between the medium and the biological activity in the landfill, we extend this approach to a two-phase flow where the gaseous phase "biogas" is included in the model. In this work, we present a new coupled model combining the biological and the mechanical aspects describing respectively the process of the biogas production and the compressible two-phase leachate-biogas flow during the anaerobic biodegradation of the organic matter in a landfill considered as a reactive porous medium. We carry out the full-discretization of the PDEs problem by using the second order BDF2 scheme in time and the P1-conforming finite element approximation in a variational framework. The numerical simulations in 2D and 3D will be given.

**Keywords:** Anaerobic biodegradation, Coupled model, Compressible two-phase flow, Biogas production, Finite element method.

**Mathematics Subject Classification:** 65N30, 65N50, 92B05, 76-10

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# Analysis of Tea Production in India: ARIMA Approach

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**Abstract:** Tea has been accepted is a popular beverage across the globe, irrespective of age-groups, geographical areas and socio-economic status. Forecasting of tea production is one of the major important requirements to individual producers, agribusiness firms and policymakers for various purposes. Time series analysis is an important part in statistics, which analyzes data set to study the characteristics of the data and helps in predicting future values of the series based on the characteristics. From the preceding analysis it is amply demonstrated that ARIMA model is a major forecasting tool for linear time series observations. The current exercise attempts to select the best fit ARIMA model to forecast export of tea in India, the model supremacy is tested with simple exponential smoothing (SES) and holt two-parameter exponential smoothing (HES). In this exercise selection of the model is based on the principle of parsimony.

**Keywords:** Forecasting, ARIMA model, Time series Analysis, Tea.

**Mathematics Subject Classification:** 62M10, 91B84

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# Combinatorial Homotopy Approach to the Topology of Data in Tracking the Persistence of Descriptively Close Shapes

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**Abstract:** For a J.H.C. Whitehead [6-7] Closure-finite Weak (CW) planar space, a path cycle hCycE is a collection of path-connected vertexes attached to each other with no end vertex so that hCycE has a nonvoid interior. For each video frame X, let H(X) be a collection of nested path cycles in a vortex that is a path nerve in X [2]. Then H(X) is a good cover of a cell complex in such a way that the union of its elements is X and all its finite nonvoid intersections are contractible. Since, hCycE has a free group presentation [5] and every free group presentation of nested 1-cycles nerve has a Betti number [1,2], we can then describe frame shapes in a concise manner in terms of their Betti numbers [3] to track the persistence of good covers of video frame shapes. This talk will be based on path cycles, good covers, and path nerve presentation [4] to introduce an application of descriptively proximal nerves in a combinatorial homotopy form of topology of data approach to detecting close good covers of time-varying video shapes [3] that appear, disappear and sometimes reappear in a sequence of video frames.

**Keywords:** Good Cover, Combinatorial Homotopy, Nerve, Path, Proximity, Topology of Data

**Mathematics Subject Classification:** 54E05, 55P57

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# Mathematical Analysis And Numerical Simulation for Periodic Problem With Discontinuous Coefficients

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**Abstract:** During the last forty years, a large number of researchers have been interested in the study of partial differential equations, their numerical simulations and their applications. A particular interest has been shown for equations with nonlinear terms. Several methods have been developed to answer different questions about the considered solutions. In particular, questions of existence, uniqueness, regularity, stability, asymptotic behavior and numerical simulation. A large literature exists on periodic equations and several researchers have been interested in the subject. Il this work we propose a new technique to mathematically analyze and numerically simulate a class of periodic parabolic equations with discontinuous coefficients. We introduce the necessary assumptions and state the definition of the weak periodic solution of the problem. We first formulate the existence problem into an equivalent optimization problem by means of a least square cost function. Then, we prove the existence of an optimal solution to the optimization problem in an appropriate admissible space. Subsequently, we use the Lagrange method to compute the derivative of the cost function with respect to the state variable. Finally, we present the discretization of our finite element problem and the presentation of the proposed numerical algorithm to solve the optimization problem. We then give some numerical examples to illustrate the efficiency of the proposed approach.

**Keywords:** weak periodic solutions; discontinuous coefficients; optimization; Lagrangian

**Mathematics Subject Classification:**

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# A Method for Controlling Series and Parallel Nonlinear and Time-Varying LRC Circuits

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**Abstract:** Series and parallel circuit systems are widely encountered in numerous electrical, electronics, control issues and differential equation applications. Accurate control of the current in series and voltage in parallel circuit systems with nonlinear time-varying inductance, resistance and capacitance is a challenge. In this study, a novel approach was proposed for the control of current in a nonlinear time-varying series circuit and of voltage in a nonlinear and time-varying parallel circuit. The proposed controller is characterized by a nonlinear algebraic equation and straining the tracking error to converge to zero. Illustrative results confirm the proposed approach for forcing the current /voltage in series and parallel nonlinear time-varying circuit to follow the targeted current /voltage trajectories efficiently. The proposed approach shows great novelty to determine the dynamics behavior of nonlinear time-varying systems. Therefore, the obtained results generalize and improve the existing conclusions. Simulations illustrate the feasibility and validity of the theoretical results.

**Keywords:** Nonlinear time-varying circuit, Tracking error, Voltage control, Current control

# Existence and Uniqueness Solution for the Integral Boundary Value Problem of Fractional Differential Equation

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**Abstract:** In this work, we will study the existence and uniqueness results of a class of nonlinear fractional differential equations with integral boundary value conditions. By using Leray-Schauder nonlinear alternative and the Banach contraction mapping principle. As an application, an example is given to prove our conclusions.

**Keywords:** Fractional differential equations, Fixed point theorem, Banach contraction theorem, uniqueness and existence solution.

**Mathematics Subject Classification:** 34B18

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# Existence of solutions to the space-fractional, degenerate nonlinear Schrodinger equation with nonlinearity

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**Abstract:** Existence of two different positive solutions to the homogeneous Dirichlet problem for a class of space-fractional weighted nonlinear Schrodinger equation
$(-\Delta)^{\alpha/2}(\omega(x)(-\Delta)^{\alpha/2}u) = v(x)u^{q-1} + \mu u^{p-1}$
on a bounded domain $\Omega \subset \mathbb{R}^n$, which satisfies certain regularity condition; the exponents $0 < \alpha < min(1, \frac{n}{2})$, $1 < p < 2 < q < 2n/(n-2\alpha)$ , the positive weight functions $\omega \in A_2$, $v \in A_\infty$ are of Muckenhoupt's class. Also the balance condition of Chanillo-Wheeden's type is assumed on pare $(v, \omega)$. To solve the cited problem two different critical points is funded for the corresponding energy functional in weighted fractional Sobolev spaces $W_0^{\alpha,2}(\omega dx, \Omega)$.

**Keywords:** fractional Laplacian, fractional Sobolev space, Shrodinger operator, degenerate elliptic equation.

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# An improved ELM-framework for dynamical systems Modeling and identification

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**Abstract:** Extreme learning machine (ELM) Modeling and Identification of Dynamical Systems presents a new approach on how to obtain the adaptive ELM models for complex systems that are typically found in real-word applications. Extreme learning machine is used in many applications such as image recognition, classification, control and system identification. In this paper, a new hybrid extreme machine-Autoregressive Moving Average (ELMARMA) and Extreme learning machine Autoregressive (ELMAR) scheme applied for dynamical systems modeling is presented. The proposed model comprises a parallel interconnection of tow sub-ELM models. The first sub-ELM model is the primary model, which represents an ordinary model with a low resolution for the dynamical system under consideration. To overcome resolution quality problem and obtain a model with higher resolution, we will introduce a second sub-ELM model called Error model which will represent a model for the error modeling between the primary model and the real nonlinear dynamic system. The method's effectiveness is evaluated through testing on the three nonlinear dynamical systems described by Narendra in the literature. In addition, a detailed comparative study with several benchmark methods will be given. Intensive computer experimentations confirm that the proposed approach can significantly improve convergence and resolution.

**Keywords:** Dynamical systems, Extreme learning machine, AR, ARMA.

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# A New PSO-ANN Scheme for Composite materials Properties prediction

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**Abstract:** In this investigation a novel PSO-ANN scheme for composite materials properties prediction is presented. It is based on neural networks which are used in many applications such as image recognition, classification, control and system identification.This approach will deal with local minima problem of the neuronal networks architecture and simultaneously preserve the fitting quality. The proposed scheme comprises a parallel interconnection of tow sub-ANN prediction systems. The first sub-ANN prediction system is the primary system, which represents an ordinary system with a low resolution for the training data under consideration (composite materials properties). To overcome resolution quality problem, and obtain a prediction system with higher resolution, we will introduce a second ANN sub model. ANN scheme Identification is achieved by innovative metaheuristic algorithm such asparticle swarm optimization (PSO). The method's effectiveness is evaluated through testing on the composite materials to predict their physical properties. Intensive computer experimentations confirm that the proposed approach can significantly improve convergence and resolution.

**Keywords:** Artificial neural network, Particle swarm optimization, Composite materials.

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# Stochastic infinity norm optimization and self-concordant primal interior-point algorithms

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**Abstract:** In this talk, we discuss the optimization problem over infinity-order cones under uncertainty. We present the algebraic properties of this cone, and use them to derive a decomposition polynomial-time algorithm for solving the problem. We also summarize some numerical experiments to highlight the efficiency of derived algorithm.

**Keywords:** Convex programming, Stochastic programming, Interior-point methods, Infinity norm optimization.

**Mathematics Subject Classification:** 90C15, 90C25, 90C30, 90C51

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# Generalized Derivations on A Prime Rings

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**Abstract:** Let $R$ be an associative ring with identity, $Z(R)$ is the center of $R$. A ring $R$ is prime if $sRt = 0$, then either $s = 0$ or $t = 0$ and $R$ is semiprime if the identity $sRs = 0$ gives $s = 0$. The $charR \neq 2$ of a ring $R$ if whenever $2s = 0$, $s \in R$, then $s = 0$. The derivation is an additive map $\delta : R \to R$ on a ring $R$ satisfies

$$\delta(st) = \delta(s)t + s\delta(t) \ \forall s, t \in R.$$

The additive map $\delta$ is said to be $(\theta, \varphi)$-derivation if

$$\delta(st) = \delta(s)\theta(t) + \varphi(s)\delta(t) \ \forall s, t \in R,$$

where, $\theta, \varphi : R \to R$ are maps on $R$.
An additive map $F : R \to R$ is called a generalized derivation associated with $\delta$ if there exists a derivation $\delta : R \to R$ such that

$$F(st) = F(s)t + s\delta(t) \ \forall s, t \in R.$$

An additive map $F : R \to R$ is called a generalized $(\theta, \varphi)$-derivation associated with $\delta$ such that $\theta, \varphi$ are maps on $R$ , if there exists a $(\theta, \varphi)$-derivation $\delta$ such that

$$F(st) = F(s)\theta(t) + \varphi(s)\delta(t) \ \forall s, t \in R.$$

In our research first we extended Ashraf's results in [1] for$(\theta, \theta)$-derivation acting as a homomorphism (resp. an anti-homomorphism) on a Jordan ideal and a subring of a prime ring $R$ with characteristic non equal two, then we extended Zaidi results in [2] for a generalized $(\theta, \theta)$-derivations. Also we find the relationship between the commutativity of a prime ring and the existence of certain specific types of generalized derivations on R.

**Keywords:** Derivation, prime ring, Generalized derivation, Jordan Ideal

**Mathematics Subject Classification:** 16W25, 16N60, 16B60

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# Integral Operators, Fractional Calculus and Modelling

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**Abstract:** In my talk I will present some new topics in the field of fractional modelling. Some illustrative examples will be presented in detail.

**Keywords:** Integral operators, Fractional calculus, Modelling

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# Dynamics of HIV/AIDS defining malignancies for diverse immune responses

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**Abstract:** : In this talk, we will show diverse growth patterns of tumors, in HIV/AIDS patients, subjected to distinct immune responses. Individuals with AIDS are more prone to develop certain cancers, namely aggressive B-cell non-Hodgkin lymphoma, and cervical cancer. The later are known as AIDS-defining malignancies. We propose a mathematical model to study the dynamics of this tumors' growths for specific immune responses. The simulations of the model suggest disparate tumor evolutions as function of the immune functions and of the tumor proliferation rates.

**Keywords:** HIV/AIDS-defining malignancies; immune response, dynamics.

**Mathematics Subject Classification:** 32-XX, 92-XX

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# Exploring the Use of Granular Flows for Energy Storage in Arid Regions

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**Abstract:** Desert regions show strong potential for the development of large-scale wind and solar generation facilities. However, electrical grids in arid regions have difficulties storing and utilizing, wind and solar generated power due to their limited water supplies. To explore a potential solution, we investigate the feasibility of using a granular flow, such as sand, as a replacement for water in a pumped hydropower system. We determine the expected power output of a turbine being spun by a granular flow by using Discrete Element Modeling (DEM) to study the interactions of granular spheres flowing out of a hopper and interacting with a turbine rotor. The results are consistent with the mass-flow-rate expression proposed by Beverloo et al. in 1961. The dependence of hopper inlet opening on power output is determined and a theoretical maximum efficiency of a granular-flow energy system proposed. We also show that a precisely controlled release rate can be achieved by varying the size of the opening onto the turbine and make suggestions on how this system could be further optimized.

# General Decay For A Viscoelastic Rotating Euler-Bernoulli Beam

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**Abstract:** In this work, we consider a viscoelastic rotating Euler-Bernoulli beam that has one end xed to a rotated motor in a horizontal plane and to a tip mass at the other end. For a large class relaxation function q, namely, $q'(t) \leq -(t)H(q(t))$, where H is an increasing and convex function near the origin and is a nonincreasing function, we establish optimal explicit and general energy decay results from which we can recover the optimal exponential and polynomial decay.

**Keywords:** general decay, viscoelastic, stabilisation, Euler-Bernoulli beam.

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# Soliton solutions of nonlinear (2 + 1)-dimensional Biswas–Milovic equation via new approach of generalized Kudryashov scheme

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**Abstract:** This paper deals with the (2+1)-dimensional Biswas–Milovic equation for soliton propagation in optical fiber utilizing the new approach of the generalized Kudryashov method. We derive a plethora of traveling wave solutions of the investigated problem. Some 3-d, 2-d and contour representations for the given special parameters are given. Moreover, the obtained results might be helpful for the future studies not only for explaining the dynamic behavior of the considered problem but also effectiveness and reliability of the proposed new approach of the method.

**Keywords:** (2+1) dimensional Biswas-Milovic equation, generalized Kudryashov method, Riccati equation, optical soliton.

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# Fractional hypergeometric functions: a survey and future perspectives

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**Abstract:** The aim of this talk is to provide the reader with the essentials of fractional calculus according to different approaches that can be useful for our applications. Here, we discuss the fractional calculus of hypergeometric functions (special functions) has significant importance and applications in various fields of science and engineering:
• Study the extensions and generalization of hypergeometric functions
• Fractional integral and differential formulas involving the extended hypergeometric type functions
• Applications
• Future Perspectives
water quality component variables and time consumption.

**Keywords:** : Fractional calculus, Fractional integral, Fractional derivative, Hypergeometric function, Mittag-Leffler function, Wright function.

**Mathematics Subject Classification:** 26A33 (main); 33E12, 33E20, 33C40, 44A10, 44A20, 45E10, 45J05, 45K05

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# A Novel Numerical Technique on Bakhvalov Type Mesh for Singularly Perturbed Problem with Integral Boundary Conditions

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**Abstract:** In this paper, we deal with the singularly perturbed linear boundary value problem with two integral boundary conditions. We propose a uniform convergence numerical method for solving singular perturbed problem with integral boundary conditions. The behavior of the exact solution and it's derivative is analyzed for this problem. A numerical method based on a finite difference scheme on a non-uniform mesh is constructed. We prove that difference scheme is first order convergent in the discrete maximum norm with respect to singular perturbation parameter. Finally, numerical results supporting the theoretical considerations are given and the effectiveness of the method is shown on examples.

**Keywords:** singular perturbation, integral boundary conditions, finite difference method, uniformly convergence.

**Mathematics Subject Classification:** 65N12; 65N30; 65N06

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# On the Uniform Convergence of the Finite Difference Scheme for Singularly Perturbed Problem with Integral Boundary Conditions

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**Abstract:** : In this paper, we consider a class of singularly perturbed non-linear differential equations of convection diffusion type with integral boundary conditions. A finite difference scheme with an appropriate Bakhvalov type mesh is suggested to solve the problem. We prove that our schemes converge almost first-order uniformly with respect to small parameter. An effective iterative algorithm for solving the non-linear difference problem and some numerical results are presented.

**Keywords:** singular perturbation, integral boundary conditions, uniformly convergence.

**Mathematics Subject Classification:** 65N12; 65N30; 65N06

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# Spectral and Scattering Properties of Eigenparameter-Dependent Discrete Dirac Equation With Point Interaction

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**Abstract:** The purpose of this study is to investigate some spectral and scattering properties of a discrete Dirac equation with both point interaction and eigenparameter-dependent boundary conditions. Firstly, we find the Jost solution and the Jost function. Secondly, we get the scattering function by using the Jost function. Then, we examine some properties of the scattering function and we obtain the resolvent operator of our problem. Then, we find the sets of eigenvalues and spectral singularities of this discrete Dirac equation with point interaction. We also obtain continuous spectrum of the problem. We also present an example on an unperturbated problem to demonstrate the application of our main results.

**Keywords:** Dirac equations, impulsive condition, Jost solution, Jost function, scattering function.

**Mathematics Subject Classification:** 34B37, 35P25, 47A75

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# Existence solutions for a couple of differential inclusions involving maximal monotone operators

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**Abstract:** In this work, we are intersted in couple of differential inclusions, both of them governed by time and state-dependent maximal monotone operators of the form

$$\begin{cases} -\dot{u}(t) \in A(t, v(t))u(t) \\ -\dot{v}(t) \in B(t, u(t))v(t) \end{cases}$$

It is assumed that both of $(t, x) \mapsto A(t, x)$ and $(t, x) \mapsto B(t, x)$ are absolutely continuous with respect to the time $t$ and Lipschitz continuous with respect to the state $x$, in the sense of Vladimirov's pseudo-distance and we solve the evolution system with multivalued upper semicontinous perturbations.

**Keywords:** Absolutely continuous variation, normal cone, maximal monotone operator, multi-valued perturbation, optimization problem, pseudo-distance.

**Mathematics Subject Classification:** 34A06, 34K27, 47B02, 28C20

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# The convergence analysis of the new CG coefficient for inconstrained optimization problems

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**Abstract:** Conjugate gradient methods have an important role in solving large scale unconstrained optimization problems. Zhang et al proposed a new coefficient but thy proved the convergence for nonconvexe minimization in strong Wolfe line search. In this presentation, we prove that this CG coefficient possesses the sufficient descent conditions and the global convergence properties under the exact line search.

**Keywords:** Sufficient descent condition, global convergence, Conjugate Gradient method, exact line search.

**Mathematics Subject Classification:** 90C26, 65H10

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# Cryptanalysis Attacks for Factoring Generalized Takagi's Scheme $N = p^r q^s$

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**Abstract:** This paper develops new strategies of factoring prime power moduli $N = p^r q^s$ also as known Generalized Takagi's scheme using method of continued fraction for $2 \leq s < r$. The paper proves that using an approximation of $\phi(N) = N + N^{\frac{r+s-2}{2r}} - 2N^{\frac{r+s-1}{2r}}$, private keys $\frac{k}{d}$ can be found from the convergents of the continued fractions expansion of $\frac{e}{N+N^{\frac{r+s-2}{2r}}-2N^{\frac{r+s-1}{2r}}}$ which leads to the factorization of the moduli $N = p^r q^s$ in polynomial time. The paper further reports two cryptanalysis attacks which exploit the security of the cryptosystem $N_i = p_i^r q_i^s$ by solving generalized key equations of the type $e_i d - k_i \phi(N_i) = 1$ and $e_i d_i - k\phi(N_i) = 1$ using simultaneous Diophantine approximation method and LLL algorithm to find the values of the unknown integers $d, k_i, \phi(N_i)$ and $d_i, k, \phi(N_i)$ respectively for $i = 1, 2, \cdots j$.

**Keywords:** Prime Power, Factorization, LLL algorithm, Diophantine approximations, Continued fraction.

# Prediction of Mechanical properties of Materials with Machine Learning

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**Abstract:** We have studied that the mechanical properties of steel casting materials can be predicted by modeling with machine learning methods, and time and cost can be saved by simulating the desired mechanical properties in the computer environment without resorting to experimental methods.In this study, the relationship between the chemical analysis of medium carbon steels, which are preferred for use in crane wheels, and the tensile strength is modeled by the multivariate regression analysis method. 205 test specimens were casted with the resin sand mold casting method. Mechanical test was performed at room temperature to determine the tensile strength of the samples. Analysis was performed using the regression learner toolbox in the MATLAB program. A total of 10 chemical analyzes were used as independent variables (Carbon, silicon, manganese, phosphorus, sulfur, chromium, nickel, molybdenum, aluminum, copper). Tensile strength was chosen as the dependent variable. The model was trained for all algorithms and the correlation coefficient (R-squared) was 0.81 in the fine tree decision tree.

**Keywords:** Prediction, Mechanical Properties, Carbon Steel, Regression Analysis.

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# Data Clustering and Its Applications in Medicine

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**Abstract:** Artificial intelligence was first mentioned back in 1956, but the biggest leap in its use has been seen in the last two decades. It goes without saying that with the in-creasing availability of artificial intelligence, one of the most important areas in which it must be applied is medicine. The purpose of this short article is to pro-vide an overview of one of the groups of machine learning, by reviewing clustering algorithms and also by reviewing the use in medicine. For an excellent interpretation, this can usually be done by finding certain groups in your data that are much easier to interpret than individual observations. This task can be solved using a clustering. In medicine, the application of clustering allows one to distinguish various groups of patients and to summarize these groups to provide much more precise recommendations [1]. Studies show that clustering algorithms can be applied to identify different diseases [2; 3; 4; 5; 6]. For example, different clustering techniques are used to identify breast cancer [7], Parkinson's disease [8], migraine [9], various psychological and psychiatric disorders [10], heart and diabetes diseases [11], Huntington's disease [12], and Alzheimer's disease [13], among many others. Given that clustering is an area of unsupervised learning, its application to physicians allows for the observation of certain exceptional cases and their subsequent analysis in much more detail. Mention may also be made of the application of clustering in medicine, where unstructured data is used. In this case, one of the possibilities of applying clustering algorithms is the analysis of various text documents. First, using various clustering algorithms, it is possible to divide various large-volume documents into certain groups. Then, these groups of documents can be summarized, thus avoiding high reading of the documents [14]. Another important example that can be particularly widely applied in medicine is image analysis. Image analysis can allow the application of clustering in a variety of different ways, but the main one is the segmentation of various X-rays, MRIs, and other images, allowing some changes to be observed in these images [15, 16, 17].

**Keywords:** machine learning, artificial intelligence, clustering, medicine.

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# Dynamics of a class of viral infection models with diffusion

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**Abstract:** Hepatitis B is a viral infection that attacks the liver. It can cause acute or chronic infection and exposes people to a high risk of death from cirrhosis and liver cancer. Infection with the hepatitis B virus (HBV) is now a major global health problem. In recent years, mathematical modeling has used partial differential equations (PDEs) to understand the dynamics of HBV infection, many authors have used virological Reaction-Diffusion Equations. Wang and Wang [1] proposed a mathematical model to simulate hepatitis B virus (HBV) infection with spatial dependence. They introduced the random mobility of viruses into the basic model proposed by Nowak, S. Bonhoeffer, AM Hill, R. Boehme, Thomas HC and H. McDade [2] and they assume that the movement of the virus follows a fickian diffusion, that is, the population flow of the virus is proportional to the concentration gradient and the proportionality constant is considered negative. In this work, we propose a generalized HBV model given by the system of nonlinear partial differential equations (or Reaction - Diffusion Equations); We aim to investigate the overall stability of equilibrium solutions to diffusion reaction systems with Neumann boundary conditions without delay. In order to consider the influences of spatial structure on virus dynamics, Wang and Wang [1] and Hattaf &Yousfi,[3] considered the HBV model with spatial dependence.

**Keywords:** Viral infection, Reaction-Diffusion Equations, Mathematical Modeling, Stability.

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# Numerical Approach using Polynomial Collocation Method for Solving Systems of Linear Integro Differential Difference Equations

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**Abstract:** In this study, an operational matrix is used to find an approximate solution to systems of linear integro differential difference equations under mixed conditions using polynomial basis function. The methods convert the integro differential equations in to systems of linear algebraic equations with unknown coefficients. Combining these matrix equations and solving the systems of equations gives the coefficient of the solution function. Convergence of the method is established and numerical examples were solved and compared with different approaches considered in literature to test accuracy and efficiency of the method in handling integro differential difference equations with high degree of accuracy in errors obtained.

**Keywords:** Integro differential equation, Operational matrix, Collocation method, Polynomial.

# Application of Fuzzy Logic and Fuzzy Similarity in Plant Sociology

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**Abstract:** In recent years, studies on plant communities with fuzzy logic and fuzzy similarity approaches have accelerated. The plant associations (plant communities) detected in studies on different habitats and different vegetation types can be interpreted more realistically by re-evaluating them with fuzzy logic and fuzzy similarity approaches. These studies can be applied on the plant associations identified in the current researches as well as directly in the field studies from the beginning.
In other words, the issue of fuzzy similarity of sets and elements in sets is used. This study brings a different perspective to new researches in plant sociology. According to this point of view, the fuzzy similarity among plant associations and relevés came to the fore.
In this talk, fuzzy logic and fuzzy similarity approaches were applied to plant communities (plant associations), which were determined in the project study conducted in the field. The results were evaluated and interpreted by comparing the findings with the classical methods.

**Keywords:** Fuzzy Logic, Fuzzy Similarity, Plant Community (Plant Association), Phytosociology, Different vegetation types and Habitats, Ecology.

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# Study on the Mikhailov-Novikov-Wang equation to Find the Exact Soliton Solutions

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**Abstract:** In this study, the novel exact solutions of the Mikhailov-Novikov-Wang equation have been examined by using Riccati-Bernoulli sub-ODE method. This equation is an integrable equation with a dynamical behavior where by the differential polynomial ring's extension, a well-known equation in nonlinear science, named Boussinesq equation, belongs to this classification [1,2]. The advantages of presented method have been discussed. All solutions have been checked with the help of Maple program. Moreover, 3D- graphs have been depicted to observe different type of solutions.

**Keywords:** Riccati-Bernoulli sub-ODE method, analytical solutions, soliton solutions.

**Mathematics Subject Classification:** 35C08, 35Qxx

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# On new analytical solutions of the (1 + 1)-dimensional symmetric regularized long wave equation

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**Abstract:** Sardar sub-equation method is applied to obtain novel analytical solutions of the (1 + 1)-dimensional symmetric regularized long wave (SRLW) equation. This method contributes a variety of soliton solutions for the presented equation. It can be declared that all solutions produced in this study satisfy the main equation. Finally, the 2D and 3D graphs of the acquired solutions are successfully charted by selecting appropriate values of parameters to demonstrate the behavior of the obtained solutions.

**Keywords:** Sardar sub-equation method, soliton solutions.

**Mathematics Subject Classification:** 35A24, 35C07, 35R11

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# Characterization of Superalloys by Artificial Neural Network Method

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**Abstract:** In this study, the use of artificial neural networks in the classification of a superalloys whose chemical analysis is performed in the quality process is investigated. In general, chemical spectro analysis method alone is not sufficient to determine which class a steel belongs to. In addition to the chemical analysis method, tests such as tensile test, hardness test or notch impact test are applied. The tests performed in addition to the chemical analysis both take time and destroy the material. The fact that an algorithm that classifies steel only according to the results of chemical analysis is not used has made destructive tests mandatory. Artificial neural networks (ANNs), usually simply called neural networks (NNs), are computing systems inspired by the biological neural networks that constitute animal brains. An ANN is based on a collection of connected units or nodes called artificial neurons, which loosely model the neurons in a biological brain. Each connection, like the synapses in a biological brain, can transmit a signal to other neurons. In our study, a total of 34 superalloy materials belonging to 6 different classes were used. Chemical composition values were determined for each superalloy sample. The appropriate artificial neural network model was determined according to the chemical composition values. A model that can predict superalloy material based on chemical composition value has been created. Weka 3.9.5 package program was used to create the artificial neural network model. The high success rate of the prediction model gave hope for the determination of the material class only with the chemical analysis method.

**Keywords:** Superalloy, Artificial Neural Networks, Weka, Chemical Composition.

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# Cybersecurity and Quantum Computations

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**Abstract:** New development of the quantum computers will significantly expand computing power, add new directions, and create new opportunities for improving cybersecurity. We think that the new technology using quantum computers will wield the power to detect the cyberattacks before they cause harm. Also, we discuss post-quantum cryptography standards and some applications will be given.

**Keywords:** Quantum computer, cyber security, cryptography

# Quasi—Stationary Optical Gaussons

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**Abstract:** This presentation will be on the dynamics of optical solitons with log—law nonlinearity, also known as optical Gaussons, in the presence of perturbation terms. The governing model is the nonlinear Schrodinger's equation that carries logartithmic form of nonlinear refractive index. A few perturbation terms are included to get a better understanding of the Gausson transmission dynamics. These include intermodal dispersion self-steepening effects, saturable amplifiers and others. The multiple—scale perturbation analysis is implemented to retrieve the quasi—stationary optical Gaussons solution. The definition of the phase that is introduced in this paper reveals a couple of resonant conditions that cannot be otherwise recovered.

**Keywords:** Optical Gaussons; multiple-scales; Fredholm's alternative; resonance.

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# Abstract Bivariate Left Fractional Pseudo-Polynomial Monotone Constrained Approximation with applications

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**Abstract:** Here we extend our earlier bivariate high order simultaneous fractional monotone constrained approximation theory by pseudo-polynomials to abstract bivariate high order simultaneous fractional monotone constrained approximation by pseudo-polynomials, with applications to bivariate Prabhakar fractional calculus and non-singular kernel fractional calculi. We cover the left side of this constrained approximation.

**Keywords:** Monotone constrained Approximation, abstract fractional derivative, abstract fractional linear differential operator, mixed modulus of smoothness, pseudo-polynomials, Prabhakar fractional calculus and non-singular kernel fractional calculi.

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# On The Geometry of The Tangent Bundle with Vertical Rescaled Generalized Cheeger-Gromoll Metric

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**Abstract:** Let (M; g) be an n-dimensional smooth Riemannian manifold. In the present paper, we introduce a new class of natural metrics denoted by Gf and called the vertical rescaled generalized Cheeger-Gromoll metric on the tangent bundle TM. We calculate its Levi-Civita connection and Riemannian curvature tensor. We study the geometry of (TM;Gf).

**Keywords:** Horizontal lift, vertical lift, Cheeger-Gromoll metric, tangent bundle.
**Mathematics Subject Classification:** 58A03, 58A05

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# Fixed Point Theorems in Banach Algebras and Application

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**Abstract:** We intend some generalizations of Darbo's fixed point theorem for multivalued mappings by considering a measure of weak noncompactness which does not necessarily have the maximum property. Moreover, we prove some fixed point theorems for multivalued mappings in Banach algebras satisfying a certain sequential condition (P). We apply such results to study the existence of a solution of a nonlinear integral inclusion.

**Keywords:** Measure of weak noncompactness, Meir-Keeler multimap, Darbo fixed point theorem, integral inclusion.
**Mathematics Subject Classification:** 47H09, 47H10, 45B05

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# A New Strategy to Obtain Solutions of a System of Non-Linear Partial Differential Equations With Mddim

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**Abstract:** Aim of this work was to appraise the magnetohydrodynamic (MHD) convection motion of MoS2 engine-oil based nanofluid in a revolving the structure. Ancillary to the same, the evolution and process of transfer of heat in the appearance of heat radiation and convective heating are investigated. Then, we illustrated these phenomena via the method of directly defined inverse mapping (MDDiM) with the approximate analytical solution firstly reported in this paper. Finding the solutions by using MDDiM is a novel idea and first time illustrated for the system of nonlinear partial differential equations. It is emphasizes by residual error (i.e, $10-1$ to $10-25$) and can easily derive deformation terms by spending very low CPU time. Based on the proposed method, the convergence rate, efficiency and best accuracy of the governing equations are studies, which exhibit meaningful structures and advantages in applied sciences. The effects of the variation of all physical parameters discussed in details lucidly..

**Keywords:** MHD, MDDiM, MoS2, nanofluid and approximate analytical solution.
**Mathematics Subject Classification:** 34A12, 34A25, 34A34, 35A20, 35C10

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# Results in Optimization, Dynamical System Theory & Networks for Mathematical Modelling

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**Abstract:** In this talk, firstly I will present my latest results on areas that I currently work on such as singular systems of differential & difference equations, mathematics of networks, optimization, and fractional calculus. Then we will discuss how these results can be applied into mathematical models related to electrical power systems, materials, gas networks, macroeconomics etc.

**Keywords:** differential, difference, networks, optimization, fractional calculus, power systems, materials, gas.

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# In Memory of Professor Jose Antonio Tenreiro Machado: The (p,q)-Calculus Died with You

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**Abstract:** Usually people play a role in other people's lives without knowing it or wanting to. Professor Jose Antonio Tenreiro Machado (1957-2021) was one of those who played an important role in the research path of many young researchers. Sometimes some research paths are chosen incorrectly. The purpose of creating the q-calculus was to use computer calculations for fractional differential equations and it seems that this was a good research path. During last years some researchers tried to generalize the notion of (p,q)-calculus. This talk is based on our very recent findings. In this talk, our aim is to show you that the (p,q)-calculus is not a real generalization for q-calculus and numerical q-calculations are better than (p,q)-calculations.

**Keywords:** q-calculus, (p,q)-calculus, discrete fractional differential equation, generalization, mathematical softwares.
**Mathematics Subject Classification:** 34A08, 39A13

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# On Classification Methods Based on Multiple Correspondence Analysis. Case Study of Distance Education in Algeria Using Python

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**Abstract:** To analyze survey questionnaire data we apply multiple correspondence analysis "MCA" as a method to help us convert data to cloud of points, but it is difficult to study it and get good results from it, so we have to do a classification to facilitate the study. Among the most useful classification methods, the CAH and the k-means. To compare them, we carried out a questionnaire on distance studies during the Corona virus , which included the opinions of 304 university professors from most universities in Algeria. In our application, we used the python programming language.

**Keywords:** Survey, ACM, classifications, Python.
**Mathematics Subject Classification:**

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# Existence Results for Fractional Partial Random Differential Equations with Delay

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**Abstract:** We provide some existence results for the Darboux problem of partial fractional random differential equations with finite delay, by applying the measure of noncompactness and a random fixed point theorem with stochastic domain.

**Keywords:** Random differential equation, left-sided mixed Riemann-Liouville integral, Caputo fractional order derivative, Darboux problem.
**Mathematics Subject Classification:** 26A33

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# Volumetric Barrier Cutting Plane Algorithms for Stochastic Linear Semi-Infinite Optimization

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**Abstract:** In this talk, we present volumetric barrier cutting plane interior-point algorithms for solving the two-stage stochastic linear semi-infinite programming with recourse, which is introduced to handle uncertainty in data defining (deterministic) linear semi-infinite programming. We will see that the dominant terms in the complexity expressions obtained are given in terms of the problem dimension and the number of realizations.

**Keywords:** Semi-infinite programming, stochastic linear programming, interior-point methods, volumetric barrier, cutting plane.
**Mathematics Subject Classification:** 90C06, 90C15, 90C34, 90C51, 90C60

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# Solution of Schrödinger Type Problem

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**Abstract:** This paper proved the existence and uniqueness of the solution of Schrödinger equation with potential and initial data are distributions (singular and regular) in the Colombeau algebra.

**Keywords:** Generalized function , semi-groups , generalized infinitesimal generators.
**Mathematics Subject Classification:** 46F10, 46S10, 35A27

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# Mathematical and Numerical Modeling of a Given Field of Temperature Influence on the Pressure Variation in a Fluid-Structure Interaction Problem

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**Abstract:** In the first part of this work we propose a mathematical model that describes the interaction between a viscous fluid (blood) and an elastic structure (vessel wall) under the influence of the temperature. Results as existence, uniqueness and regularity for the unknowns of the problem: velocity, displacement, temperature, pressure are established by associating to the coupled physical system a variational problem.In the second part, using some numerical methods, we analyze the blood pressure variation depending on the ambient temperature. It is known that in real life, lower values of ambient temperature determine higher values of blood pressure. We consider an equivalent variational problem involving the pressure and we approximate it by means of an Uzawa algorithm. The purpose of the numerical simulations is to determine the blood pressure field corresponding to different given values of ambient temperature. We find that these simulations are in agreement with the situation of real life.

**Keywords:** Thermal fluid-structure interaction, coupled system, variational formulation, existence and uniqueness, Uzawa algorithm.
**Mathematics Subject Classification:** 74F10, 74F05, 35A15, 65M60

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# A Hybrid Generalized Fractional Q-Differential Equations Under the Generalized Version of Darbo's Criterion

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**Abstract:** In this work, we investigate a new hybrid nonlinear fractional q-differential equations in the sense of $\psi$-Caputo. To emphasize the novelty of the manuscript, a pure technique of the non-compactness measures is applied for a hybrid system based on the notion of the modulus of continuity in Darbo's criterion which covers the existing results of other works published before. The Ulam–Hyers and generalized Ulam–Hyers stabilities are explored for the given neutral non-hybrid nonlinear problem. An application is prepared in the framework of an example to ensure the validity of theorems for different cases.

**Keywords:** Measure of non-compactness, Darbo's criterion, hybrid generalized problem, Caputo q-derivation.
**Mathematics Subject Classification:** 34A08, 34B16, 39A13

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# On One Evolutionary System of Equations Like the Bean Model with Damping

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**Abstract:** The paper considers the Cauchy problem for a degenerate quasi-linear evolutionary system of equations. The compactness of the support of the solution to Cauchy problem is established, which is equivalent to the finiteness speed of propagation of disturbances. Estimates are found for the radius of the support of solutions under the condition that the initial data has a compact support. Special weight spaces and weight energy estimates are introduced to take into account the convective addition in quasi-linear equation, which plays the role of a damper. Weak and strong solutions of the Cauchy problem are obtained.

**Keywords:** Quasi-linear evolutionary system, speed of propagation of disturbances, weak and strong solutions.
**Mathematics Subject Classification:** 35M99

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# On Fundamental Properties of Integro-Differential Equations

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**Abstract:** In this work, we investigate some fundamental properties solutions of a certain integro- differential equation. A few new results are obtained on the fundamental properties solutions of the considered equation. The Lyapunov-Krasovkii method is used as a basic technique to prove the main results of this work. An example is given to illustrate applications of the given results.

**Keywords:** Lyapunov -Krasovskii method, stability, boundedness, instability, integrability.
**Mathematics Subject Classification:** 34D05, 34K20, 45J05.

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# Multiple Solitons in a Square Optical Lattice with a Negative Line Defect

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**Abstract:** The fundamental, dipole and multiple solitons on a line defect is explored in a cubic medium with a square optical lattice. The model equation for generation of these solitons is the nonlinear Schrödinger equation (NLSE) with an external lattice [1-5]. The two-dimensional external lattice is chosen as a square lattice with a vacancy defect that is defined in [6, 7].
In this study, multiple soliton solutions of the NLSE are calculated numerically by the square operator method (SOM) [8], and the nonlinear stability of these solitons are studied by the direct simulation of the model equation. It has been demonstrated that the multiple solitons can exist on negative line defect, and these solitons can stay stable during the evolution by the selection of convenient lattice depth and frequency.

**Keywords:** Multiple solitons, line-defect, lattice solitons, soliton stability.
**Mathematics Subject Classification:** 35J10, 78A60, 81Q05

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# Dynamics of COVID-19 Disease with Lockdown and Immunized Population

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**Abstract:** We propose a new compartmental mathematical model describing the transmission and the spreading of COVID-19 epidemic with a special focus on the non-total immunity. The model (called SIARD) is given by a system of differential equations which model the interactions between five populations "susceptible", "reported infectious", "unreported infectious", "recovered with/without non total immunity" and "death". Depending on the basic reproduction number, we prove that the total immunity induces local stability-instability of equilibria and the epidemic may disappear after a first epidemic wave and more epidemic waves may appear in the case of non-total immunity. Using the sensitivity analysis we identify the most sensitive parameters. Numerical simulations are carried out to illustrate our theoretical results. As an application, we found that our model fits well the Moroccan epidemic wave, and predicts more than one wave for French case.

**Keywords:** Covid19, SIARD model, ODE, basic reproduction number, stability.
**Mathematics Subject Classification:** 34D05, 65L08, 92B05

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# Geometry Bi-Harmonic on Kenmotsu Manifolds

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**Abstract:** The present study is devoted to biharmonic maps who is critical point of the bi-energy functional on Kenmotsu manifolds. An example for biharmonic map of a three-Kenmotsu manifold is constructed for illustration.

**Keywords:** Biharmonic map, almost contact Riemanian manifold, Kenmotsu manifold.
**Mathematics Subject Classification:** 35Q35, 37K45

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# Some Issues of Constitutive Fractional Modelling: Basic Principles

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**Abstract:** The chapter addresses constitutive fractional modelling based on basic thermodynamic principles with emphasis on applications of fractional operators with singular and non-singular memory kernels. The Boltzmann superposition and the fading memory principles form the fundament of the developed models and refer to the formulation of diffusion and viscoelastic phenomena. The models developed are exemplified by pro and contra examples showing clearly the main differences between the correctly formulated models and those formally fractionalized.

**Keywords:** Constitutive models, fading memory, Boltzmann superposition, diffusion, viscoelasticity.

# The Fractional Calculus as Instrument to Model Nonlocal Phenomena Either in Space or Time

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**Abstract:** We present a basic introduction to the Fractional Calculus and the associated new mathematical scenarios. In this context, we show some relevant applications ranging from the Mars exploration to the study of new materials.

**Keywords:** Fractional calculus, Competition of scales, Interpolating equations, New materials, Mars exploration.
**Mathematics Subject Classification:** 34A08, 35R11, 34L40.

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# Sharp Oscillation Conditions for Delay Difference Equations

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**Abstract:** Consider the first-order linear difference equation

$$\Delta x(n) + p(n)x(n-k) = 0, \quad n \geq 0, \tag{1.1}$$

where $\Delta$ denotes the forward difference operator, i.e. $\Delta x(n) = x(n+1) - x(n)$, $\{p(n)\}_{n=0}^{\infty}$ is a nonnegative sequence of reals and $k$ is a natural number.
Sharp conditions for the oscillation of all solutions to this equation are presented when the well-known oscillation conditions

$$A := \limsup_{n\to\infty} \sum_{i=n-k}^{n} p(i) > 1. \tag{1.2}$$

or

$$\alpha := \liminf_{n\to\infty} \sum_{i=n-k}^{n-1} p(i) > \left(\frac{k}{k+1}\right)^{k+1}. \tag{1.3}$$

are not satisfied. In the case that the sequence $A(n) = \sum_{i=n-k}^{n-1} p(i)$ is *slowly varying at infinity* then under mild additional assumptions

$$B := \limsup_{n\to\infty} A(n) = \limsup_{n\to\infty} \sum_{i=n-k}^{n-1} p(i) > \left(\frac{k}{k+1}\right)^{k+1}. \tag{1.4}$$

is a sharp condition for the oscillation of all solutions to Eq.(1.1). This result is also extended to the following linear difference equation with several variable delays

$$\Delta x(n) + \sum_{i=1}^{k} p_i(n)x(\tau_i(n)) = 0, \tag{1.1}'$$

where $p_i(n)\colon \mathbb{N} \to [0,\infty)$, the retarded arguments $\tau_i\colon \mathbb{N} \to \mathbb{Z}$ satisfy $n - N \leq \tau_i(n) \leq n-1$ for all $1 \leq i \leq k$ and $n \in \mathbb{N}$.

# Artificial Neural Network Modeling and Control for Dynamical and Statistical Characteristics of Photonic Quantum Memristor

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**Abstract:** Quantum memristor is a quantum device that accounts for the memory, with the decoherence mechanism controlled by a feedback algorithm [1]. Last year, a variety of prototypes for quantum memristors has been proposed: superconducting circuits and platforms, different realizations of Josephson junctions, photonic systems [2]. Nevertheless, still there is a lack of theoretical methods modeling efficient exploitation of such devices. A new statistical physics technique based on Artificial Neural Networks (ANNs) has been developed recently in [3]. Such ANNs are well-trained with the data collected from the 'Ab Initio' experiment or numerical simulations to mimic the microscopic statistical states, and then to extrapolate the results for evaluation macroscopic states of the quantum system, its phase structures, and thermodynamic characteristics. Here we study a novel procedure based on the small-scale networks of Hodgkin-Huxley neurons [4] to model statistical micro-states and to control over dynamical characteristics (particularly, the purity) of photonic quantum memristor [2]. We compare the pros and cons of a few alternative control algorithms: Fradkov's speed gradient [5] and Kolesnikov's target attractor feedback [6]. We discuss also possible applications of our approach to memristor-based reservoir computing.

**Keywords:** Artificial neural networks, quantum dynamical systems, feedback control.
**Mathematics Subject Classification:** 68T07, 37N20, 93B52

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# The Block MINRES with Breakdowns Method for Solving Symmetric Linear Systems with Multiple Right-Hand Sides

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**Abstract:** The Bl-MINRES is a Block Krylov solver for solving a symmetric system of linear equations with multiple right-hand sides This method is classically implemented first, by applying the Bl-Lanczos process to create a basis for the block Krylov subspace generated by the matrix of the system from the initial residual. Next, this method is solving a block least-squares problem, which is equivalent to solving several least-squares problems implying the same tridiagonal matrix. These latter are usually solved by using a block updating procedure for the QR decomposition based on givens rotations of the tridiagonal matrix or the Housholder decomposition. The kind problem of these type of methods is when exist breakdowns. In this present work, we develop a new block version of the Bl-MINRES method referred to us New-Bl-MINRES with breakdowns hereafter based on a new process named New-Bl-Lanczos with breakdown. By using a simple implementation, we will give a technical to avoid breakdown problems. Several numerical experiments are providing to illustrate the performance of the new implementation.

**Keywords:** Linear system, Krylov subspace method, projector, pseudo-inverse, MINRES method.
**Mathematics Subject Classification:** MSC65F, MSC15A

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# Monte Carlo and Latin Hypercube Sampling Simulation of the proposed Fixed-Size Batch Service Lateness Queues with Multiple Vacations

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**Abstract:** We have analyzed a new model of a single server queue with service lateness which consists on duration of the control followed by the duration of service, both are done in batches of fixed size $K$. We assume that initially there are $K$ customers in the system, so thier control starts directly. After completion of the control time, this batch enters the service immediately. Once the service is completed, if there are $K$ or more customers in the queue, then the first $K$ customers will be selected and their control will start, else, if there are less than $K$ customers, the server goes for a multiple vacations until at its return, it finds at least $K$ customers waiting in the system to start their control. Explicit formulas using appropriate generating functions were discussed, which gives the steady state probabilities and some performance measures of the system. A simulation was performed to the studied model using both the Monte Carlo (MC) method, and the Latin Hypercube Sampling (LHS) generator in MATLAB language.

**Keywords:** Batch service, multiple vacations, service lateness, Monte Carlo, latin hypercube sampling, simulation.
**Mathematics Subject Classification:** 60K25, 90B22, 68M20, 00A72, 11K45

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# The Numerical Solutions of Fractional Differential Equations with Beta-Derivative

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**Abstract:** In this study, the Fibonacci collocation method is investigated to obtain the numerical solution of the fractional order differential equations based on the beta-derivative. The problem is firstly reduced into an algebraic system, later the unknown coefficients of the approximate solution function are obtained. To illustrate the efficiency of the proposed method, an example is solved, and the obtained results are compared with the exact solutions.

**Keywords:** Collocation method, Fibonacci polynomials, beta derivative
**Mathematics Subject Classification:** 26A33, 34B15

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# The Soliton Solutions of (2+1)-Dimensional Nonlinear Two-Coupled Maccari Equation with Complex Structure via New Kudryashov Scheme

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**Abstract:** In this study, we tackle with the (2+1)-dimensional complex nonlinear two-coupled Maccari equation which is a kind of Schrodinger equation and presented by Attilio Maccari in 1996. The Maccari equations has an importance to define the motion of isolated waves concentrated in a tiny region of space in a variety of fields, including hydrodynamics, plasma physics, quantum field theory, nonlinear optics, the sonic Langmuir solitons and so on. We utilized the popular method namely new Kudryashov scheme, we obtained the soliton solutions, graphical representations and made its physical interpretation..

**Keywords:** Coupled-Maccari system, new Kudryashov scheme, soliton solutions
**Mathematics Subject Classification:**

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# Phase II Profile Monitoring via Robust Semi-Parametric MCUSUM

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**Abstract:** Most of the previous studies in Phase II analysis in real-life applications focused on monitoring profiles assuming that there is no model misspecification in the estimated models and control limits. However, these estimated models may not perfectly fit the relationship between the response variable and the independent variable(s). Thus, this research proposes a new robust Multivariate CUSUM control chart, namely, a semi-parametric technique for performing Phase II profile monitoring using linear mixed models. The proposed multivariate CUSUM control chart will help in detecting different shift's sizes in the slope parameter, considering different numbers of the profiles, sample sizes, and different levels of misspecifications for in-control and out-of-control scenarios for uncorrelated and correlated profiles. The performance of the proposed control chart compared to other classical parametric and non-parametric approaches are investigated using comprehensive simulation studies and a real-life application, where Average Run Length (ARL) and Extra Quadratic Loss (EQL) criteria are used for these comparisons. It is found that the multivariate CUSUM based on the semi-parametric technique has the best performance and higher sensitivity in detecting different shifts compared to the parametric and the non-parametric approaches.

**Keywords:** Model misspecification, profile monitoring, model robust regression, MCUSUM, ARL, EQL.
**Mathematics Subject Classification:**

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# Solving nonlinear fractional PDEs using novel wavelet collocation method

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**Abstract:** In this paper, we consider an effective technique based on wavelet collation method for solving fractional differential equations. We transform the fractional differential equation to an algebraic equations system by using a wavelet operational matrices of fractional integrals. We give some illustrative examples to show the applicability and effectiveness of the method.
**Keywords:** Wavelets, fractional differential equations

**Mathematics Subject Classification: 65T60, 65T60, 65T60**

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# On novel exact solutions of nonlinear PDEs using sin – cos / sinh – cosh methods

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**Abstract:** In this study, we investigate a class of nonlinear PDEs by using the extended rational sin – cos / sinh – cosh methods. Some new exact solutions such as singular periodic wave, periodic wave, topological, and singular soliton solutions are constructed. We present 2- and 3-dimensional graphs of some solutions for suitable variables. The considered methods are efficient and powerful tools that can be applied for various wave solutions to different nonlinear physical models.
**Keywords:** Exact solutions, nonlinear PDEs, extended rational sin – cos / sinh – cosh methods

**Mathematics Subject Classification: 35M86, 32W50**

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# Analytic Expression of Local Particle Density of an Ultracold Fermion Gas

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**Abstract:** Theoretical studies carried out on the properties of quantum gases of fermions confined by a harmonic trap potential have grown since the experimental observation by DeMarco and Jin of a fermion quantum atom gas degenerate [1]. Experimentally, temperatures of the order of a fraction of the Fermi temperature have been achieved. Research on quantum gases, either for boson or fermion gases, is a field of research in full development, and this is manifested by the impressive number of works published on the subject. In fact, the systems of ultra-cold atomic quantum gases provide an opportunity to study concepts essential to condensed matter theory, and this is thanks to the sophisticated experiments that have been developed over the past two decades and provide study environments with controllable conditions across light beams (lasers). Thus, interactions such as spin-orbit coupling and periodic potentials that can take place in a solid state environment will be simulated by those that occur in systems formed by cold atoms.

Rotating degenerate fermion gas systems are typically treated with rotating cues to eliminate the time dependence of the potential confining and thus the Hamiltonian of the system. At this turning point, it is well known that the Hamiltonian of the system is modified by the addition $-\overrightarrow{\Omega}\overrightarrow{L}$ [2], $\overrightarrow{\Omega}$ is the angular speed of rotation and $\overrightarrow{L}$ represents the angular momentum of the total system. This angular momentum of the gas exhibits quantum oscillations in function of the number of particles [3], which are analogous to the Haas-van Alphen oscillations of the magnetization in solid state systems [4]. These quantum oscillations are macroscopic manifestations due to the quantification of the energy levels of electrons in the presence of the magnetic field. In fact, an electron gas in a metal shows an oscillatory dependence of physical observables as a function of field strength magnetic, as for example, the oscillations of resistivity (Shubnikov oscillations-de Haas), the oscillation of magnetization, Hall resistance, or specific heat. Those oscillations were predicted by Landau [5] and observed by de Haas-van Alphen. Of the important quantum phenomena that can occur in a fermionic gas degenerate, the quantum Hall effect presents an excellent example because of its relation to the training of Landau levels. For two-dimensional electron gas systems, subjected to an intense magnetic field, the energy levels organize themselves into a structure strongly degenerated by Landau levels. Theoretical studies carried out on degenerate fermions have confirmed that a similar organization occurs in a gas of neutral fermions under the effect of a very fast rotation, leading to various macroscopic quantum effects [6]. It is important to mention that Dalibard et al. used atom-light interaction to "simulate" acting artificial gauge fields on neutral matter in a similar way to particles in a repository rotating [7]. Our study here is focused on a degenerate fermion system confined by a potential harmonic isotropic and rotating at zero temperature. We will look for the exact expression of the local density $\rho\left(\overrightarrow{r}\right)$. We will use a mathematical tool called the Bloch density matrix MDB.
**Keywords:** Potential Harmonic Isotropic, Fermion System, Quantum Oscillations, Bloch Density Matrix, Local Density

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# The fractal flows via scaling-law vector calculus

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**Abstract:** In this report, the scaling-law vector calculus is considered to model the fractal flows. The strong and weak conjectures for the scaling-law flows are obtained in detail. The obtained result is as a potentially mathematical tool proposed to develop an important way of approaching this challenge for the scaling-law flows. This work was accepted for publication in [1].

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# New Exact Solutions of the Time-Fractional Foam Drainage Equation via a Riccati-Bernoulli Sub Ode Method

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**Abstract:** In this paper, a Riccati-Bernoulli sub Ode method is proposed to obtain new exact solutions of the time-fractional Foam Drainage equation. The fractional derivative is considered in the sense of Jumarie's modified Riemann-Liouville fractional derivative (JMRFD). A traveling wave transformation and Riccati-Bernoulli equation are firstly utilized to convert the nonlinear fractional partial differential equation (NFPDE) into a set of algebraic equations. Thereafter exact solutions of the nonlinear fractional partial differential equations can be found by solving a set of algebraic equations. Also, a Bäcklund transformation of the Riccati-Bernoulli equation is applied to get an infinite sequence of solutions of the above equation. Our main intention in this present paper is to indicate that the suggested method is appropriate to obtain the new exact solutions of fractional partial differential equations. It can be said that the main advantage of the mentioned scheme is very simple and easy to apply. As a result, all the obtained results are presented in the paper.

**Keywords:** Bäcklund transformation, Fractional calculus, Jumarie's modified Riemann-Liouville fractional derivative, Nonlinear fractional partial differential equation, Travelling wave transformation
**Mathematics Subject Classification:** 35G25, 26A33, 37K35

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# Generalized Derivations of Ternary Hom-Jordan Algebras

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**Abstract:** The purpose of the talk is to study the relationships between a Hom-Jordan algebras and its induced ternary Hom-Jordan algebras. We introduce the notion of alpha-k-derivation, alpha-k -quasiderivation and generalized alpha-k-derivation of ternary Hom-Jordan algebras, and we give some construction of ternary Hom-Jordan algebras.

**Keywords:** Generalized derivations, quasiderivations, centroid, ternary Hom-Jordan algebras.
**Mathematics Subject Classification:** 17B40, 17C10, 17C99

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# On The Asymptotic Stability of Delayed Discrete Dynamical Systems

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**Abstract:** In this communication, we consider a discrete dynamical system with delay and employ discrete variant of exponential dichotomy together with fixed point theory to prove asymptotic stability of the zero solution under sufficient conditions. The main result of the paper is also implemented to an infinite delayed Volterra difference system.

**Keywords:** Asymptotic stability, Contraction, Discrete dynamical system, Exponential dichotomy
**Mathematics Subject Classification:** 39A10, 39A30, 39B72

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# Warped Product Lagrangian Submanifolds in Complex Space

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**Abstract:** The procedure of constructing a Lagrangian immersion in the complex projective space, starting with two other Lagrangian immersions into complex projective spaces of lesser dimension is known as a Calabi product, motivated by the similar construction in the affine differential geometry. In particular, one may consider a point instead of the one of the immersions, and in both cases the submanifold has a warped product structure of the interval and one or two Lagrangian immersions. Such Lagrangian submanifold then admits a splitting of the tangent bundle into orthogonal subbundles defined in terms of the corresponding second fundamental form, in case of a point and an immersion decomposition consists of two components and in case of a proper Calabi product, decomposition has three components. The generalization of this notion was investigated for Lagrangian immersions in non-flat complex space forms. Here we study the flat case and investigate the properties of the Lagrangian immersions with tangent bundle admitting the decomposition in question and we give explicit expressions for such immersions.

**Keywords:** Lagrangian submanifolds, warped product, complex space.
**Mathematics Subject Classification:** 53B25, 53B20

# Analysis and Control of Neural Dynamics States

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**Abstract:** We have proposed a new strategy using predictive control to remove periodic orbits and higher-order orbits when the dynamics evolve in continuous time and in which the gain matrix of the state feedback controller is determined from sufficient conditions for global asymptotic stability. This new result is obtained by using the approximation of spectral radius of matrix theory. The suppression of stable periodic orbits of neural dynamics forms the basis of our approach to the prevention of epilepsy. Simulated solutions are given via Matlab at the end of the analytical study.

**Keywords:** Predictive control, Global asymptotic stability, Spectral radius, FitzHugh-Nagumo FHN Model, Periodic orbits.

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# High Dimensional Data: Generalized Partially Linear Single-Index Model for Functional Non-Parametric Data

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**Abstract:** Generalized linear models (GLM) provide a unified framework of likelihood for parametric regression analysis and are also an extension of linear models. Indeed, they allow to model, in a parametric way, the relation between a transformation of the average response and some covariates. In this paper, we consider the estimation of generalized functional non-parametric partially linear single-index models with an unkown and smooth function (single-index link function), an unknown single-index coefficient vector, an unknown regression operator and with a known link function in the generalized model. These models would be called "Non-Parametric Generalized Partially Linear Single-Index Models For Functional Data" (NPGPLSIMFD), where the systematic component in the model has a flexible semi-parametric form with a general link function. We propose an efficient and practical approach to (i) estimate the single-index link function, (ii) estimate the single-index coefficients as well as (iii) the non-parametric component of the model. The estimation procedure is developed by applying the quasi-likelihood estimation methodology. After constructing the estimators of the function and the coefficient described above, we present a wide range of properties of these estimators and we give the rates of convergence of the constructed estimators. Moreover, by making use of normalized cubic B-splines basis approximation and Fisher score iterations, we show the efficiency and the practicality of our estimation methodology on some simulated and real data.

**Keywords:** Asymptotic normality, functional data analysis (FDA), polynomial splines, quasilikelihood, nonparametric regression, single-index model.
**Mathematics Subject Classification:**

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# Design of Hybrid Sliding Mode Control via Fuzzy Logic Synthesized by Backstepping Approach for Double Star Induction Motor

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**Abstract:** This paper proposes design of a novel scheme control based on sliding mode control via fuzzy logic synthesized by backstepping (FBSMC), to regulate the speed of a dual star induction machine (DSIM), to make guaranteeing the stability and the robustness performance machine system. For this purpose, an appropriate combination of the Backstepping sliding mode controller (BSMC) Improved by fuzzy logic is adopted. This control strategy based on the hybrid controllers are used to guaranteeing the machine stable, preferment and robust to obtain a better dynamic response and anti-disturbance performance. We present and discuss the results of the simulations obtained with MatLab/Simulink.

**Keywords:** Hybrid controllers, sliding mode, DSIM, backstepping, fuzzy logic.

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# Hilbert-Schmidt Numerical Radius for Operator Matrices

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**Abstract:** This work deals with several Hilbert-Schmidt numerical radius inequalities of $2\times 2$ operator matrices of class $C_2$. More generally, the Hilbert-Schmidt numerical radius of $n \times n$ operator matrices. This numerical radius defined as $w_2(A) = sup_{\theta\in R}\|Re(e^{i\theta}A\|_2$, with $A\epsilon B(H)$, the $C^*$-algebra of all bounded linear operator matrices on a Hilbert space H, which is a case of the new generalization of numerical radius defined by Abu-Omar and Kittaneh in [1] as $w_N(A) = sup_{\theta\in R}N(Re\left(e^{i\theta}A\right)), whenN(\bullet) = \|\bullet\|_2(theHilbert - Schmidtnorm)$.

**Keywords:** Numerical radius, Hilbert-Schmidt norm, operator matrix, Inequalities
**Mathematics Subject Classification:** 47A12, 47A30 , 47A63

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# Upper and Lower Bounded of Numerical Radius for Certain Operators

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**Abstract:** This work deals with several inequalities for the numerical radius of bounded linear operators in Hilbert space. Some results for vectors in inner product space due to recent papers such as works of Omar Hirzallah, Fuad Kittaneh and all, who introduced new inequalities. Further, we present some upper bounded for numerical radius of the sum and the product of bounded linear operators.

**Keywords:** Numerical radius, operator norm, operator adjoint, operator normal
**Mathematics Subject Classification:** 46Cxx, 47A12, 15A60

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# On the Stable F-Harmonic Maps

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**Abstract:** We prove that any stable f-harmonic map from sphere $S^n$ to Riemannian manifold (N; h) is constant, where f is a smooth positive function on $S^n \times N$ satisfying one condition with $n > 2$. We also prove that any stable f-harmonic map $\varphi$ from a compact Riemannian manifold $(M; g)$ to $S^n(n > 2)$ is constant where, in this cas, f is a smooth positive function on M $\times S^n$ satisfying $\Delta^{S^n}(f)o\varphi \leq 0$.

**Keywords:** harmonic maps, f-harmonic maps, stable f-harmonic maps
**Mathematics Subject Classification:** 53C43, 58E20

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# Fractional p-Laplacian Problem Involving Critical Growth

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**Abstract:** In this work we study a nonhomogeneous problem involving p-fractional Laplacian and critical exponent with changing sign data. Using variational methods, we establish the existence of at least two positive solutions under suitable conditions.

**Keywords:** Fractional p-Laplacian, fractional critical exponent, variational method
**Mathematics Subject Classification:** 35R11, 35J60, 35J62, 35J20

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# Optimal Consumption and Investment for Exponential Utility Function

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**Abstract:** We investigate an optimal consumption and investment problem for Black-Scholes type financial market on the whole investment interval $[0, T]$. We formulate various utility maximization problem, which can be solved explicitly. The method of solution uses the convex dual function (Legendre transform) of the utility function. Related to this concept, we introduce and study the convex dual of the value function for our problem.

**Keywords:** Portfolio optimization, Black-scholes model, optimal consumption, exponential utility, utility maximization
**Mathematics Subject Classification:** 91B28, 93E20

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# On the Toeplitz Determinants whose elements are the coefficients of the functions belong to the class of alpha-close-to-convex functions

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**Abstract:** Let $\mathcal{A}$ be the class of analytic functions in the unit disc $\mathbb{U}$ which are of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. For $0 \leq \alpha < 1$, and for a starlike function $\phi(z)$, let $C_\alpha$, be the class of all functions $f \in \mathcal{A}$ satisfying the condition $\mathrm{Re}\left\{(1-\alpha)\frac{zf'(z)}{\phi(z)} + \alpha\frac{(zf'(z))'}{\phi'(z)}\right\} > 0$. In this paper we consider initial Toeplitz matrices whose elements are the coefficients $a_n$ of the function $f$ in the class $C_\alpha$ and obtain their upper bounds.

**Keywords:** Starlike function, convex function, close-to-convex function, Hankel determinant and Toeplitz determinant
**Mathematics Subject Classification:** 30C45

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# Ulam-Hyers-Rassias Stability of a Fractional Boundary Value Problem with Caputo Derivative

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**Abstract:** In this talk, we investigate the Ulam-Hyers stability and Ulam-Hyers-Rassias stability of a class of fractional four-point boundary value problem involving Caputo derivative and with a given parameter. By using contraction principles, sufficient conditions are obtained to guarantee the uniqueness of solution. Therefore, we obtain sufficient conditions for the stability of that class of nonlinear fractional boundary value problem on the continuous function space. The presented results improve and extend some previous research.

**Keywords:** Fractional boundary value problems, Caputo derivative, Ulam-Hyers stability, Ulam-Hyers- Rassias stability
**Mathematics Subject Classification:** 26A33, 34B15, 34D20, 47H10

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# Customers' Equilibrium Behavior in the observable M/M/1 Queue with Differentiated Vacations

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**Abstract:** We study the strategic behavior of customers based on a reward-cost structure in a single server queueing system with differentiated vacations. The server takes type 1 vacation after a busy period and type 2 vacation after returning from a vacation (type 1 or type 2), while there is no customer waiting in the system. The customers decide whether to join or not the queue based on the available information upon their arrival.We consider two cases: Fully observable queue and Almost observable queue.For each case, we evaluate the equilibrium strategies and analyze the stationary behavior of the system. Through numerical results, we show the impact of information level and some system parameters on the equilibrium thresholds and social benefit.

**Keywords:** Queueing systems, vacations, equlibrium, strategies, social benefit
**Mathematics Subject Classification:** 35Q35, 37K45

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# Performance of a Single Unreliable Server Retrial Queueinq System with Negative Arrivals

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**Abstract:** In this paper, we consider a single repairable server retrial queueing system with negative arrivals and two types of breakdowns (passive and active). We assume that when an active breakdown occurs the customer in service is lost. After repair, the server becomes free. In order to derive the system performance measures we use the generating function method. We also study the reliability of the server. Finally, we show numerically the effects of some parameters on the mean number of customers in the orbit and the server state probabilities.

**Keywords:** Retrial Queues, Negative Arrivals, reliability

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# A Comparative Study between Fuzzy Logic and Artificial Neural Network Models for Reservoir Property Estimation from Well Logs Data: A Case Study of Hassi Berkine Oil Field

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**Abstract:** Porosity and permeability are the most important characteristics in reservoirs characterization. In recent years, a new estimation methods have been introduced which are more precise and applicable than conventional methods as well as artificial intelligence. In this study a Fuzzy logic has shown very reliable results in the field of petroleum characterization to describe the properties of the reservoir. The fuzzy model of Mamdani was formulated to predict the porosity and permeability in the studied area of the oil field of Hassi Berkine in order to determine the membership functions where the Fuzzy k-Means algorithm are used with a center of gravity (COG) method in the Deffuzziffication phase. In addition, artificial neural networks have been applied to predict porosity and permeability where the well-logs data are subdivided into three parts: 70% of the data is used for the Training phase and 15% for the validation and 15% for the testing phase. To show the effectiveness of fuzzy logic technique,compared to other methods, the Multiple correlation coefficient (R2) was calculated between the porosity and permeability values predicted by the two methods and the values measured.

**Keywords:** Fuzzy logic, artificial neural networks, Porosity, Permeability, Hassi Berkine

# Generalized Stochastic Petri Nets for Performance Analysis of a K-out-of-n System

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**Abstract:** We study a K-out-of-M + W + C retrial system with two reliable repairers, using Generalized Stochastic Petri Nets (GSPNs). The life times of primary components and warm standby components are assumed to be exponentially distributed random variables. Upon arrival, a failed component is repaired immediately if one of the two repairers is found free, and the failed component enters a retrial orbit if the two repairers are busy. From the GSPN model, we derive a Continuous Time Markov Chain (CTMC) , then we obtain the main stationary probabilities and some performance measures

**Keywords:**Stochastic petri nets, Retrial systems, Performance analyses

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# A Practical Approach to Solve the Gardner and the Potential Kdv Equations

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**Abstract:** In this article, we present the exponential expansion method as an analytical method to give traveling wave solutions of two nonlinear evolution equations such as Gardner and potential KdV equations, which have various applications in physics and applied mathematics. The simulation of the obtained results is presented in the forms of graphs to illustrate the reliability and efficiency of our method.

**Keywords:** Exponential expansion method, Nonlinear evolution equation, Gardner equation, Potential Korteweg–de Vries equation, Traveling wave solution
**Mathematics Subject Classification:** 35Q53, 35G20, 35C07

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# Boundedness of Non Regular Pseudodifferential Operators on Variable Besov Spaces

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**Abstract:** We study the boundedness of non regular pseudo differential operators, with symbols belonging to certain vector-valued Besov space, on Besov spaces with variable smoothness and integrabilty. These symbols include the classical Hörmander classes.

**Keywords:** Variable Besov spaces, pseudodifferential operators, non regular symbols
**Mathematics Subject Classification:** 46E35, 47B30, 35S50

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# A Dirichlet-Transmission Boundary Value Problem for Darcy-Forchheimer-Brinkman and Navier-Stokes Equations in Bounded Lipschitz Domains

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**Abstract:** A boundary value problem for the nonlinear Darcy-Forchheimer-Brinkman and Navier-Stokes equations in two adjacent bounded Lipschitz domains from $\mathbb{R}^n$, (n=2,3), has been studied when one of the domains was completely enclosed within the other and the transmission conditions on the common interface were linear while the condition on the exterior boundary was of Dirichlet type. First, the Dirichlet-Transmission problem associated with the Brinkman and Stokes linear equations is solved by applying potential theory techniques. Then, using the well-posedness of the aforementioned problem and involving a fixed point theorem, we proved the existence and uniqueness of the weak solution of the Dirichlet-Transmission problem for nonlinear Darcy-Forchheimer-Brinkman and Navier-Stokes, when boundary data are sufficiently small. A numerical simulation of a special case of the considered problem, corresponding to the lid-driven cavity with internal porous square block, is also performed using Comsol software.

**Keywords:** Dirichlet-Transmission boudary value problem, bounded Lipschitz domains, Darcy-Forchheimer-Brinkman equation, Navier-Stokes equation, potential theory, Sobolev spaces
**Mathematics Subject Classification:** 35J25, 76D05, 76S05, 76M99

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# The New Solutions of Conformable Fractional Kuramoto-Sivashinsky Equations

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**Abstract:** We have studied conformable q-homotopy analysis transform method for obtaining the new solutions of conformable fractional Kuramoto-Sivashinsky equations. It has been been generated a new method which is combined with q-homotopy analysis transform method and conformable fractional derivative. The graphs of the solutions for conformable fractional Kuramoto-Sivashinsky equations are plotted via MAPLE software.

**Keywords:** Conformable q-homotopy analysis transform method, conformable fractional Kuramoto-Sivashinsky equation, conformable fractional derivative
**Mathematics Subject Classification:** 35R11, 35C05, 65R10

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# Global Asymptotic Stability of Kind of Nonlinear Neutral Dynamic Equations on Time Scales

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**Abstract:** This paper is mainly concerned the global asymptotic stability of the zero solution of a class of nonlinear neutral dynamic equations in C1rd. By converting the nonlinear neutral dynamic equation into an equivalent integral equation, our main results are obtained via the Banach contraction mapping principle.

**Keywords:** Fixed points, neutral dynamic equations, asymptotic stability, timescales
**Mathematics Subject Classification:** 34K20, 34K30, 34K40

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# The Asymptotic Formula for the Sum of the Fourth Degrees of the Negative Eigenvalues of the Second Order Differential Operator in the Semi-Axis

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**Abstract:** In this paper we study the negative spectrum of a second order differential equation an semi-axis. Asymptotic formula are proved for the distribution function of negative eigenvalues.

**Keywords:** Differential operator, spectrum, eigenvalues, distribution function, Hilbert space, self-adjoint operator, lower bounded operator
**Mathematics Subject Classification:** 35J30, 35P05,47A10

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# Convergence of Branching Processes with Migration and Continuous Time

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**Abstract:** The homogeneous branching process with migration and continuous time is considered. The generating function of the process and limit theorem for subcortical branching process with migration are found.

**Keywords:** Branching process, migration, continuous time, generating function
**Mathematics Subject Classification:** 60G10, 60J28, 93B40

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# Existence and Uniqueness of Solution for a Fractional Order SIR Model

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**Abstract:** This work is about existence and uniqueness of solution for a fractional order SIR model. We use the fractional order derivative in Caputo sense and the concerned result have been establish by using fixed point theory approach.

**Keywords:** SIR model, fractional derivative

**Mathematics Subject Classification:** 26A33

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# Periodic Solutions of a Planar Perturbed System via the Melnikov Method and the Averaging Theory

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**Abstract:** In this work, we study the maximum number of limit cycles bifurcating from the periodic solutions of a center. We determine an upper bound for the maximum number of limit cycles of a perturbed equation in the four cases where m and n are even and odd. The main tools used are the averaging theory of first order and the Melnikov method.

**Keywords:** Periodic solution, averaging method, Melnikov method
**Mathematics Subject Classification:** 34C25, 34C29, 34A26

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# Baranchik-Type Estimator of Spherical Symmetry with Residual Under Balanced Loss Functions

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**Abstract:** This article considers the problem of estimating the mean vector of $d-$dimensional spherically symmetric distributed $X$ when the scale parameter is known but when a residual vector $U$ is available: more precisely, let $(X, U) \in R^{d+k}$ be a random vector around $(\theta, 0) \in R^{d+k}$. The loss functions is assumed to be modifications of balanced loss functions the form: **(i)** $\omega\rho(\|\delta-\delta_0\|^2)+(1-\omega)\rho(\|\delta-\gamma(\theta)\|^2)$ and **(ii)** $\ell(\omega\|\delta-\delta_0\|^2+(1-\omega)\|\delta-\gamma(\theta)\|^2)$ where $\delta_0$ is a target estimation of $\gamma(\theta)$, and where $\rho$ and $\ell$ are increasing and concave functions. For $d \geq 4$ and the target estimator $\delta_0 = X$, we provide Baranchik-type estimators that dominate $\delta_0 = X$ and are minimax.

**Keywords:** Spherical symmetry with residual, balanced loss functions, Baranchik-type estimators

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# Unimodular Matrix on Shallow Water Wave Theory Unimodularity Through Matrix Method

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**Abstract:** The matrix method used in many studies of wave theory, for easy calculations, is described and used in the shallow water breaking theory. We deal with shallow water wave breaking and the method of unimodular matrix. Introducing a class of nonlinear water waves, the solution in the stratification regions using matrix method through unimodularity is given.

In this paper, we present the stratification approximated by n-layers. Our method firstly involves a 2x2 matrix. Taking the production of n 2x2 matrices, we choose the layers with linear variation. The main part of our work covers the fact that energy conservation law is satisfied. For this, the unimodularity of the matrices is used. The models are tested against experiments concerning periodic wave transformation. The density and the speed of waves vary exponentially with depth. We conclude with experiments and some important conclusions.

**Keywords:** Shallow water waves, wave breaking, unimodular matrix, approximation problem

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# Hermite-Hadamard-Fejer Type Inequalities for B-convex Functions

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**Abstract:** In this work, some inequalities for a class of abstract convexity are given [1]. Inequalities are important applications of the convexity. One of the most remarkable is Hermite-Hadamard-Fejer, which has applications to the trigonometric functions and mathematical physics [2]. The concept of this study is examining types of the Hermite-Hadamard-Fejer Inequality for B-convex functions that is an abstract convexity type[3,4,5].

**Keywords:** Convexity, Abstract Convexity, Hermite-Hadamard-Fejer Inequality, B-convexity
**Mathematics Subject Classification:** 46N10, 52A20

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# An Examination of the Gauss-Jacobi Numerical Method for an Abstract Convexity Type

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**Abstract:** Numerical methods are using to obtain an approximation in general [1]. The method of Gauss-Jacobi is one of them, which can be examine for the convex functions [2]. In this work, we study this numerical method for B-convex functions [3]. The abstract convexity type B-convexity has some applications to the mathematical economy, thus it stands out in recently [4,5]. Since both of the applications to convexity type and numerical method, this study also holds significance.

**Keywords:** Gauss-Jacobi, Abstract Convexity, Numerical Method, Inequality
**Mathematics Subject Classification:** 47N10, 52A20 90C25

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# Two-class Motor Imagery EEG Signal Classification Using Curvelet, PCA and K-NN

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**Abstract:** The most important signal for brain-computer interfaces (BCI) is the electroencephalogram (EEG). Motor imagery based BCI is a widely used approach nowadays. To categorize two-class motor imagery EEG signals (left and right hands motor imagery), this research suggests a method based on the combination of curvelet transform, principal component analysis, and k-NN. The signal recordings from the C3, C4 and Cz electrodes form BCI Competition III dataset IIIa are filtered 5-30Hz, and curvelet transform is applied to the signals. Feature data is created by calculating the mean, standard deviation, entropy, median, maximum value, kurtosis, skewness and log-variance over the transformation coefficients. Classification is carried out in two ways; by presenting each computed feature data to the classifiers separately, and by performing feature selection on whole feature dataset using PCA. Classification is performed using k-NN, and the best classification results are 71,67% for both standard deviation and log-variance, 68,33% with PCA.

**Keywords:**BCI, EEG, Motor imagery, PCA, Curvelet transform, k-NN

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# Two-class Motor Imagery EEG Signal Classification Via Four Frequency Range and Curvelet Transform

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**Abstract:** BCI (brain–computer interface) technology is a way of human–computer interaction, and the classification of electroencephalogram (EEG) signals is critical for BCI systems. This study aims to investigate the success of features obtained by curvelet transform to classify left and right-hand motor imagery task. The log-variance values calculating over the curvelet coefficients for each EEG channel (C3, Cz, and C4) separated into four frequency range are used to build feature sets. After investigating the classification success of these feature sets created for each frequency range, classification success is re-evaluated by combining all feature sets and applying PCA. The best classification results come from features in the 8-13Hz frequency band, and the entire collection of features with PCA is set. Those are 68.33% and 73.33%, respectively.

**Keywords:** Motor imagery EEG, PCA, Curvelet transform, k-NN

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# A Quasistatic Contact Problem for Viscoelastic Materials with Damage and Thermal Effects

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**Abstract:** In this work, we consider a quasi-static contact problem for a thermo visco-elastic body with damage between a thermo-viscoelastic body and a rigid obstacle. The contact is frictional and bilateral. Witch results the damage of contacting surface. The evolution of the damage is described by an inclusion of parabolic type. We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem. The proof is based on a classical existence and uniqueness result on parabolic inequalities, differential equations and fixed point argument.

**Keywords:** Thermo viscoelastic, variational inequality, damage field, fixed point
**Mathematics Subject Classification:** 74F05, 74M15, 74H20, 74R20, 74H25

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# An Optimized Hybrid Block Method with at least Fifth-order Convergence under Fixed and Adaptive Stepsize Formulation

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**Abstract:**We have developed a new one-step optimized block method with at least fifth algebraic order of convergence for numerically solving first- and higher-order initial-value problems in ordinary differential equations. It will be constructed using a continuous approximation achieved through interpolation and collocation techniques at three intra-step points chosen by optimizing the local truncation errors of the main formulae. Furthermore, suitable reformulation and implicit five-stage RK-type versions of the proposed block method are also developed. The theoretical analysis, including A-stability, consistency, convergence, and order stars, is considered. Both fixed and adaptive stepsize approaches are presented of the proposed methods. When tested on challenging differential systems, the proposed strategy gives better accuracy, as revealed through the efficiency plots and the error distribution tables, including the machine time measured in seconds.

**Keywords:** Collocation; A-stability; Adaptive stepsize
**Mathematics Subject Classification:** 65L05, 65L07, 65L20

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# On The Karci's Fractional Derivative

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**Abstract:** This study aims to give information about Karci's new fractional derivative. Its properties are given by comparison with popular fractional derivative approaches. In addition, some examples illustrating the concept of Karci's fractional derivative are given. Finally, some new properties are obtained for Karci's fractional derivative.

**Keywords:** Fractional Calculus, Fractional derivative, Fractional integral
**Mathematics Subject Classification:** 26A24, 26A33

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# Application of the Discrete Geometrical Invariants for detection of the trace mixtures in electrochemical measurements

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**Abstract:** Previous work has determined the discrete geometrical invariant (DGI) in 2D space,which allows the analysis of a pair of random sequences ([$r1k$, $r2k$], $k$ = 1,2,...$N$)containing an equal number of data points to be reduced to eight "universal" parametersthat present different inter-correlations between the two compared sequences. Theseeight parameters can serve as a "universal" platform for comparison of various randomsequences of different natures. In this paper, we derive mathematical expressions for theDGI in 3D space, which represent three random sequences in the form of a "trajectory" ofan "imaginary" particle in 3D space. The DGI is of the fourth order in 3D space and allowsthree random sequences ({$r1k$, $r2k$, $r3k$}, $k$ = 1,2,...$N$) to be compared with one another.This unified and "universal" platform identifies (in total) six surfaces and 13 reduced andcompact parameters obtained from 28 basic moments and their intercorrelations up tothe fourth order, inclusively. The transcendental numbers $\pi$ and E (Euler constant) are considered as examples of the use of the DGI method, and their 3D images are derivedtogether with values for the 13 quantitative parameters that differentiate them from eachother. An application of the method, specifying electrochemical data,is also presented to illustrate the potentially wide application of the approach to theidentification and classification of nominally random data sequences.

# Fractional logistic differential equations

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**Abstract:** We review some general aspects of differential equations of fractional order. As a model we focus on the fractional logistic differential equation by using different fractional calculus and approaches. A compartmental model of the COVID-19 epidemic is presented as an application.

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# On Λ-Fractional Analysis and Mechanics

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**Abstract:** Λ-Fractional Analysis has been introduced just to fill up the mathematical gap exhibited in fractional calculus, where the various fractional derivatives fail to fulfill the prerequisites demanded by Differential Topology. Nevertheless, the various advantages exhibited by the fractional derivatives, and especially their non-local character, attracted the interest of the physicists, although the majority of them try to avoid it. The introduced Λ-fractional analysis can generate Fractional Geometry since the Λ-fractional derivatives generate differentials. The Λ-fractional analysis is introduced to Mechanics, just to formulate non-local response problems with the demanded mathematical accuracy. Further, fractional peridynamic problems with horizon are suggested.

**Keywords:** Fractional-order, fractal dimension, Fractional Integral, Fractional Derivative, Riemann-Liouville Fractional derivative, Λ-fractional derivative, left and right Λ-spaces, Cantor set, fractional horizon.

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# A numerical scheme for fractional order population dynamics model

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**Abstract:** In this work few real models with integer and arbitrary orders such as logistic equation, predator-prey problem SEIRS epidemic, Ebola disease, coronavirus (COVID-19) have been shown by different type of fractional derivatives.

After that a numerical method is presented for solving these type of fractional order problems. The method is based on three-step Adams-Bashforth scheme. The fractional order logistic equation and predator-prey problem are solved as test examples when the fractional derivative is in the Caputo sense.

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# Mathematical modeling in Fractional calculus and its application in current scenario

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**Abstract:** Mathematical models are mainly used to depict real world problems that humans encounter in their daily explorations, investigations and activities. However, these mathematical models have some limitations as indeed the big challenges are the conversion of observations into mathematical formulations. If this conversion is inefficient, then mathematical models will provide some predictions with deficiencies. A specific real-world problem could then have more than one mathematical model, each model with its advantages and disadvantages. In the last months, the spread of covid-19 among humans have become fatal, destructive and have paralyzed activities across the globe. The lockdown regulations and many other measures have been put in place with the hope to stop the spread of this deathly disease that have taken several souls around the globe. Nevertheless, to predict the future behavior of the spread, humans rely on mathematical models and their simulations. While many models, have been suggested, it is important to point out that all of them have limitations therefore newer models can still be suggested. In this paper, we examine an alternative model depicting the spread behavior of covid-19 among humans. Different differential and integral operators are used to get different scenarios.

**Keywords:** Mathematical model, COVID-19, Corona virus, Numerical Scheme

# Multiple positive solutions for a nonlocal PDE with critical Sobolev-Hardy and singular nonlinearities via perturbation method

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**Abstract:** We prove the existence of multiple positive solutions of a non local problem. The main idea is to look for critical points of the associated energy functional. The first solution existence is proven as a local minimizer of the energy functional. The second one is the limit of a sequence of solutions of a perturbed problem.

**Keywords:** Nonlocal operator, singular nonlinearities, Hardy Sobolev exponent, variational and approximation method

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(Online) International Symposium on Applied Mathematics and Engineeringwas held online due to COVID-19 Pandemic. At the Symposium, people from different parts of the world and from different countries had the opportunity to work together. This Symposium is a prime example of how people can contribute to science together.

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