

Distance based similarity measures for interval-valued intuitionistic fuzzy soft sets and its application

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Abstract: In this article we have introduced the concept of similarity measure for interval-valued intuitionistic fuzzy soft sets based on distance between two interval-valued intuitionistic fuzzy soft sets, some examples and basic properties are also studied. An algorithm is developed in intuitionistic fuzzy soft set setting for a decision making method. Lastly a fictitious numerical example is given to illustrate possible application in a medical diagnosis problem.

Keywords: Soft set, fuzzy soft set, intuitionistic fuzzy soft set, interval-valued intuitionistic fuzzy soft set, Hamming Distance, Euclidean Distance, similarity Measure.

1 Introduction

Similarity measure is an important topic in the fuzzy set [28] theory. Similarity measure indicates degree of similarity between two fuzzy sets. Wang [24] first introduced the concept of similarity measure of fuzzy sets and gave a computational formula. Since then, similarity measure of fuzzy sets has attracted several researchers' interest and has been investigated more. Similarity measure of fuzzy sets is now being extensively applied in many research fields such as fuzzy clustering, image processing, fuzzy reasoning, fuzzy neural network, pattern recognition, medical diagnosis, game theory, coding theory and several problems that contain uncertainties.

Similarity measure between two fuzzy sets (interval-valued fuzzy sets, interval-valued fuzzy soft sets, intuitionistic fuzzy sets, intuitionistic fuzzy soft sets, interval-valued intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy soft sets) have been defined by many authors [4,8,11,19,20,21,22,23,25,26]. There are different techniques for measuring similarity measure between two fuzzy sets or between two fuzzy soft sets. Some of them are based on distances and some others are based on matching function. There are techniques based on set-theoretic approach also. Some properties are common to these measures and some are not, which influence the choice of the measure to be used in several applications. One of the significant differences between similarity measure based on matching function S and similarity measure S' based on distance is that if $A \cap B = \emptyset$, then $S(A, B) = 0$ but $S'(A, B)$ may not be equal to zero, where A and B are two fuzzy sets. Therefore, distance-based measures are also popular.

Distances between two intuitionistic fuzzy sets A and B is defined by Szmidt and Kacprzyk [22]. In some real life problems it is often needed to compare two sets. The sets may be fuzzy, may be vague etc. For example in a problem of image recognition we need to know whether two patterns or images are identical or approximately identical or at least to

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what degree they are identical. Several researchers like Chen [4,5,6], Hu and Li[7] etc. have studied the problem of similarity measure between fuzzy sets and vague sets. P. Majumdar and S. K. Samanta [12,13,14] have studied the similarity measure of soft sets, fuzzy soft sets and intuitionistic fuzzy soft sets. Cagman and Deli[3] studied similarity measure of intuitionistic fuzzy soft sets. Recently A. Mukherjee and S. Sarkar[20] introduced several similarity measures for interval valued intuitionistic fuzzy soft sets.

In this work several distances between two interval-valued intuitionistic fuzzy soft sets are defined and based on these distances similarity measure between two interval-valued intuitionistic fuzzy soft sets are proposed. An application of similarity measure between two interval-valued intuitionistic fuzzy soft sets in a medical diagnosis problem is also illustrated.

2 Preliminaries

In this section we briefly review some basic definitions related to interval-valued intuitionistic fuzzy soft sets which will be used in the rest of the paper.

Definition 2.1 [28] Let X be a non empty collection of objects denoted by x . Then a *fuzzy set* (in short notation FS) α in X is a set of ordered pairs having the form, $\alpha = \{(x, \mu_\alpha(x)) : x \in X\}$ where the function $\mu_\alpha : X \rightarrow [0, 1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in α . The interval $M = [0, 1]$ is called membership space.

Definition 2.2 [29] Let $D[0, 1]$ be the set of closed subintervals of the interval $[0, 1]$. An *interval-valued fuzzy set* in X , $X \neq \phi$ and $Card(X) = n$, is an expression A given by $A = \{(x, M_A(x)) : x \in X\}$, where $M_A : X \rightarrow D[0, 1]$.

Definition 2.3 [1] Let X be a non empty set. Then an *intuitionistic fuzzy set (IFS for short)* A is a set having the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$ where the functions $\mu_A : X \rightarrow [0, 1]$ and $\gamma_A : X \rightarrow [0, 1]$ represents the degree of membership and the degree of non-membership respectively of each element $x \in X$ and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.4 An *interval valued intuitionistic fuzzy set* A over a universe set U is defined as the object of the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in U\}$, where $\mu_A(x) : U \rightarrow D[0, 1]$ and $\gamma_A(x) : U \rightarrow D[0, 1]$ are functions such that the condition: $\forall x \in U, \sup \mu_A(x) + \sup \gamma_A(x) \leq 1$ is satisfied (where $D[0, 1]$ is the set of all closed subintervals of $[0, 1]$).

Definition 2.5 [10,15] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair is called a *soft set* over U , where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 2.6 [16] Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and AB . Then the pair (F, A) is called a *fuzzy soft set* over U , where F is a mapping given by $F : A \rightarrow I^U$.

Definition 2.8 [27] Let U be an initial universe and E be a set of parameters, a pair (F, E) is called an *interval valued-fuzzy soft set* over $F(U)$, where F is a mapping given by $F : E \rightarrow F(U)$, where $F(U)$ is the set of all interval-valued

fuzzy sets of U .

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U , thus, its universe is the set of all interval-valued fuzzy sets of U , i.e. $F(U)$. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $F(U)$, $\forall e \in E, F(U)$ is referred as the interval fuzzy value set of parameters e , it is actually an interval-valued fuzzy set of U where $x \in U$ and $e \in E$, it can be written as: $F(e) = \{(x, \mu_{F(e)}(x)) : x \in U\}$ where $F(U)$ is the interval-valued fuzzy membership degree that object x holds on parameter.

Definition 2.9 [9] Let U be an initial universe and E be a set of parameters. Let $IVIFS^U$ be the set of all interval valued intuitionistic fuzzy sets on U and . Then the pair (F,A) is called an *interval valued intuitionistic fuzzy soft set (IVIFSS)* over U , where F is a mapping given by $F : A \rightarrow IVIFS^U$.

3 Similarity measures of interval-valued intuitionistic fuzzy soft sets

In this section we define some distances between two interval-valued intuitionistic fuzzy soft sets (IVIFSSs) and similarity measures of IVIFSSs . Some properties and examples are also studied.

Definition 3.1 Let be the universe, $E = \{e_1, e_2, e_3, \dots, e_m\}$ be the set of parameters, and (F,A) , be two IVIFSSs on U with their intuitionistic fuzzy approximation functions $F_A(e_i) = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$ and $G_B(e_i) = \{(x, \mu_B(x), \nu_B(x)) : x \in U\}$ respectively, where $\mu_A(x): U \rightarrow D[0,1]$ and $\nu_A(x): U \rightarrow D[0,1]$ are functions such that the condition: $\forall x \in U, \sup \mu_A(x) + \sup \nu_A(x) \leq 1$ and $D[0,1]$ is the set of all closed subintervals of $[0,1]$. If $A = B$ then we define the following distances between (F,A) and (G,B) :

1. Hamming distance;

$$d_H(F, G) = \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \{ |\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j)| + |\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j)| \}$$

2. Normalized Hamming distance;

$$d_{NH}(F, G) = \frac{1}{2n} \sum_{i=1}^m \sum_{j=1}^n \{ |\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j)| + |\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j)| \}$$

3. Euclidean distance;

$$d_E(F, G) = \left[\frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \{ (\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j))^2 + (\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j))^2 \} \right]^{\frac{1}{2}}$$

4. Normalized Euclidean distance :

$$d_{NE}(F, G) = \left[\frac{1}{2n} \sum_{i=1}^m \sum_{j=1}^n \left\{ (\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j))^2 + (\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j))^2 \right\} \right]^{\frac{1}{2}}.$$

Where

$$\bar{\mu}_F(e_i)(x_j) = \frac{1}{2} \left\{ \sup \mu_F(e_i)(x_j) + \inf \mu_F(e_i)(x_j) \right\},$$

$$\bar{\mu}_G(e_i)(x_j) = \frac{1}{2} \left\{ \sup \mu_G(e_i)(x_j) + \inf \mu_G(e_i)(x_j) \right\},$$

$$\bar{\nu}_F(e_i)(x_j) = \frac{1}{2} \left\{ \sup \nu_F(e_i)(x_j) + \inf \nu_F(e_i)(x_j) \right\},$$

$$\bar{\nu}_G(e_i)(x_j) = \frac{1}{2} \left\{ \sup \nu_G(e_i)(x_j) + \inf \nu_G(e_i)(x_j) \right\}.$$

Definition 3.2 Let U be universe and E be the set of parameters and (F, A) , (G, B) be two IVIFSSs on U , where $A = B \subseteq E$. Then based on the distances defined in definition 3.1 similarity measure of (F, A) and (G, B) is defined as

$$SM(F, G) = \frac{1}{1 + d(F, G)} \quad (1)$$

Another similarity measure of (F, A) and (G, B) can also be defined as

$$SM(F, G) = e^{-\alpha d(F, G)}. \quad (2)$$

Where $d(F, G)$ is the distance between the IVIFSSs (F, A) and (G, B) and α is a positive real number, called steepness measure.

Definition 3.3 Let (F, A) and (G, B) be two IVIFSSs defined on the universe U . Then the distance between (F, A) and (G, B) is defined as follows

$$d(F, G) = \left[\frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \left\{ |\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j)|^k + |\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j)|^k \right\} \right]^{\frac{1}{k}}, \quad (3)$$

and

$$d(F, G) = \left[\frac{1}{2n} \sum_{i=1}^m \sum_{j=1}^n \left\{ |\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j)|^k + |\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j)|^k \right\} \right]^{\frac{1}{k}}, \quad (4)$$

where $k > 0$. If $k = 1$ then equation (3) and (4) are respectively reduced to Hamming distance and Normalized Hamming distance. Again if $k = 2$ then equation (3) and (4) are respectively reduced to Euclidean distance and Normalized Euclidean distance.

The weighted distance is defined as

$$d^w(F, G) = \left[\frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n w_j \left\{ |\bar{\mu}_F(e_i)(x_j) - \bar{\mu}_G(e_i)(x_j)|^k + |\bar{\nu}_F(e_i)(x_j) - \bar{\nu}_G(e_i)(x_j)|^k \right\} \right]^{\frac{1}{k}}, \tag{5}$$

where $w = (w_1, w_2, w_3, \dots, w_n)^T$ is the weight vector of $x_j, (j = 1, 2, 3, \dots, n)$ and $k > 0$.

Especially, if $k = 1$ then (5) is reduced to the weighted Hamming distance and If $k = 2$, then (5) is reduced to the weighted Euclidean distance.

Definition 3.4 Based on the weighted distance between two IVIFSSs (F, A) and (G, B) given by equation (5) , the similarity measure between (F, A) and (G, B) is defined as

$$SM(F, G) = \frac{1}{1 + d^w(F, G)}. \tag{6}$$

Example 3.5 Let $U = \{x_1, x_2, x_3\}$ be the universal set and $A = \{e_1, e_2, e_3\}$ be the set of parameters. Let (F, A) and (G, A) be two interval-valued intuitionistic fuzzy soft sets over U such that their tabular representations are as follows:

(F, A)	e_1	e_2	e_3
x_1	[0.2,0.3],[0.5,0.7]	[0.7,0.8],[0.1,0.2]	[0.4,0.6],[0.0,0.3]
x_2	[0.0,0.2],[0.5,0.6]	[0.3,0.6],[0.1,0.4]	[0.6,0.8],[0.1,0.2]
x_3	[0.3,0.4],[0.5,0.6]	[0.8,0.9],[0.0,0.1]	[0.5,0.6],[0.2,0.3]

(G, A)	e_1	e_2	e_3
x_1	[0.3,0.4],[0.2,0.3]	[0.6,0.8],[0.1,0.2]	[0.6,0.7],[0.2,0.3]
x_2	[0.4,0.5],[0.1,0.2]	[0.2,0.4],[0.4,0.5]	[0.6,0.7],[0.2,0.3]
x_3	[0.5,0.7],[0.2,0.3]	[0.5,0.6],[0.0,0.3]	[0.4,0.6],[0.1,0.2]

Now by definition 3.1 the Hamming distance between (F, A) and (G, A) is given by $d_H(F, G) = 1.55$ and by equation (1) similarity measure between (F, A) and (G, A) is given by $S(F, G) \cong 0.39$.

Example 3.6 Let $U = \{x_1, x_2, x_3, x_4\}$ be the universal set and $A = \{e_1, e_2, e_3\}$ be the set of parameters. Let (F_1, A) and (G_1, A) be two interval-valued intuitionistic fuzzy soft sets over U such that their tabular representations are as follows:

(F_1, A)	e_1	e_2	e_3
x_1	[0.3,0.4],[0.5,0.6]	[0.1,0.2],[0.6,0.7]	[0.0,0.3],[0.5,0.7]
x_2	[0.6,0.7],[0.1,0.2]	[0.4,0.5],[0.2,0.4]	[0.0,0.3],[0.5,0.7]
x_3	[0.7,0.8],[0.0,0.2]	[0.2,0.4],[0.4,0.5]	[0.5,0.7],[0.2,0.3]
x_4	[0.5,0.8],[0.0,0.2]	[0.3,0.6],[0.1,0.4]	[0.2,0.6],[0.3,0.4]

(G_1, A)	e_1	e_2	e_3
x_1	[0.2,0.35],[0.55,0.65]	[0.05,0.2],[0.5,0.7]	[0.1,0.4],[0.5,0.6]
x_2	[0.6,0.7],[0.1,0.2]	[0.4,0.6],[0.0,0.3]	[0.1,0.2],[0.6,0.7]
x_3	[0.65,0.75],[0.1,0.2]	[0.1,0.3],[0.4,0.5]	[0.5,0.7],[0.1,0.2]
x_4	[0.5,0.7],[0.1,0.2]	[0.4,0.5],[0.1,0.35]	[0.3,0.6],[0.25,0.35]

Now by definition 3.1 the Hamming distance between (F_1, A) and (G_1, A) is given by $d_H(F_1, G_1) = 0.525$ and by equation (1) similarity measure between (F_1, A) and (G_1, A) is given by $S(F_1, G_1) \cong 0.65$.

Theorem 3.7 If $S(F, G)$ be the similarity measure between two IVIFSSs (F, E) and (G, E) then

1. $S(F, G) = S(G, F)$
2. $0 \leq S(F, G) \leq 1$
3. $S(F, G) = 1$ if and only if $(F, E) = (G, E)$.

Proof: Obvious from the definition 3.2.

Definition 3.8 Let (F, A) and (G, B) be two IVIFSSs over U . Then (F, A) and (G, B) are said to be α -similar, denoted by $(F, A) \stackrel{\alpha}{\cong} (G, B)$ if and only if $S((F, A), (G, B)) > \alpha$ for $\alpha \in (0, 1)$. We call the two IVIFSSs significantly similar if $S((F, A), (G, B)) > \frac{1}{2}$.

Example 3.9 In example 3.4 $S(F, G) = 0.39 < \frac{1}{2}$ and in example 3.5. Therefore the IVIFSSs (F, E) and (G, E) are not significantly similar but (F_1, E) and (G_1, E) are significantly similar.

4 Application in medical diagnosis problem

In this section we construct a decision making method for a medical diagnosis problem based on similarity measure of two interval-valued intuitionistic fuzzy soft sets (IVIFSSs). The algorithm of this method is as follows:

Step 1: construct a IVIFSS (F, A) over the universe U based on an expert.

Step 2: construct a IVIFSS (G, A) over the universe U based on a responsible person for the problem.

Step 3: calculate the distances of (F, A) and (G, A) .

Step 4: calculate similarity measure of (F, A) and (G, A) .

Step 5: estimate result by using the similarity.

Here we are giving an example of a decision making method. The similarity measure of two IVIFSSs based on Hamming distance can be applied to detect whether a ill person is suffering from a certain disease or not. In this problem we will try to estimate the possibility that an ill person having certain symptoms is suffering from brain cancer. For this we first construct a model IVIFSS for illness and another IVIFSS for ill person. Then we find the similarity measure of these two IVIFSSs. If they are significantly similar then we conclude that the person is possibly suffering from cancer.

Example 4.1 Let U be the universal set, which contains only two elements x_1 (brain cancer) and x_2 (not brain cancer) i.e. $U = \{x_1, x_2\}$. Here the set of parameters E is a set of certain visible symptoms. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, where $e_1 = \text{headache}$, $e_2 = \text{seizures}$, $e_3 = \text{nausea and vomiting}$, $e_4 = \text{vision or hearing problems}$, $e_5 = \text{weakness}$ and $e_6 = \text{behavioural and cognitive problems}$.

Step 1: Construct a IVIFSS (F, A) over U for brain cancer as given below, which can be prepared with the help of an expert (a doctor or a medical person).

(F, A)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	[0.4,0.5],[0.3,0.4]	[0.3,0.4],[0.4,0.6]	[0.2,0.3],[0.4,0.7]	[0.4,0.6],[0.2,0.4]	[0.4,0.5],[0.4,0.5]	[0.6,0.7],[0.2,0.3]
x_2	[0.2,0.3],[0.6,0.7]	[0.5,0.6],[0.2,0.4]	[0.5,0.6],[0.3,0.4]	[0.7,0.8],[0.0,0.2]	[0.4,0.6],[0.2,0.4]	[0.1,0.4],[0.5,0.6]

Step 2: Construct a IVIFSS (G, B) over U based on data of a ill person as given below

(G, B)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	[0.35,0.5],[0.25,0.45]	[0.2,0.4],[0.5,0.6]	[0.15,0.35],[0.4,0.65]	[0.4,0.6],[0.2,0.4]	[0.3,0.5],[0.4,0.5]	[0.5,0.7],[0.1,0.3]
x_2	[0.1,0.25],[0.5,0.71]	[0.4,0.6],[0.3,0.4]	[0.4,0.65],[0.3,0.45]	[0.6,0.71],[0.1,0.2]	[0.3,0.5],[0.1,0.4]	[0.0,0.3],[0.65,0.7]

where $A = B = E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

Step 3: Calculate Hamming distance of (F, A) and (G, B) :

Now by definition 3.1 the Hamming distance between (F, A) and (G, B) is given by

$$d_H(F, G) = 0.525$$

Step 4: Calculate similarity measure between (F, A) and (G, B) :

Now by equation (3.1) similarity measure between (F, A) and (G, B) is given by

$$S(F, G) \cong 0.65 > \frac{1}{2}.$$

Step 5: Here the two IVIFSSs i.e. two sets of symptoms (F, A) and (G, B) are significantly similar, therefore we conclude that the person is possibly suffering from brain cancer.

Example 4.2 Now we consider example 4.1 with a different ill person.

Step 1: Construct a IVIFSS (F, A) over U for brain cancer as in example 4.1, which can be prepared with the help of a medical person.

Step 2: Construct a IVIFSS (H, C) over U based on data of a ill person as given below

(H,C)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	[0.0,0.3],[0.5,0.7]	[0.2,0.3],[0.6,0.7]	[0.0,0.1],[0.7,0.8]	[0.2,0.3],[0.5,0.7]	[0.1,0.3],[0.6,0.7]	[0.3,0.4],[0.5,0.6]
x_2	[0.6,0.8],[0.1,0.2]	[0.3,0.4],[0.5,0.6]	[0.3,0.4],[0.35,0.55]	[0.3,0.5],[0.4,0.5]	[0.2,0.3],[0.6,0.7]	[0.0,0.1],[0.7,0.8]

Step 3: Calculate Hamming distance between (F,A) and (H,C) :

Now by definition 3.1 the Hamming distance between (F,A) and (H,C) is given by

$$d_H(F,H) = 3.1$$

Step 4: Calculate similarity measure of (F,A) and (H,C) :

Now by equation (1) similarity measure between (F,A) and (H,C) is given by

$$S(F,H) \cong 0.24 < \frac{1}{2}$$

Step 5: Here the two IVIFSSs i.e. two sets of symptoms (F,A) and (H,C) are not significantly similar, therefore we conclude that the person is not possibly suffering from brain cancer.

5 Conclusion

In this paper we have defined distances between two IVIFSSs and proposed similarity measures of two IVIFSSs based on distances between two IVIFSSs. Then we construct a decision making method based on similarity measures. Finally we give two simple examples to show the possibilities of diagnosis of a certain diseases. In these examples if we use the other distances, we can obtain similar results. Thus we can use the method to solve the problem that contain uncertainty such as problem in social, economic system, pattern recognition, medical diagnosis, game theory, coding theory and so on.

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