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Heat transfer analysis of a fin with temperature-dependent thermal conductivity and heat transfer coefficient

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Abstract: In this paper Least Square Method (LSM), Collocation Method (CM) and new approach which called Akbari-Ganji's Method (AGM) are applied to solve the nonlinear heat transfer equation of fin with power-law temperature-dependent both thermal conductivity and heat transfer coefficient. The major concern is to achieve an accurate answer which has efficient approximation in accordance to Ruge-Kutta numerical method. Results are presented for the dimensionless temperature distribution and fin efficiency for different values of the problem parameters which for the purpose of comparison, obtained equation were calculated with mentioned methods. It was found the proposed solution is very accurate, efficient, and convenient for the discussed problem, furthermore convergence problems for solving nonlinear equations by using AGM appear small so the results demonstrate that the AGM could be applied through other methods in nonlinear problems with high nonlinearity.

Keywords: Akbari-Ganji Method, Collocation Method, Heat Transfer Equation, Least Square Method.

1 Introduction

Fins are used to increase the heat transfer of heating systems such as, refrigeration, cooling of the oil carrying pipe, cooling, electric transformers, cooling of computer processor and air conditioning. A review about the extended surfaces and its industrial applications is presented by Kern and Krause [1].

Group classification of the differential equation of fin has been analyzed using symmetry analysis [2-3]. In another work, Pakdemirli and Sahin [4] investigated nonlinear equation of fin with general temperature-dependent thermal conductivity. A simple state which the thermal conductivity and heat transfer coefficient are constant, the exact analytical solution is existent. But if a large temperature difference exists within a fin, heat transfer coefficient and thermal conductivity are not constant. Because of this, in general, thermal conductivity and heat transfer coefficient are functions of temperature. As obtaining the exact solution of these nonlinear problems is difficult, so, researchers have focused on finding different solutions for these types of equations, therefore they used semi-analytical methods such as the perturbation method (PM) [5], homotopy perturbation method (HPM) [6-7], variational iteration method (VIM) [8-9], homotopy analysis method (HAM) [10], differential transform method (DTM) [11] and Adomian Decomposition Method (ADM) [12]. In this section, a short review about the related works is presented. This report specially includes the solution of the heat transfer equation of a fin with temperature-dependent thermal conductivity and/or temperature-dependent heat transfer coefficient using accurate semi-analytical methods such as Akbari-Ganji's Method (AGM) [13-14], Collocation Method

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(CM) [15-16], Least Square Method (LSM) [17-18] and comparing them with each other. The oldest works in this subject have been done by Aziz and Hug [19] and Aziz and Benzies [20]. They solved the heat transfer equation of a convective fin with linear temperature-dependent thermal conductivity using perturbation method. In this work, a nonlinear fin with the power-law temperature-dependent both thermal conductivity and heat transfer coefficient is considered. Then the mentioned differential equation solved with Akbari-Ganji's Method (AGM), Collocation Method (CM), Least Square Method (LSM) and then the obtained results would compare with the exact solution.

Collocation Method (CM) has been used by Stern and Rasmussen [21] for solving a third order linear differential equation. Vaferi et al. [22] have studied the feasibility of applying of an orthogonal Collocation Method to solve diffusivity equation in the radial transient flow system. Recently Hatami et al. [23] used collocation method for heat transfer study through porous fins.

Least Square Method (LSM) is introduced by Aziz and Bouaziz [24-25] for predicting the performance of longitudinal fins. They found that least square method is simple compared with other analytical methods .Shaoqin and Huoyuan [26] developed and analyzed Least-Squares approximations for the incompressible magneto-hydrodynamic equations also Hatami et al. [27–29], Hatami and Ganji [30–32], Hatami and Domairry [33-34] and Ahmadi et al. [35] applied these analytical methods in different engineering problems.

The main purpose of this paper is introducing Akbari-Ganji's Method (AGM) as new methods and by comparing it with listed methods we can precisely conclude that the AGM has high efficiency and accuracy for solving nonlinear problems with high nonlinearity. It is necessary to mention that a summary of the excellence of this method in comparison with the other approaches can be considered as follows: Boundary conditions are needed in accordance with the order of differential equations in the solution procedure but when the number of boundary conditions is less than the order of the differential equation, this approach can create additional new boundary conditions in regard to the own differential equation and its derivatives. Therefore, it is logical to mention that AGM is operational for miscellaneous nonlinear differential equations in comparison with the other methods.

Nomenclature

AGM	Akbari-Ganji's Method
LSM	Least Square Method
CM	Collocation Method
T_∞	Free convection
q_x	In Heat Flux
$q_{x+\Delta x}$	Out Heat Flux
$\theta(x)$	Heat transfer function

2 Heat Transfer Equation

Heat transfer equation of a fin with linear temperature dependent thermal conductivity and power-law temperature-dependent heat transfer coefficient has been considered by some researchers. It's differential equation and boundary conditions are in the following form [36]:

$$(1+\beta\cdot\theta(x))\frac{d^2\theta(x)}{dx^2}-M^2(\theta(x))^{n+1}+\beta\left(\frac{d\theta(x)}{dx}\right)^2=0 \tag{1}$$

With the following boundary conditions:

$$\theta'(0) = 0,\ \ \theta(1) = 1 \tag{2}$$

3 Basic Idea of Collocation Method (CM)

Suppose that a differential operator D acting on a function u to produce a function p[1]:

$$D(u(x)) = p(x) \tag{3}$$

Function u can be approximated by a function$\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set as:

$$u \cong \tilde{u} = \sum_{i=1}^{n} C_i \varphi_i \tag{4}$$

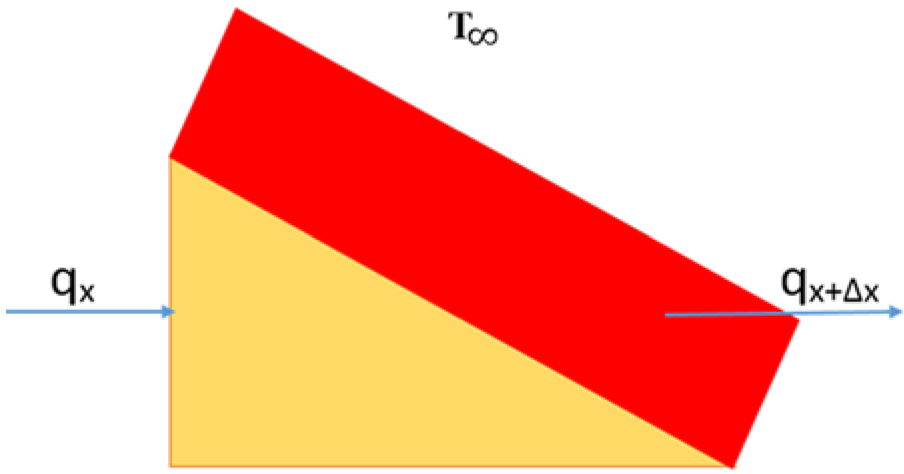

Fig. 1. Geometry of the problem

Substituting $\tilde{u}$ from Eq. (4) into the differential operator D (i.e., Eq. (3)) does not necessarily result in $p(x)$ due to the existence of error or residual as:

$$E(x) = R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{5}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x)\, W_i(x) = 0,\ \ i = 1, 2, ..., n \tag{6}$$

Where the number of weight functions W_i are exactly equal to the number of unknown constants C_i in$\tilde{u}$. This results in a set of n algebraic equations for the unknown constantsC_i. In collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain (i.e.$W_i(x) = \delta(x-x_i)$). The Dirac δ function is defined as:

$$\delta(x-x_i) = \begin{cases} 1 & if\, x = x_i \\ 0 & Otherwise \end{cases} \tag{7}$$

The main aim of this method is to force the residual function in Eq. (5) becomes equal to zero.

3.1 Solving the differential equation with CM

Consider the trial function as:

$$\theta(x) = 1 + c_1\left(1-x^2\right) + c_2\left(1-x^3\right) + c_3\left(1-x^4\right) + c_4\left(1-x^5\right) \tag{8}$$

Eq.(8)) satisfies the boundary condition mentioned in Eq. (2). Substituting $\theta(x)$ into Eq. (1) leads to the residual function, $R(c_1,c_2,c_3,c_4,x)$ as follow:

$$\begin{aligned}
&R(c_1,c_2,c_3,c_4,x) = \\
&45\beta x^8 c_4^2 + 72\beta x^7 c_3 c_4 + \left(56\beta c_2 c_4 + 28\beta c_3^2\right) x^6 \\
&+\left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 + 42\beta c_1 c_4 + 42\beta c_2 c_3\right) x^5 \\
&+\left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 + 30\beta c_1 c_3 + 15\beta c_2^2\right) x^4 \\
&+\begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\ +20\beta c_1 c_2 - 20\beta c_1 c_4 - 20\beta c_2 c_4 - 20\beta c_3 c_4 - 20\beta c_4^2 - 20\beta c_4 - 20 c_4 \end{pmatrix} x^3 \\
&+\begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\ +6\beta c_1^2 - 12\beta c_1 c_3 - 12\beta c_2 c_3 - 12\beta c_3^2 - 12\beta c_3 c_4 - 12\beta c_3 - 12 c_3 \end{pmatrix} x^2 \\
&+\left(-6\beta c_1 c_2 - 6\beta c_2^2 - 6\beta c_2 c_3 - 6\beta c_2 c_4 - 6\beta c_2 - 6c_2\right) x \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n \\
&-2\beta c_1^2 - 2\beta c_1 c_2 - 2\beta c_1 c_3 - 2\beta c_1 c_4 - 2\beta c_1 - 2c_1 = 0
\end{aligned} \tag{9}$$

The residual function must be close to zero. For reaching this importance, four specific points in the domain $x \in [0,1]$should be chosen as:

$$x_1 = \frac{1}{5},\quad x_2 = \frac{2}{5},\quad x_3 = \frac{3}{5},\quad x_4 = \frac{4}{5} \tag{10}$$

Finally by substitutions these points into the residual function,$R(c_n,x)$, a set of four equations and seven unknown coefficients can be obtained.

As an example, the approximate solution of the third order for the specific case of $n = 2, M = 5, \beta = 2$ is obtained and for this particular case, the constants (c_1,c_2,c_3,c_4) are calculated as:

$$c_1 = -0.3034072316, c_2 = -0.3877945372, c_3 = 0.4909619407, c_4 = -0.4074813190 \tag{11}$$

Then the answer of the equation would easily obtained by inserting these unknown parameters (c_1, c_2, c_3, c_4) achieved by the collocation method into trial function:

$$\theta(x) = 0.3922788529 + 0.3034072316x^2 + 0.3877945372x^3 - 0.4909619407x^4 + 0.4074813190x^5 \tag{12}$$

4 Least Square Method (LSM)

There existed an approximation technique for solving differential equations called the Least Square Method (LSM). Suppose a differential operator D is acted on a function u to procedure a functions p:

$$D(u(x)) = p(x) \tag{13}$$

It is considered that u is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is:

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \tag{14}$$

Now, when substituted into the differential operator,D, the result of the operations generally isn't $p(x)$. Hence an error or residual will exist:

$$R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{15}$$

The notion in LSM is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x)W_i(x) = 0 i = 1,2,\dots,n \tag{16}$$

Where the number of weight functions W_i is exactly equal the number of unknown constants c_i in$\tilde{u}$. The result is a set of n algebraic equations for the unknown constantsc_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM's name can be seen. In other words, a minimum of:

$$S = \int_x R(x)R(x)dx = \int_x R^2(x)dx \tag{17}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is:

$$\frac{\partial S}{\partial c_i} = 2\int_x R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{18}$$

Comparing with Eq. (17), the weight functions are seen to be:

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{19}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants:

$$W_i = \frac{\partial R}{\partial c_i} \tag{20}$$

4.1 Solving the differential equation with LSM

Because trial function must satisfy the boundary conditions in Eq. (2), so it will be considered as:

$$\theta(x) = 1 + c_1\left(1-x^2\right) + c_2\left(1-x^3\right) + c_3\left(1-x^4\right) + c_4\left(1-x^5\right) \tag{21}$$

In this problem, according to the Eq. (16), residual function will be as:

$$\begin{aligned}
R(x) &= 45\beta x^8 c_4^2 + 72\beta x^7 c_3 c_4 + \left(56\beta c_2 c_4 + 28\beta c_3^2\right)x^6 \\
&+ \left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 + 42\beta c_1 c_4 + 42\beta c_2 c_3\right)x^5 \\
&+ \left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 + 30\beta c_1 c_3 + 15\beta c_2^2\right)x^4 \\
&+ \begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\ +20\beta c_1c_2 - 20\beta c_1c_4 - 20\beta c_2c_4 - 20\beta c_3c_4 - 20\beta c_4^2 - 20\beta c_4 - 20c_4 \end{pmatrix} x^3 \\
&+ \begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\ +6\beta c_1^2 - 12\beta c_1c_3 - 12\beta c_2c_3 - 12\beta c_3^2 - 12\beta c_3c_4 - 12\beta c_3 - 12c_3 \end{pmatrix} x^2 \\
&+ \left(-6\beta c_1c_2 - 6\beta c_2^2 - 6\beta c_2c_3 - 6\beta c_2c_4 - 6\beta c_2 - 6c_2\right)x \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n \\
&- 2\beta c_1^2 - 2\beta c_1c_2 - 2\beta c_1c_3 - 2\beta c_1c_4 - 2\beta c_1 - 2c_1 = 0
\end{aligned} \tag{22}$$

By substituting the residual function, $R(x)$ into Eq. (19), a set of equation with four equations will appear and by solving this system of equations, coefficients $c_1 - c_4$ will be determined. For example, using Least Square Method for this problem when $n = 2, M = 5, \beta = 2$,$\theta(x)$ is as follows:

$$\theta(x) = 0.3863431577 + 0.3085306360x^2 + 0.5301979592x^3 - 0.8043042649x^4 + 0.5792325120x^5 \tag{23}$$

5 Basic Idea of Akbari-Ganji's Method (AGM)

Boundary conditions and initial conditions are required for analytical methods of each linear and nonlinear differential equation according to the physics of the problem. Therefore, we can solve every differential equation with any degrees. In order to comprehend the given method in this paper, two differential equations governing on engineering processes will be solved in this new manner.

In accordance with the boundary conditions, the general manner of a differential equation is as follows:

The nonlinear differential equation of p which is a function of u, the parameter u which is a function of x and their derivatives are considered as follows:

$$p_k : f\left(u, u', u'', ..., u^{(m)}\right) = 0; u = u(x) \tag{24}$$

Boundary conditions:

$$\begin{cases} u(x) = u_0\,,\ u'(x) = u_1\,, ...,\ u^{(m-1)}(x) = u_{m-1} \quad at \quad x = 0 \\ u(x) = u_{L_0}\,,\ u'(x) = u_{L_1}\,, ...,\ u^{(m-1)}(x) = u_{L_{m-1}} \quad at \quad x = L \end{cases} \tag{25}$$

To solve the first differential equation with respect to the boundary conditions in x=L in Eq.(2), the series of letters in the nth order with constant coefficients which is the answer of the first differential equation is considered as follows:

$$u(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x^1 + a_2 x^2 + ... + a_n x^n \tag{26}$$

The more precise answer of Eq. (25), the more choice of series sentences from Eq. (27). In applied problems, approximately five or six sentences from the series are enough to solve nonlinear differential equations. In the answer of differential Eq. (27) regarding the series from degree (n), there are (n+1) unknown coefficients that need (n+1) equations to be specified. The boundary conditions of Eq. (26) are used to solve a set of equations which is consisted of (n+1) ones.

The boundary conditions are applied on the functions such as follows:

a) The application of the boundary conditions for the answer of differential Eq. (27) is in the form of:
When x = 0:

$$\begin{cases} u(0) = a_0 = u_0 \\ u'(0) = a_1 = u_1 \\ u''(0) = a_2 = u_2 \\ \vdots \end{cases} \tag{27}$$

and when x = L:

$$\begin{cases} u(L) = a_0 + a_1 L + a_2 L^2 + ... + a_n L^n = u_{L_0} \\ u'(L) = a_1 + 2a_2 L + 3a_3 L^2 + ... + n a_n L^{n-1} = u_{L_1} \\ u''(L) = 2a_2 + 6a_3 L + 12a_4 L^2 + ... + n(n-1)\, a_n L^{n-2} = u_{L_{m-1}} \\ \vdots \end{cases} \tag{28}$$

b) After substituting Eq. (29) into Eq. (25), the application of the boundary conditions on differential Eq. (25) is done according to the following procedure:

$$\begin{array}{ll} p_0 : & f\left(u(0), u'(0), u''(0), ..., u^{(m)}(0)\right) \\ p_1 : & f\left(u(L), u'(L), u''(L), ..., u^{(m)}(L)\right) \\ \vdots & \end{array} \tag{29}$$

With regard to the choice of n sentences from Eq. (27) and in order to make a set of equations which is consisted of *(n+1)* equations and *(n+1)* unknowns, we confront with a number of additional unknowns which are indeed the same coefficients of Eq. (27). Therefore, to remove this problem, we should derive m times from Eq. (27) according to the additional unknowns in the afore-mentioned sets of differential equations and then applying the boundary conditions of Eq.(2) on them.

$$\begin{array}{l} p'_k: \;\; f(u', u'', u''', \dots, u^{(m+1)}) \\ p''_k: \;\; f(u'', u''', u^{(IV)}, \dots, u^{(m+2)}) \\ \vdots \end{array} \tag{30}$$

c) Application of the boundary conditions on the derivatives of the differential equation in Eq. (31) is done in the form of:

$$p'_k: \begin{cases} f\left(u'(0), u''(0), u'''(0), \dots, u^{(m+1)}(0)\right) \\ f\left(u'(L), u''(L), u'''(L), \dots, u^{(m+1)}(L)\right) \end{cases} \tag{31}$$

$$p''_k: \begin{cases} f\left(u''(0), u'''(0), \dots, u^{(m+2)}(0)\right) \\ f\left(u''(L), u'''(L), \dots, u^{(m+2)}(L)\right) \end{cases} \tag{32}$$

$(n+1)$Equations can be made from Eq. (27) to Eq. (31) so that $(n+1)$unknown coefficients of Eq. (26) take for example will be computed. The answer of the nonlinear differential Eq. (24) will be gained by determining coefficients of Eq. (26).

5.1 Solving the differential equation with AGM

First of all we rewrite the problem Eq. (24) in the following order:

$$g(x) = (1 + \beta \cdot \theta(x)) \frac{d^2}{dx^2}\theta(x) - M^2 \cdot (\theta(x))^{n+1} + \beta\left(\frac{d}{dx}\theta(x)\right)^2 = 0 \tag{33}$$

In AGM, the answer of the differential equation is considered as a finite series of polynomials with constant coefficients, as follows:

$$\begin{array}{l} \theta(x) = \sum\limits_{i=0}^{13} a_i \cdot x^i = a_{13}x^{13} + a_{12}x^{12} + a_{11}x^{11} + a_{10}x^{10} + a_9x^9 + a_8x^8 + a_7x^7 + a_6x^6 \\ +a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x^1 + a_0 \end{array} \tag{34}$$

It is notable that in the afore-mentioned equation, the constant coefficientsa_0 to a_{13} are obtained by applying the introduced boundary conditions.

For this part to write the equations which will be obtained through the solving procedure because the equations become prolongation we write the equations with the mentioned physical amounts as the previous part which is:

$$n = 2, M = 5, \beta = 2 \tag{35}$$

5.2 Application of the boundary conditions

In AGM, the boundary conditions are applied in order to compute constant coefficients of Eq. (35) in two ways as follows:

a) Applying the boundary conditions on Eq. (35) is expressed as follows:

$$\theta = \theta\,(B.C) \tag{36}$$

It is notable that BC is the abbreviation of boundary conditions.

According to the above explanations, the boundary conditions are applied on Eq. (34) in the following form:

$$\theta\,(1) = 1 \rightarrow a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} = 0 \tag{37}$$

$$\theta'\,(0) = 0 \rightarrow a_1 = 0 \tag{38}$$

b) Applying the boundary conditions on the main differential equation, which in this case study is Eq. (1), and also on its derivatives is done after substituting Eq. (34) into the main differential equation as follows:

$$g(\theta(x)) \rightarrow g(\theta(B.C)) = 0, g'(\theta(B.C)) = 0, \ldots \tag{39}$$

So, after substituting Eq. (34) which has been considered as the answer of the main differential equation into Eq. (1), the initial conditions are applied on the obtained equation and also on its derivatives on the basis of Eq. (39) as follows:

$$g(\theta'(0)):\ \rightarrow 2\,(1+2a_0)\,a_2 - 25a_0^3 + 2a_1^2 = 0 \tag{40}$$

$$g'(\theta'(0)):\ \rightarrow 12a_1a_2 + 6\,(1+2a_0)\,a_3 - 75a_0^2a_1 = 0 \tag{41}$$

$$g''(\theta'(0)):\ \rightarrow 24a_2^2 + 48a_1a_3 + 24\,(1+2a_0)\,a_4 - 150a_0a_1^2 - 150a_0^2a_2 = 0 \tag{42}$$

$$\begin{aligned} &g'''(\theta'(0)):\ \rightarrow 240a_3a_2 + 240a_1a_4 + 120\,(1+2a_0)\,a_5 \\ &-150a_1^3 - 900a_0a_1a_2 - 450a_0^2a_3 = 0 \end{aligned} \tag{43}$$

$$\begin{aligned} &g^{(4)}(\theta'(0)):\ \rightarrow 1440a_4a_2 + 720a_3^2 + 1440a_1a_5 + 720\,(1+2a_0)\,a_6 \\ &-1800a_1^2a_2 - 1800a_0a_2^2 - 3600a_0a_1a_3 - 1800a_0^2a_4 = 0 \end{aligned} \tag{44}$$

$$\begin{aligned} &g^{(5)}(\theta'(x)):\ \rightarrow 10080a_5a_2 + 10080a_4a_3 + 10080a_1a_6 + 5040\,(1+2a_0)\,a_7 \\ &-9000a_1a_2^2 - 9000a_1^2a_3 - 18000a_0a_2a_3 - 18000a_0a_1a_4 - 9000a_0^2a_5 = 0 \end{aligned} \tag{45}$$

$$\begin{aligned} &g^{(8)}(\theta'(0)):\ \rightarrow -6048000a_1a_2a_5 - 6048000a_0a_3a_5 - 6048000a_0a_2a_6 - 3024000a_2^2a_4 \\ &-6048000a_1a_4a_3 - 3024000a_1^2a_6 - 6048000a_0a_7a_1 - 3024000a_4^2a_0 + 7257600a_3a_7 + 7257600a_2a_8 \\ &+7257600a_1a_9 + 3628800\,(1+2a_0)\,a_{10} - 3024000a_0^2a_8 + 3628800a_5^2 + 7257600a_4a_6 - 3024000a_2a_3^2 = 0 \end{aligned} \tag{46}$$

$$\begin{aligned} &g^{(9)}(\theta'(0)):\ \rightarrow -54432000\mathrm{a}_1a_3a_5 - 54432000a_1a_2a_6 - 54432000a_0a_4a_5 \\ &-54432000a_0a_3a_6 - 54432000a_0a_2a_7 - 54432000a_2a_4a_3 - 54432000a_0a_8a_1 - 9072000a_3^3 \\ &+79833600a_3a_8 + 79833600a_3a_8 - 27216000a_1^2a_7 - 27216000a_2^2a_5 - 27216000a_4^2a_1 \\ &+79833600a_1a_{10} + 39916800\,(1+2a_0)\,a_{11} - 27216000a_0^2a_9 + 79833600a_5a_6 + 79833600a_4a_7 = 0 \end{aligned} \tag{47}$$

$$
\begin{aligned}
&g^{(10)}(\theta'(0)): \rightarrow -544320000a_0a_9a_1 - 544320000a_0a_8a_2 \\
&-544320000a_1a_2a_7 - 544320000a_0a_3a_7 - 544320000a_1a_3a_6 \\
&-544320000a_2a_3a_5 - 544320000a_1a_4a_5 - 544320000a_0a_4a_6 \\
&+958003200a_7a_5 + 958003200a_1a_{11} + 479001600(1+2a_0)a_{12} \\
&-272160000a_2a_4^2 - 272160000a_{10}a_0^2 - 272160000a_8a_1^2 \\
&-272160000a_6a_2^2 - 272160000a_0a_5^2 - 272160000a_4a_3^2 \\
&+958003200a_3a_9 + 958003200a_2a_{10} + 958003200a_4a_8 \\
&+479001600a_6^2 = 0
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
&g^{(11)}(\theta'(0)): \rightarrow 6227020800(1+2a_0)a_{13} - 5987520000a_1a_3a_7 \\
&-5987520000a_1a_8a_2 - 5987520000a_0a_4a_7 - 5987520000a_0a_3a_8 \\
&-5987520000a_0a_2a_9 - 5987520000a_0a_5a_6 - 5987520000a_2a_4a_5 \\
&-5987520000a_0a_{10}a_1 - 5987520000a_2a_6a_3 - 5987520000a_1a_4a_6 \\
&+12454041600a_2a_{11} + 12454041600a_1a_{12} + 12454041600a_3a_{10} \\
&-2993760000a_3a_4^2 + 12454041600a_9a_4 - 2993760000a_{11}a_0^2 \\
&-2993760000a_7a_2^2 - 2993760000a_5a_3^2 - 2993760000a_9a_1^2 \\
&-2993760000a_1a_5^2 + 12454041600a_6a_7 + 12454041600a_5a_8 = 0
\end{aligned} \tag{49}
$$

By solving a set of algebraic equations which is consisted of thirteen equations with thirteen unknowns from Eq.s (37-38) and Eq.s (40-49), the constant coefficients of Eq. (34) can easily be gained.

$$
\begin{aligned}
&a_0 = 0.3866158339,\ a_1 = 0 \\
&a_2 = 0.4073647483,\ a_3 = 0 \\
&a_4 = 0.1210296500,,\ a_5 = 0 \\
&a_6 = 0.06034929948,\ a_7 = 0 \\
&a_8 = 0.01663667227,\ a_9 = 0 \\
&a_{10} = 0.006320188146,\ a_{11} = 0 \\
&a_{12} = 0.001683607937,\ a_{13} = 0
\end{aligned} \tag{50}
$$

Eq. (34) which is the solution of the proposed problem is rewritten in the form of

$$
\begin{aligned}
\theta(x) = &\ 0.001683607937x^{12} + 0.006320188146x^{10} \\
&+0.01663667227x^8 + 0.06034929948x^6 \\
&+0.1210296500x^4 + 0.4073647483x^2 \\
&+0.3866158339
\end{aligned} \tag{51}
$$

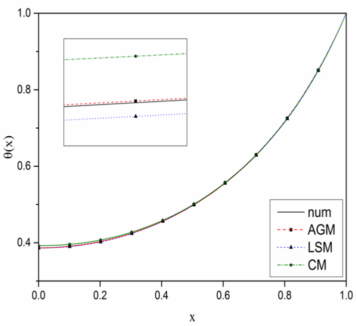

Fig. 1: A comparison of the results of the methods.

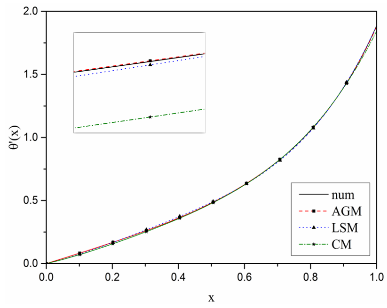

Fig. 2: A comparison between the first derivatives of the methods.

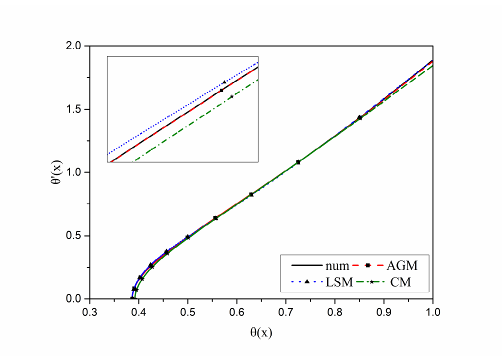

Fig. 3: A comparison between the obtained $\theta(x)$ and $\theta'(x)$of the methods.

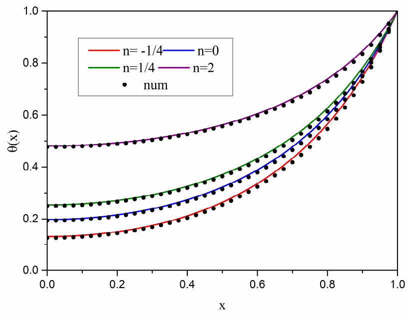

Fig. 4: A comparison amongst the obtained $\theta(x)$ by AGM for various amounts of n by assumption of $M=2,\beta=-1/2$

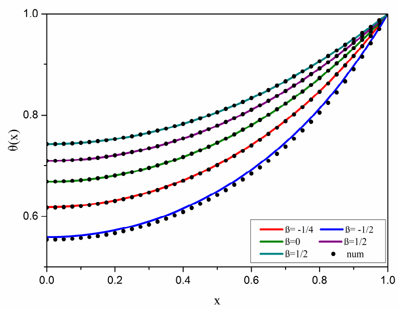

Fig. 5: A comparison amongst the obtained $\theta(x)$ by AGM for various amounts of $\beta$by assumption of $M=2, \mathrm{n}=1/4$.

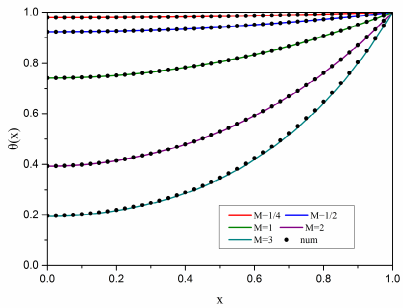

Fig. 6: A comparison amongst the obtained $\theta(x)$ by AGM for various amounts of M by assumption of $\beta=1/2, \mathrm{n}=1/4$.

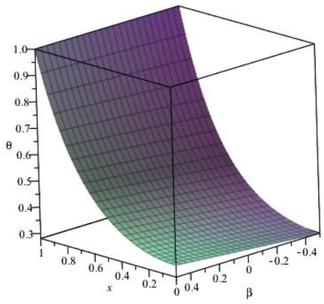

Fig. 7: A comparison amongst the obtained $\theta(x)$ by AGM for various ranges of β by assumption of $n=2, \mathrm{M}=5$.

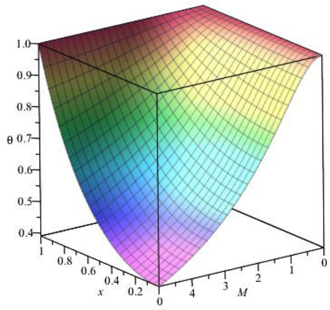

Fig. 8: A comparison amongst the obtained $\theta(x)$ by AGM for various ranges of M by assumption of $n=2, \beta=2$.

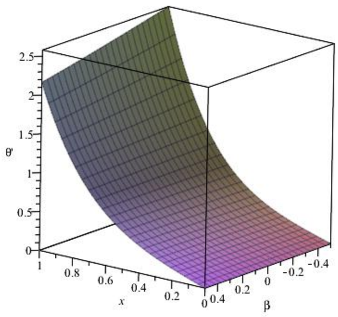

Fig. 9: A comparison amongst the obtained $\theta'(x)$ by AGM for various ranges of β by assumption of $n=2, \mathrm{M}=5$.

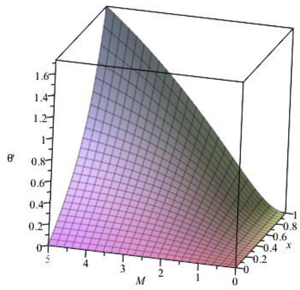

Fig. 10: A comparison amongst the obtained $\theta'(x)$ by AGM for various ranges of M by assumption of $n=2, \beta=2$.

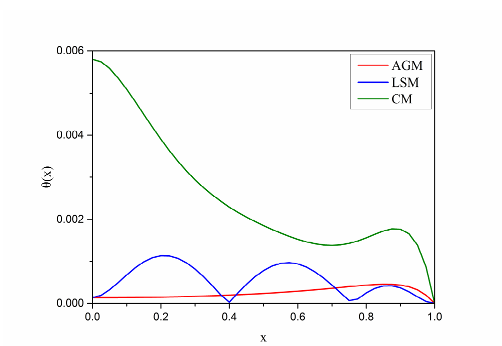

Fig. 11: A comparison between Errors of the methods.

Table.1. The obtained Errors through different methods in comparison with numerical answer.

x	Error of AGM	Error of LSM	Error of CM
0	0.0001341593	0.0001385169	0.0057971783
0.2	0.0001488816	0.0011346608	0.0039032582
0.4	0.0001965689	0.0000271123	0.0022920751
0.6	0.0002900837	0.0009407614	0.0015266843
0.8	0.0004353797	0.0002493342	0.0015624152
1	0	0	0

6 Results and discussion

The current study presents that AGM is efficient and accurate in comparison with numerical method (Ruge-Kutta,R_4) and the other methods such as LSM and CM by utilizing this method through mentioned problem in heat transfer course of study. It would be obvious that AGM have the strong ability to solve nonlinear equations.

For Least Square Method and Collocation Method the trial function has chosen in order to satisfy the boundary conditions and by continuing the solving process the final answer obtained based on the trial function, the important point which we could mentioned in this investigation is that by choosing more terms for the trial function the answer would be a bit accurate but the process of solving would be difficult and in some cases it would be Incalculable.

In the other hand the assumed function is much simpler so obtaining the answer and calculating the coefficients are much easier and by using AGM for solving differential equation. For showing this fact we choose 14 terms function as trial function. In this procedure by adding more terms from the beginning of this assumption it is obvious that the obtained answer is closing to the exact amount of the equation.

7 Conclusion

In this paper investigation has been done through nonlinear heat transfer equation of a fin with the power-law temperature-dependent both thermal conductivity and heat transfer coefficient, in this paper the process of solving mentioned nonlinear differential equation has done through strong analytical methods and the comparison has been done with numerical method.

The mentioned procedure shows the accuracy and simplicity of AGM which is obvious in the relevant plots and table, we could certainly claim that the answer that is obtained by this method is too close to numerical solution. It should be noted that among this methods AGM and LSM could be more accurate for obtaining the answer but what is obvious is that AGM is much simpler for solving these kinds of nonlinear equations.

Therefore, it is logical to say that AGM is a very applicable and suitable approach for solving nonlinear differential equations. In addition to the afore-mentioned explanations after applying boundary conditions on the considered answer, we exit from the field of differential equation into a set of algebraic equations. Then, by solving a set of algebraic equations which is a simple procedure, the constant coefficients of the considered answer can easily be obtained.

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