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A new methodological development for solving linear bilevel integer programming problems in hybrid fuzzy environment

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Abstract: This paper deals with fuzzy goal programming approach to solve fuzzy linear bilevel integer programming problems with fuzzy probabilistic constraints following Pareto distribution and Frechet distribution. In the proposed approach a new chance constrained programming methodology is developed from the view point of managing those probabilistic constraints in a hybrid fuzzy environment. A method of defuzzification of fuzzy numbers using $\alpha-$cut has been adopted to reduce the problem into a linear bilevel integer programming problem. The individual optimal value of the objective of each DM is found in isolation to construct the fuzzy membership goals. Finally, fuzzy goal programming approach is used to achieve maximum degree of each of the membership goals by minimizing under deviational variables in the decision making environment. To demonstrate the efficiency of the proposed approach, a numerical example is provided.

Keywords: Bilevel programming, fuzzy goal programming, ranking function, stochastic programming, triangular fuzzy number.

1 Introduction

The concept of Bilevel Programming Problem (BLPP) was introduced by Candler and Townsley [1] in 1982. The BLPP is considered as a hierarchical decision making problem with a structure of two levels in a highly conflicting decision making situation. The executions of decisions are sequential from upper level to lower levels. In BLPP, a decision maker (DM) at the upper level is termed as the leader and the lower level is termed as the follower [2]. In the context of BLPP, the decisions maintain a hierarchy from leader to the follower. BLPPs have been successfully applied to various hierarchical decision making situations such as traffic planning [3], pricing and fare optimization in the airline industry [4], management of hazardous materials [5], aluminum production process [6], pollution control policy determination [7], tax credits determination for biofuel producers [8], pricing in competitive electricity markets [9], supply chain planning [10], facility location [11], defense problem [12] and so forth. Most of the developments on BLPPs are based on vertex enumeration method [1] and transformation approaches [2] which are effective only for very simple types of problems. The main pitfall of these methods is that the decision makers (DMs) have no cooperating attitude with each other. So, these methods fail to give a decision acceptable to both the DMs. Also the above approaches give a dissatisfactory solution to the DMs if the parameter values contain some degree of uncertainty.

Uncertainties that frequently occur in the real life decision making situations may be fuzzily or stochastically described. The context that arises due to the simultaneous presence of randomness and fuzziness is known as hybrid fuzzy environment. To resolve the randomness, Dantzig [13] introduced stochastic programming using the concept of probability theory. There are two main approaches of stochastic programming, namely, chance constrained programming (CCP) and two- stage programming. Charnes and Cooper [14] first developed the CCP models. The concept of CCP

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technique for solving different type of problems was further extended by several researchers [15,16] in recent past. With this advancement in computational resources and scientific computing techniques many complicated optimization models can now be solved efficiently.

The possibilistic uncertainty involved inherently with the DMs ambiguous understanding of the nature of parameters associated with the problem. Fuzzy numbers, introduced by Zadeh [17], are used to handle fuzzy uncertainty. Zimmermann [18] showed that the solutions obtained by fuzzy linear programming are always efficient. Sakawa et al. [19] formulated cooperative fuzzy bilevel programming problems and proposed an interactive fuzzy programming approach to solve the problems. Shih and Lee [20] applied fuzzy set theory to overcome the computational difficulties in solving bilevel problems. Recently, Zhang et al. [21] studied fuzzy bilevel programming problem, which focuses on the situation where the leader or the follower has multiple objectives with fuzzy parameters and all followers share their decision variables. Moitra and Pal [22] adopted fuzzy goal programming (FGP) approach for solving linear BLPP. Abo-Sinha [23] discussed multi-objective optimization for solving non-linear multi-objective bi-level programming problems in fuzzy environment. Osman et al. [24] extended fuzzy approaches [23] for solving non-linear bi-level and tri-level multi-objective decision making under fuzziness. Baky [25] studied FGP algorithm for solving decentralized bi-level multi-objective programming problems. Arora and Gupta [26] presented interactive FGP approach for linear BLPP with the characteristics of dynamic programming. Satisfactory solution is derived by updating the satisfactory degree of the decision makers with the consideration of overall satisfactory balance between both the levels. Deng et. al. [27] developed a method for solving the fuzzy BLPP with multiple followers through structured element method.

Considering simultaneous occurrence of randomness and fuzziness in BLPP, Modak and Biswas [28] developed an FGP approach for solving bilevel stochastic programming problems. But FGP approaches to fuzzy linear bilevel integer programming problem (FLBLIPP) with Pareto distributed and Frechet distributed fuzzy random variables (FRVs) are yet to appear in literature.

In the present study a methodology for solving FLBLIPP with Pareto distributed and Frechet distributed FRVs in right hand side parameters of the constraints are developed. Also the parameter of the objectives and the left hand side parameters of the constraints are taken as triangular fuzzy numbers. At first the probabilistic uncertainty is removed from the constraints by applying CCP technique. Then using a method of defuzzification of triangular fuzzy numbers [29] the problem is reduced to linear bilevel integer programming problem (LBLIPP). The individual optimal value of the objective of each DM is found in isolation to construct the fuzzy membership goals of each of the DMs. Finally FGP approach is used to achieve maximum degree of each of the membership goals of the DMs by minimizing under-deviational variables in the decision making environment.

2 Basic concepts

The preliminary ideas such as $\alpha-$cut of fuzzy set, fuzzy number, triangular fuzzy number, defuzzification method for finding the expected value of fuzzy number, fuzzy random variable following Pareto distribution and Frechet distribution which are necessary in the treatise of formulating the proposed model are described in this section.

Definition 1. *$\alpha-$cut of a fuzzy set $\tilde{A}$ is a crisp set, denoted by $\tilde{A}[\alpha]$ and is defined by $\tilde{A}[\alpha] = \{x : x \in X \text{ and } \mu_{\tilde{A}}(x) \geq \alpha\}$, $(0 < \alpha \leq 1)$, where X represents the set on which the fuzzy set $\tilde{A}$ is defined.*

Definition 2. *A fuzzy set $\tilde{A}$ defined on the set of real numbers, $\mathbb{R}$, is said to be a fuzzy number if*

(1) *$\tilde{A}$ is a normal fuzzy set. i.e., there exists a point $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$.*
(2) *The $\alpha-$cut of $\tilde{A}$ i.e. $\tilde{A}[\alpha]$ is a convex set for all $\alpha \in (0,1]$.*
(3) *The support of $\tilde{A}$ is a bounded set.*

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Definition 3. *A fuzzy number $\tilde{A}$ is said to be a triangular fuzzy number if its membership function is expressed as*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a^L}{a-a^L} & \text{if } a^L \le x \le a \\ \frac{a^R-x}{a^R-a} & \text{if } a \le x \le a^R \\ 0 & \text{otherwise} \end{cases}$$

A triangular fuzzy number $\tilde{A}$ is written in the form $\tilde{A} = (a^L, a, a^R)$. Geometrically a triangular fuzzy number is presented as,

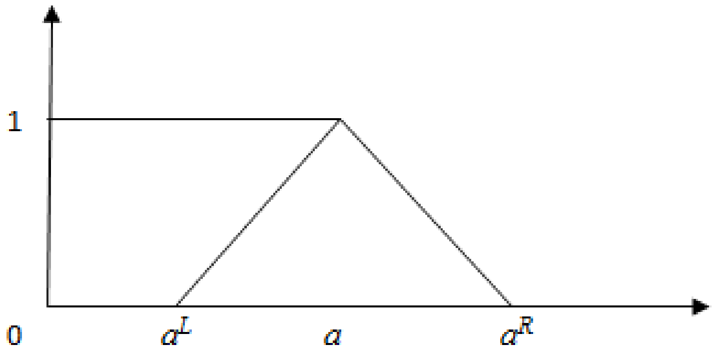

Fig. 1

Definition 4. *Let $\tilde{A} = (a^L, a, a^R)$ be a triangular fuzzy number. Then the equivalent crisp value of $\tilde{A}$ is obtained by finding the expected value using the α- cut of the triangular fuzzy number. The membership function of the triangular fuzzy number $\tilde{A} = (a^L, a, a^R)$ is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a^L)}{(a-a^L)} & if\ a^L \le x \le a \\ \frac{(a^R-x)}{(a^R-a)} & if\ a \le x \le a^R \\ 0 & otherwise \end{cases}.$$

The $\alpha-$cut of the triangular fuzzy number $\tilde{A} = (a^L, a, a^R)$ is written as

$$\tilde{A}[\alpha] = [a^L + (a - a^L)\,\alpha,\ a^R - (a^R - a)\alpha].$$

Then the equivalent crisp value of the triangular fuzzy number $\tilde{A}$ is calculated as

$$\begin{aligned} V(\tilde{A}) &= \int_0^1 (a^L + (a - a^L)\,\alpha)\,\alpha d\alpha + \int_0^1 (a^R - (a^R - a)\alpha)\alpha d\alpha \\ &= \left[\frac{a^L}{2} + \frac{(a - a^L)}{3} + \frac{a^R}{2} - \frac{(a^R - a)}{3}\right] \\ &= \frac{[a^L + 4a + a^R]}{6}. \end{aligned}$$

Definition 5. *Let X be a continuous random variable with probability density function $f(x, \theta)$, where θ is the parameter of the probability density function. If θ is uncertain in nature, then θ may be chosen as fuzzy number, $\widetilde{\theta}$. Then a continuous random variable with fuzzy number $\widetilde{\theta}$ as parameter is known as continuous FRV $\tilde{X}$. The probability density function of the continuous FRV $\tilde{X}$ is denoted by $f(x\,;\widetilde{\theta})$.*

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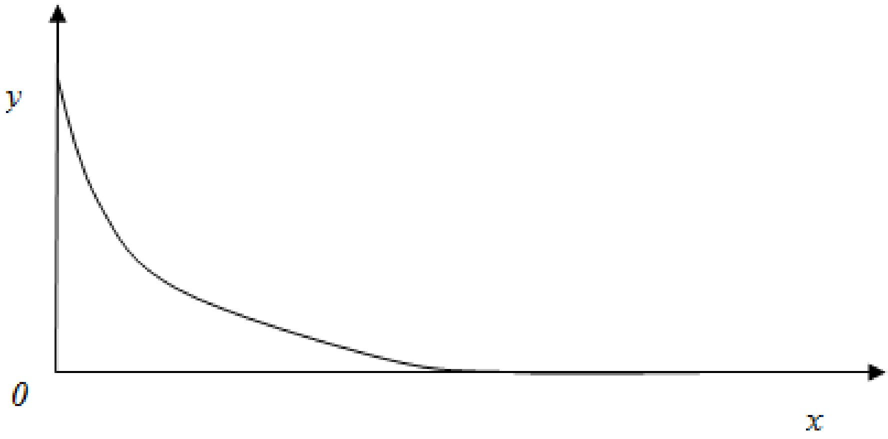

Fig. 2

2.1 Fuzzy Random Variable following Pareto Distribution

Let $\tilde{b}_i$ is a Pareto distributed FRV. Its probability density function is written as

$$f\left(b_i\,;\widetilde{\beta}_i\,,\,\widetilde{\lambda}_i\right)=\frac{u_i {v_i}^{u_i}}{{b_i}^{u_i+1}},u_i\in\widetilde{\lambda}_i\,[\alpha]\,,\;v_i\in\;\widetilde{\beta}_i[\alpha]$$

where $b_i \geq v_i$. Here $\widetilde{\lambda}_i\,[\alpha]\,,\,\widetilde{\beta}_i[\alpha]$ are the α–cut of fuzzy numbers $\widetilde{\lambda}_i,\,\widetilde{\beta}_i$ whose support are the set of positive real numbers. The probability density curve of the Pareto distribution is shown in Figure 2.

2.2 Fuzzy Random Variable following Frechet Distribution

Let $\tilde{b}_i$ is a Frechet distributed FRV, its probability density function is written as

$$f\left(b_i\,;\,\widetilde{\mu}_i\,,\,\widetilde{\delta}_i,\,\widetilde{\eta}_i\right)=\frac{r_i}{q_i}\left(\frac{b_i-d_i}{q_i}\right)^{-1-r_i}e^{-\left(\frac{b_i-d_i}{q_i}\right)^{-r_i}},r_i\in\widetilde{\mu}_i\,[\alpha]\,,\;q_i\in\;\widetilde{\delta}_i\,[\alpha]\,,\;d_i\in\widetilde{\eta}_i[\alpha]$$

where $b_i \geq d_i$. Here $\widetilde{\mu}_i\,[\alpha]\,,\;\widetilde{\delta}_i[\alpha]$ and $\widetilde{\eta}_i[\alpha]$ are the α–cut of fuzzy numbers $\widetilde{\mu}_i,\;\widetilde{\delta}_i$ and $\widetilde{\eta}_i$ respectively. The support of $\widetilde{\mu}_i,\;\widetilde{\delta}_i$ are the set of positive real numbers and the support of $\widetilde{\eta}_i$ is the set of real numbers. The probability density curve of the Frechet distribution is given in Figure 3.

3 Fuzzy linear BLICCP model

The general form of FLBLIP problem under probabilistic environment is expressed as
Find $X(x_1,\;x_2,\;\ldots,\;x_n)$ so as to $\text{Max}_{X_1}\,\tilde{Z}_1\,(X)=\sum_{j=1}^{n}\tilde{c}_{1j}x_j$ (Leader's Problem), where for given X_1 ; X_2 solves $\text{Max}_{X_2}\,\tilde{Z}_2\,(X)=\sum_{j=1}^{n}\tilde{c}_{2j}x_j$ (Follower's Problem), subject to $\Pr\left(\sum_{j=1}^{n}\tilde{a}_{ij}x_j\leq\tilde{b}_i\right)\;\geq 1-p_i\,;\,i=1,2,\ldots,\;m$

$$\sum_{j=1}^{n}\tilde{a}_{tj}x_j\leq\tilde{b}_t;t=1,2,\ldots,s,\;x_j\geq 0\text{ and are integers};\;j=1,2,\ldots,\;n. \quad (1)$$

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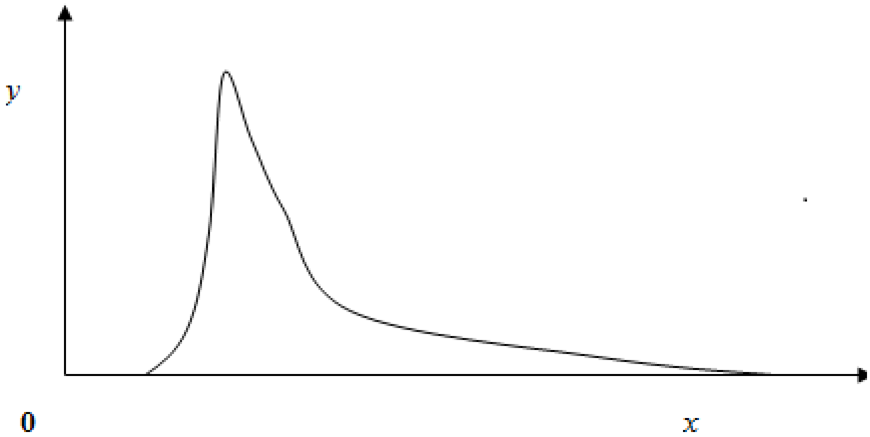

Fig. 3

Here some $\tilde{b}_i\ (i=1,2,..,l)$ represents Pareto distributed FRVs and the others $\tilde{b}_i\ (i=l+1,\ l+2,\ldots,\ m)$ follows Frechet distributed FRVs. The coefficients of the objectives $\tilde{c}_{ij}$ and $\tilde{c}_{2j}$ are taken as triangular fuzzy numbers. The coefficients of the constraints $\tilde{a}_{ij},\ \tilde{a}_{tj},\ \tilde{b}_t\ \ (i=1,2,\ldots,\ m\ ;j=1,2,\ldots,\ n\ ;t=1,2,\ldots,\ s)$ are also considered as triangular fuzzy numbers. Again the parameters of the FRVs are triangular fuzzy numbers and $p_i\ \in[0,1]$.

It is to be mentioned here that the decision vector $X_1=(x_{11},\ x_{12},\ \ldots,\ x_{1n_1})$ is controlled by the leader and the decision vector $X_2=(x_{21},\ x_{22},\ \ldots,\ x_{2n_2})$ is controlled by follower. Also $X_1\cup X_2=X=(x_1,\ x_2,\ \ldots,\ x_n)\in\mathbb{R}^n$ with $n_1+n_2=n$.

3.1 Chance Constrained Programming Technique

In this subsection CCP methodology is applied to convert the probabilistic constraints into fuzzy constraints. There are l constraints in which the FRVs follow Pareto distribution and the FRVs in the remaining $m-l$ probabilistic constraints follow Frechet distribution.

At first the CCP technique is applied to the l constraints involving Pareto distributed FRVs as follows

$$\Pr\left(\sum_{j=1}^{n}\tilde{a}_{ij}x_j\le\tilde{b}_i\right)\ \ge 1-p_i;(i=1,2,..,l)$$

i.e., $\Pr\left(\tilde{A}_i\le\tilde{b}_i\right)\ \ge 1-p_i$, where $\tilde{A}_i=\sum_{j=1}^{n}\tilde{a}_{ij}x_j$, i.e., $\int_{k_i}^{\infty}\frac{u_i {v_i}^{u_i}}{{b_i}^{u_i+1}}\,db_i\ge 1-p_i$; $u_i\in\widetilde{\lambda}_i[\alpha]$, $v_i\in\ \widetilde{\beta}_i[\alpha]$, $k_i\in\tilde{A}_i[\alpha]$and $k_i\ge v_i$, i.e., $k_i\le\frac{v_i}{(1-p_i)^{\frac{1}{u_i}}}$; $u_i\in\widetilde{\lambda}_i[\alpha]$, $v_i\in\ \widetilde{\beta}_i[\alpha]$, $k_i\in\tilde{A}_i[\alpha]$ and $k_i\ge v_i$.
Since, this inequality is true for all $\alpha\in(0, 1]$, the expression can be written in terms of α-cut as

$$\tilde{A}_i[\alpha]\le\frac{\widetilde{\beta}_i[\alpha]}{(1-p_i)^{\frac{1}{\widetilde{\lambda}_i[\alpha]}}}\ \text{and}\ \tilde{A}_i[\alpha]\ge\widetilde{\beta}_i[\alpha];\ (i=1,2,..,l). \tag{2}$$

Now, using first decomposition theorem, the above equation is reduced to the following form as

$$\tilde{A}_i\le\frac{\widetilde{\beta}_i}{(1-p_i)^{\frac{1}{\widetilde{\lambda}_i}}}\ \text{and}\ \tilde{A}_i\ge\widetilde{\beta}_i;\ \ (i=1,2,..,l). \tag{3}$$

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Now, applying CCP technique in the remaining $m-l$ probabilistic constraints involving Frechet distributed FRVs, the constraints take the following form as

$$\Pr\left(\sum_{j=1}^{n}\tilde{a}_{ij}x_j \le \tilde{b}_i\right) \ge 1-p_i; (i=l+1,l+2,..,m)$$

i.e., $\Pr\left(\tilde{A}_i \le \tilde{b}_i\right) \ge 1-p_i$, where $\tilde{A}_i = \sum_{j=1}^{n}\tilde{a}_{ij}x_j$, i.e. $\int_{h_i}^{\infty}\frac{r_i}{q_i}\left(\frac{b_i-d_i}{q_i}\right)^{-1-r_i}e^{-\left(\frac{b_i-d_i}{q_i}\right)^{-r_i}}db_i \ge 1-p_i$; $r_i \in \widetilde{\mu}_i[\alpha]$, $q_i \in \widetilde{\delta}_i[\alpha]$, $d_i \in \widetilde{\eta}_i[\alpha]$ and $h_i \in \tilde{A}_i[\alpha]$ and $h_i \ge d_i$, i.e., $h_i \le d_i + q_i\frac{1}{\ln\left(\frac{1}{p_i}\right)^{\frac{1}{r_i}}}$; $r_i \in \widetilde{\mu}_i[\alpha]$, $q_i \in \widetilde{\delta}_i[\alpha]$, $d_i \in \widetilde{\eta}_i[\alpha]$and $h_i \in \tilde{A}_i[\alpha]$ and $h_i \ge d_i$.

Since, this inequality is true for all $\alpha \in (0, 1]$, the expression can be written in terms of α-cut as

$$\tilde{A}_i[\alpha] \le \widetilde{\eta}_i[\alpha] + \widetilde{\delta}_i[\alpha]\frac{1}{\ln\left(\frac{1}{p_i}\right)^{\frac{1}{\widetilde{\mu}_i[\alpha]}}} \text{ and } \tilde{A}_i[\alpha] \ge \widetilde{\eta}_i[\alpha];\ (i=l+1,l+2,..,m). \tag{4}$$

Now using first decomposition theorem, the above equation is reduced to the following form as

$$\tilde{A}_i \le \widetilde{\eta}_i + \widetilde{\delta}_i\frac{1}{\ln\left(\frac{1}{p_i}\right)^{\frac{1}{\widetilde{\mu}_i}}} \text{ and } \tilde{A}_i \ge \widetilde{\eta}_i;\ (i=l+1,l+2,..,m). \tag{5}$$

Hence the FLBLICCP model (1), is converted into the equivalent FLBLIPP problem by using the derived methodology as; Find $X(x_1,\ x_2,\dots,\ x_n)$ so as to $\text{Max}_{X_1}\tilde{Z}_1(X) = \sum_{j=1}^{n}\tilde{c}_{1j}x_j$ (Leader's Problem), where for given X_1; X_2 solves $\text{Max}_{X_2}\tilde{Z}_2(X) = \sum_{j=1}^{n}\tilde{c}_{2j}x_j$ (Follower's Problem), subject to

$$\begin{aligned}
&\sum_{j=1}^{n}\tilde{a}_{ij}x_j \le \frac{\widetilde{\beta}_i}{(1-p_i)^{\frac{1}{\widetilde{\lambda}_i}}};\ i=1,2,..,l\\
&\sum_{j=1}^{n}\tilde{a}_{ij}x_j \ge \widetilde{\beta}_i\ i=1,2,..,l\\
&\sum_{j=1}^{n}\tilde{a}_{ij}x_j \le \widetilde{\eta}_i + \widetilde{\delta}_i\frac{1}{\ln\left(\frac{1}{p_i}\right)^{\frac{1}{\widetilde{\mu}_i}}};\ i=l+1,l+2,..,m\\
&\sum_{j=1}^{n}\tilde{a}_{ij}x_j \ge \widetilde{\eta}_i;\ i=l+1,l+2,..,m\\
&\sum_{j=1}^{n}\tilde{a}_{tj}x_j \le \tilde{b}_t;\ t=1,2,\dots,s,\ x_j \ge 0 \text{ and are integers; } j=1,2,\dots,\ n.
\end{aligned} \tag{6}$$

Here $\tilde{c}_{1j}, \tilde{c}_{2j}, \tilde{a}_{ij}, \tilde{a}_{tj}, \tilde{b}_t$ $(j=1,2,\dots,n;\ \ i=1,2,\dots,\ m;\ t=1,2,\dots,s)$ are taken as triangular fuzzy numbers. Also the parameters $\widetilde{\beta}_i$, $\widetilde{\lambda}_i$,$\widetilde{\eta}_i$, $\widetilde{\delta}_i$, $\widetilde{\mu}_i$ of the Pareto and Frechet distributed FRVs are considered as triangular fuzzy numbers. Then $\frac{1}{\widetilde{\lambda}_i}$ and $\frac{1}{\widetilde{\mu}_i}$ are also taken as triangular fuzzy numbers. So, these triangular fuzzy numbers can be expressed as

$$\begin{aligned}
&\tilde{c}_{1j} = \left(c_{1j}^L,\ c_{1j},\ c_{1j}^R\right);\ \tilde{c}_{2j} = \left(c_{2j}^L,\ c_{2j},\ c_{2j}^R\right);\ \tilde{a}_{ij} = \left(a_{ij}^L,\ a_{ij},\ a_{ij}^R\right);\ \tilde{a}_{tj} = \left(a_{tj}^L,\ a_{tj},\ a_{tj}^R\right);\\
&\tilde{b}_t = \left(b_t^L,\ b_t,\ b_t^R\right);\ \widetilde{\beta}_i = \left(\beta_i^L,\ \beta_i,\ \beta_i^R\right);\ \widetilde{\eta}_i = \left(\eta_i^L,\ \eta_i,\ \eta_i^R\right);\ \widetilde{\delta}_i = \left(\delta_i^L,\ \delta_i,\ \delta_i^R\right);\\
&\frac{1}{\widetilde{\lambda}_i} = \left(\left(\frac{1}{\lambda_i}\right)^L,\ \frac{1}{\lambda_i},\ \left(\frac{1}{\lambda_i}\right)^R\right);\ \frac{1}{\widetilde{\mu}_i} = \left(\left(\frac{1}{\mu_i}\right)^L,\ \frac{1}{\mu_i},\ \left(\frac{1}{\mu_i}\right)^R\right).
\end{aligned} \tag{7}$$

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3.2 Fuzzy linear BLIPP model

The fuzzy numbers are defuzzified in this subsection to find an equivalent deterministic model of the given problem. Defuzzification of fuzzy numbers is a process that maps a fuzzy number to a crisp number. The crisp values associated with the fuzzy numbers of model (6) are obtained by the method of defuzzification [29] of the triangular fuzzy number using $\alpha-$ cut are given as

$$V\left(\tilde{c}_{1j}\right)=\frac{c_{1j}^{L}+4c_{1j}+\ c_{1j}^{R}}{6};\ V\left(\tilde{c}_{2j}\right)=\frac{c_{2j}^{L}+4c_{2j}+\ c_{2j}^{R}}{6};\ V\left(\tilde{a}_{ij}\right)=\frac{a_{ij}^{L}+4a_{ij}+\ a_{ij}^{R}}{6};$$
$$V\left(\tilde{a}_{tj}\right)=\frac{a_{tj}^{L}+4a_{tj}+\ a_{tj}^{R}}{6};\ V\left(\tilde{b}_{t}\right)=\frac{b_{t}^{L}+4b_{t}+\ b_{t}^{R}}{6};\ V\left(\widetilde{\beta}_{i}\right)=\frac{\beta_{i}^{L}+4\beta_{i}+\ \beta_{i}^{R}}{6};$$
$$V\left(\widetilde{\eta}_{i}\right)=\frac{\eta_{i}^{L}+4\eta_{i}+\ \eta_{i}^{R}}{6};\ V\left(\widetilde{\delta}_{i}\right)=\frac{\delta_{i}^{L}+4\delta_{i}+\ \delta_{i}^{R}}{6};\ V\left(\frac{1}{\widetilde{\lambda}_{i}}\right)=\frac{\left(\frac{1}{\lambda_{i}}\right)^{L}+4\,\frac{1}{\lambda_{i}}+\ \left(\frac{1}{\lambda_{i}}\right)^{R}}{6};$$
$$V\left(\frac{1}{\widetilde{\mu}_{i}}\right)=\frac{\left(\frac{1}{\mu_{i}}\right)^{L}+4\,\frac{1}{\mu_{i}}+\ \left(\frac{1}{\mu_{i}}\right)^{R}}{6};\ \ (j=1,2,\ldots,n;\ i=1,2,\ldots,\ m;\ t=1,2,\ldots,s). \tag{8}$$

Thus the equivalent deterministic model of the FLBLIPP (6) is stated as; Find $X(x_1,\ x_2,\ \ldots,\ x_n)$ so as to $\mathrm{Max}_{X_1} V(\tilde{Z}_1(X))=\sum_{j=1}^{n}V(\tilde{c}_{1j})x_j$ (Leader's Problem), where for given X_1; X_2 solves $\mathrm{Max}_{X_2} V(\tilde{Z}_2(X))=\sum_{j=1}^{n}V(\tilde{c}_{2j})x_j$ (Follower's Problem), subject to

$$\sum_{j=1}^{n}V(\tilde{a}_{ij})x_j\le\frac{V(\widetilde{\beta}_i)}{(1-p_i)^{V\left(\frac{1}{\lambda_i}\right)}};\ \ i=1,2,..,l$$
$$\sum_{j=1}^{n}V(\tilde{a}_{ij})x_j\ge V(\widetilde{\beta}_i);\ \ i=1,2,..,l$$
$$\sum_{j=1}^{n}V(\tilde{a}_{ij})x_j\le V(\widetilde{\eta}_i)+V(\widetilde{\delta}_i)\frac{1}{\ln\left(\frac{1}{p_i}\right)^{V\left(\frac{1}{\mu_i}\right)}};\ \ i=l+1,l+2,..,m$$
$$\sum_{j=1}^{n}V(\tilde{a}_{ij})x_j\ge V(\widetilde{\eta}_i);\ \ i=l+1,l+2,..,m$$
$$\sum_{j=1}^{n}V(\tilde{a}_{tj})x_j\le V(\tilde{b}_t);\ \ t=1,2,\ldots,s;\ \ x_j\ge 0 \text{ and are integers};\ j=1,2,\ldots,\ n. \tag{9}$$

4 Fuzzy goals and membership functions

In a bilevel system, it can reasonably be assumed that both the DMs are motivated to cooperate with each other and each one tries to optimize his/her own benefit paying serious attention to the benefit of the others. Now, both the leader and follower optimize their objective independently under the same set of system constraints defined in (9). Let

$$\left[x_k^b\,;V\left(Z_k\right)^b\right]=\left[x_{k1}^b,x_{k2}^b,..,x_{kn}^b\,;V\left(Z_k\right)^b\right]$$

and

$$\left[x_k^w\,;V\left(Z_k\right)^w\right]=\left[x_{k1}^w,x_{k2}^w,..,x_{kn}^w\,;V\left(Z_k\right)^w\right],$$

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$(k=1,2)$ be the best and worst independent solutions of the objectives of the respective DMs. Hence, the fuzzy objective goal for each of the corresponding DMs is expressed as:

$$V(Z_k) \gtrsim V(Z_k)^b \text{ for } k=1,2. \tag{10}$$

In a bilevel decision making context, it is to be realized that the full achievement of the respective goal values of the DMs are not always possible due to conflicting nature of the objectives of the DMs and also due to the scarcity of limited resources in the decision making context. Again values lower than the worst values of the objectives of the DMs are completely unacceptable to the DMs. Hence the membership functions of the defined fuzzy goals are formulated as

$$\mu_{V(Z_k(x))} = \begin{cases} 0 & \text{if } \quad V(Z_k) \le V(Z_k)^w \\ \frac{V(Z_k)-V(Z_k)^w}{V(Z_k)^b-V(Z_k)^w} & \text{if } V(Z_k)^w \le V(Z_k) \le V(Z_k)^b \quad (k=1,2). \\ 1 & \text{if } \quad V(Z_k) \ge V(Z_k)^b \end{cases} \tag{11}$$

The membership function defined above are now converted into the membership goals by introducing under- and over-deviational variables and assigning the highest membership value (unity) as the aspiration level to each of them.

5 FGP model

The FGP model of the corresponding linear bilevel integer programming problem (9) is presented as: Find $X(x_1,x_2,\ldots.x_n)$ so as to Min D $=\sum_{k=1}^{2} w_k d_k^-$ and satisfy

$$\frac{V(Z_1)-V(Z_1)^w}{V(Z_1)^b-V(Z_1)^w} + d_1^- - d_1^+ = 1,$$
$$\frac{V(Z_2)-V(Z_2)^w}{V(Z_2)^b-V(Z_2)^w} + d_2^- - d_2^+ = 1,$$

subject to

$$\begin{aligned} &\sum_{j=1}^{n} V(\tilde{a}_{ij})x_j \le \frac{V(\widetilde{\beta}_i)}{(1-p_i)^{V\left(\frac{1}{\lambda_i}\right)}};\ i=1,2,..,l \\ &\sum_{j=1}^{n} V(\tilde{a}_{ij})x_j \ge V(\widetilde{\beta}_i);\ i=1,2,..,l \\ &\sum_{j=1}^{n} V(\tilde{a}_{ij})x_j \le V(\widetilde{\eta}_i)+V(\widetilde{\delta}_i)\frac{1}{\ln\left(\frac{1}{p_i}\right)^{V\left(\frac{1}{\mu_i}\right)}};\ i=l+1,l+2,..,m \\ &\sum_{j=1}^{n} V(\tilde{a}_{ij})x_j \ge V(\widetilde{\eta}_i);\ i=l+1,l+2,..,m \\ &\sum_{j=1}^{n} V(\tilde{a}_{tj})x_j \le V(\tilde{b}_t); t=1,2,\ldots,s;\ x_j \ge 0 \text{ and are integers};\ j=1,2,\ldots,\ n \\ &d_1^-,d_1^+,d_2^-,d_2^+ \ge 0 \text{ with } d_1^-.d_1^+ = d_2^-.d_2^+ = 0 \end{aligned} \tag{12}$$

where $w_k = 1/(V(Z_k)^b - V(Z_k)^w)$ (k = 1, 2) represents fuzzy weight corresponding to the membership goals of the DMs. The derived model (12) is then solved using *minsum* goal programming to achieve most compromise solution in a decision making context.

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The derived methodology for solving FLBLICCP model can be summarized by the following algorithm.

Step 1: Using CCP technique, the fuzzy probabilistic constraints are converted into constraints involving fuzzy numbers.

Step 2: Defuzzification of fuzzy number using $\alpha-$cut is applied to find the expected value of the fuzzy numbers.

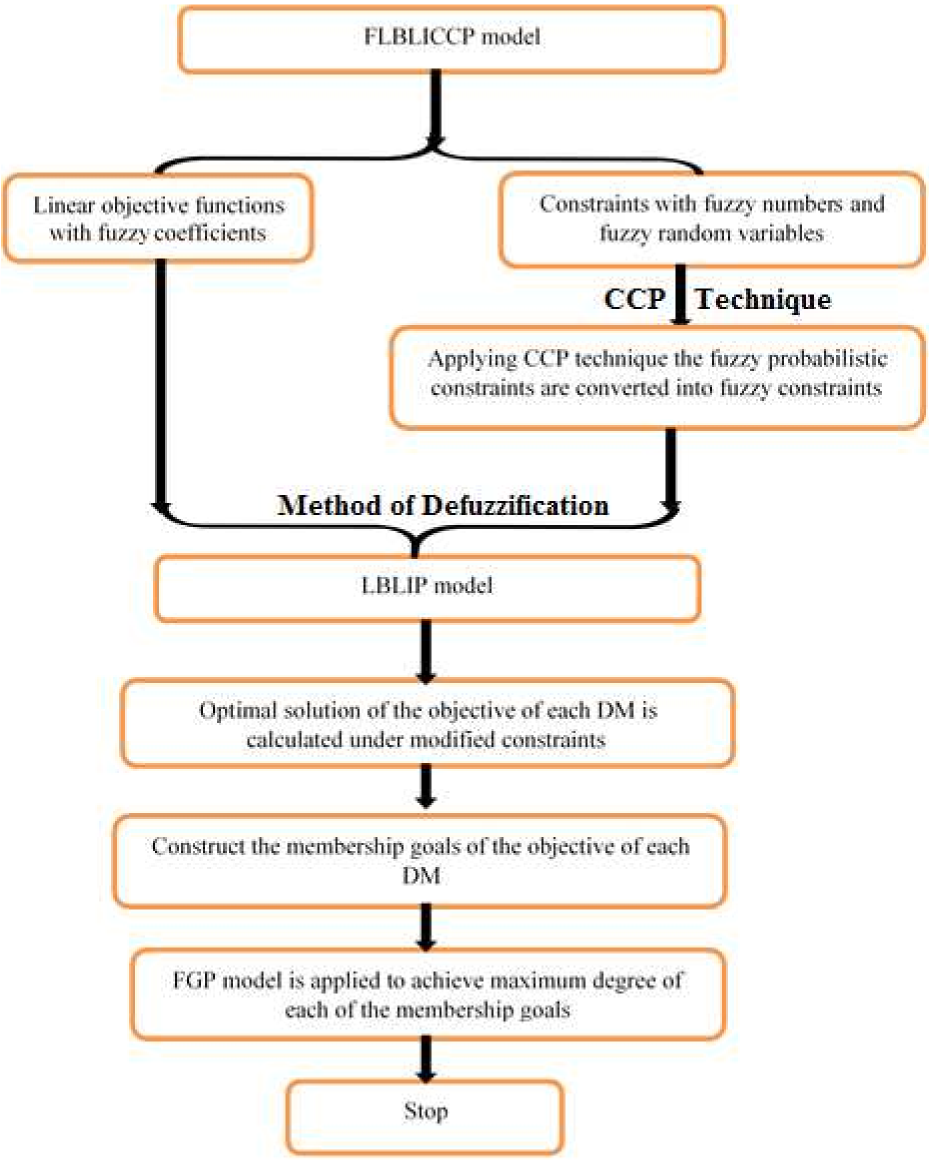
Step 3: The individual optimal value of the objective of each DMs is found in isolation.

Step 4: The fuzzy membership goals of each of each DMs are constructed.

Step 5: FGP approach is deployed to achieve maximum degree of each of the membership goals.

Step 6: Stop.

The solution process for solving FLBLICCP model can also be presented using the following flow chart.



To illustrate the proposed approach, a numerical example is solved in the next section.

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6 Numerical example

The following QFBLCCPP is considered to explore the application potentiality of the proposed approach: Find $X(x_1,x_2,\ldots,x_n)$ so as to, Max $\tilde{Z}_1 = \tilde{9}x_1 + \widetilde{12}x_2 + \tilde{1}x_3$, Max $\tilde{Z}_2 = \widetilde{11}x_2 + \widetilde{10}x_3$. Subject to

$$
\begin{aligned}
&Pr\left(\tilde{3}x_1 + \tilde{6}x_2 + \tilde{4}x_3 \le \tilde{b}_1\right) \ge 1 - p_1 \\
&Pr\left(\tilde{2}x_1 + \tilde{3}x_2 + \tilde{1}x_3 \le \tilde{b}_2\right) \ge 1 - p_2 \\
&\tilde{9}x_1 + \tilde{1}x_2 + \tilde{1}x_3 \le \widetilde{32} \\
&x_1,\ \ x_2,\ \ x_3\ \ge 0 \text{ and integers.}
\end{aligned} \tag{13}
$$

Here $\tilde{b}_1$ represents Pareto distributed fuzzy random variable and $\tilde{b}_2$ represents Frechet distributed fuzzy random variable. The values of scale and shape parameter of the Pareto distribution are given as follows.

Scale Parameter	Shape parameter	Specific probability level
$\widetilde{\beta}_1 = \widetilde{23}$	$\frac{1}{\widetilde{\lambda}_1} = \tilde{3}$	$p_1 = 0.09$

Table 1: Value of Scale and Shape parameter of the Pareto distribution.

Also, the values of scale, location and shape parameter of the Frechet distribution are presented as follows.

Scale parameter	Location Parameter	Shape Parameter	Specific probability level
$\widetilde{\delta}_2 = \tilde{6}$	$\widetilde{\eta}_2 = \widetilde{11}$	$\frac{1}{\widetilde{\mu}_2} = \widetilde{0.5}$	$p_2 = 0.20$

Table 2: Value of Scale, location and Shape parameter of the Frechet distribution

The triangular fuzzy numbers related to the parameters of the distributions are taken with the form as

$$
\begin{aligned}
&\widetilde{\beta}_1 = \widetilde{23} = (22.5,\ 23,\ 23.5); \frac{1}{\widetilde{\lambda}_1} = \tilde{3} = (2.95, 3, 3.05); \\
&\widetilde{\delta}_2 = \tilde{6} = (5.8, 6, 6.2); \widetilde{\eta}_2 = \widetilde{11} = (10, 11, 12); \frac{1}{\widetilde{\mu}_2} = \widetilde{0.5} = (0.3, 0.4, 0.5).
\end{aligned}
$$

Also the coefficients of the objectives and the constraints are also taken as triangular fuzzy number with the values

$$
\begin{aligned}
&\tilde{9} = (8.5, 9, 9.5); \widetilde{12} = (11, 12, 13); \tilde{1} = (0.95, 1, 1.05); \widetilde{11} = (10.6, 11, 11.4); \widetilde{10} = (9, 10, 11) \\
&\tilde{3} = (2.95, 3, 3.05); \tilde{2} = (1.5, 2, 2.5); \tilde{6} = (5, 6, 7); \tilde{4} = (2, 4, 6); \widetilde{32} = (31, 32, 33).
\end{aligned}
$$

Now, applying CCP technique to the probabilistic constraints and then using the defuzzification technique of fuzzy number to the objectives and modified constraints the model (13) reduces to:

$$
MaxV(\tilde{Z}_1) = 9x_1 + 12x_2 + x_3,\ \ MaxV(\tilde{Z}_2) = 11x_2 + 10x_3,
$$

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Subject to

$$\begin{aligned}
&3x_1+6x_2+4x_3\geq 23\\
&3x_1+6x_2+4x_3\leq 30.67\\
&2x_1+3x_2+x_3\geq 11\\
&2x_1+3x_2+x_3\leq 15.74\\
&9x_1+x_2+x_3\leq 32;\ x_1,\ x_2,\ x_3\ \geq 0 \text{ and integers.}
\end{aligned} \tag{14}$$

Now each DM considers their objective independently and then solve with respect to the system constraints in (14) to find the best and worst values of the objectives. The results are obtained as $x_1=3,\ x_2=3\ ,\ \ x_3=0$ with $V(\tilde{Z}_1)^b=63$; and $x_1=0,\ x_2=3\ ,\ \ x_3=3$ with $V(\tilde{Z}_2)^b=63$. The worst values of the objective of the respective DMs are calculated as

$$V(\tilde{Z}_1)^w=39 \quad \text{and } V(\tilde{Z}_2)^w\ =33.$$

Thus the fuzzy goals of the objective of the DMs are found as:

$$V(\tilde{Z}_1)^b \gtrsim 63$$

$$V(\tilde{Z}_2)^b \gtrsim 63.$$

On the basis of the tolerance limits of the objective of the DMs, the membership functions of the leader and follower are expressed as

$$\mu_{V(Z_1(x))}=\frac{9x_1+12x_2+x_3-39}{24}$$

$$\mu_{V(Z_2(x))}=\frac{11x_2+10x_3-33}{30}$$

Thus the FGP model is constructed, after converting the membership function into membership goals by assigning under and over-deviational variables to the membership functions and by minimizing the under deviational variables as

Minimize $D=0.042d_1^-+0.033d_2^-$, subject to

$$\begin{aligned}
&0.042(9x_1+12x_2+x_3-39)+d_1^--d_1^+=1\\
&0.033(11x_2+10x_3-33)+d_2^--d_2^+=1\\
&3x_1+6x_2+4x_3\geq 23\\
&3x_1+6x_2+4x_3\leq 30.67\\
&2x_1+3x_2+x_3\geq 11\\
&2x_1+3x_2+x_3\leq 15.74\\
&9x_1+x_2+x_3\leq 32\\
&x_1,\ x_2,\ x_3\ \geq 0 \text{ and integers.}
\end{aligned} \tag{15}$$

Now the above FGP model is solved using *software* LINGO (*Ver. 11*) to find the compromise solution in the decision making context. The solutions which are achieved are present through the following Table.

Solution	Expected value of Objective	Membership Value
$x_1=0, x_2=5, x_3=0$	$V\left(\tilde{Z}_1\right)=60$	$\mu_{V(Z_1(x))}=0.875$
	$V\left(\tilde{Z}_2\right)=55$	$\mu_{V(Z_2(x))}=0.733$

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It is to be noted here that if the problem is solved without considering the integer programming, the achieved solutions are almost the same as like this case. However, some situations arises in which the DMs concentrates on integer values. From that point of view, the developed methodology has been presented.

7 Conclusions

In this paper an innovative technique for solving FLBLICCP model is discussed in a hierarchical decision making environment for finding most satisfactory solution to all the DMs for overall benefit of the organization. The proposed procedure can be extended to solve hierarchical decision making problems with quadratic, fractional type of objectives. Also this methodology can be used to solve nonlinear decision making problems in a fully fuzzified domain. The suggested technique can also be used to solve the FLBLICCP model with coefficients taken as trapezoidal fuzzy numbers. The proposed methodology can be applied to different real life problems for obtaining most satisfactory solution in a hierarchical decision making environment. However, it is hoped that the proposed procedure may open up new vistas into the way of making decision in the decision making arena.

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References

[1] W. Candler, R. Townsley, A linear two-level programming problem, Computers and Operations Research, 9(1982), 59 – 76.
[2] W.F. Bialas, M.H. Karwan, Two-level linear programming, Management and Science, 30(1984), 1004 – 1020.
[3] A. Migdalas, Bilevel programming in traffic planning: Models, methods and challenge, Journal of Global Optimization, 7(1995), 381–405.
[4] J.P. Cote, P. Marcotte, G. Savard, A bilevel modeling approach to pricing and fare optimization in the airline industry, Journal of Revenue and Pricing Management, 2(2003), 23–36.
[5] B.Y. Kara, V. Verter, Designing a road network for hazardous materials transportation, Transportation Science, 38(2004), 188–196.
[6] M.G. Nicholls, The applications of non-linear bi-level programming to the aluminum industry, Journal of Global Optimization, 8(1996), 245–261.
[7] M.A. Amouzegar, K. Moshirvaziri, Determining optimal pollution control policies: an application of bilevel programming, European Journal of Operational Research, 119(1999), 100–120.
[8] S. Dempe, J.F. Bard, Bundle trust-region algorithm for bilinear bilevel programming, Journal of Optimization Theory and Applications, 110(2001), 265–288.
[9] M. Fampa, L.A. Barroso, D. Candal, L. Simonetti, Bilevel optimization applied to strategic pricing in competitive electricity markets, Computational Optimization and Applications, 39(2008), 121–142.
[10] E. Roghanian, S.J. Sadjadi, M.B. Aryanezhad, A probabilistic bilevel linear multi-objective programming problem to supply chain planning, Applied Mathematics and Computation,188(2007), 786–800.
[11] T. Uno, H. Katagiri, Single- and multi-objective defensive location problems on a network, European Journal of Operational Research, 188(2008), 76–84.
[12] T. Uno, K. Katagiri, K. Kato, A multi-dimensionalization of competitive facility location problems, International Journal of Innovative Computing, Information and Control, 7(2011), 2593–2601.
[13] G. B. Dantzig, Linear programming under uncertainty, Management Science,1(1955), 197-206.
[14] A. Charnes, W.W. Cooper, Chance-constrained programming, Management Science, 6(1959), 73-79.
[15] S. Kataoka, A stochastic programming model, Econometrica, 31(1963), 181–196.
[16] A.M. Geoffrion, Stochastic programming with aspiration or fractile criterion, Management Science, 13(1967), 672–679.
[17] L.A. Zadeh, Fuzzy sets, Information and Control,8(1965), 338–353.
[18] H.J. Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems, 1(1978), 45 – 55.

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[19] M. Sakawa, H. Katagiri, T. Matsui, Stackelberg solutions for fuzzy random two-level linear programming through probability maximization with possibility, Fuzzy Sets and Systems,188(2012), 45–57.

[20] H. Shih, E. S. Lee, Compensatory fuzzy multiple level decision making, Fuzzy Sets and Systems, 114(1) (2000), 71–87.

[21] G. Zhang, J. Lu, Y. Gao, Fuzzy bilevel programming: multiobjective and multi-follower with shared variables, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems,16 (2008),105–133.

[22] B. B. Pal, B. N. Moitra, A fuzzy goal programming procedure for solving quadratic bilevel programming problems, International Journal of Intelligence Systems, 18(5) (2003),529-540.

[23] M. A. Abo-Sinha, A bi-level non-linear multi-objective decision making under fuzziness, Operation Research Society of India (OPSEARCH), 38 (2001), 484–495.

[24] M. S.Osman, M. A.Abo-Sinna, A. H.Amer,A multi-level non-linear multi-objective decision-making under fuzziness, Applied Mathematics and Computation, 153(1)(2004), 239-252.

[25] I. A. Baky, Fuzzy goal programming algorithm for solving decentralized bi-level multi-objective programming problems, Fuzzy Sets Systems, 160(2009), 2701–2713.

[26] S. R. Arora, R. Gupta, Interactive fuzzy goal programming approach for bilevel programming problem, European Journal of Operational Research, 194(2) (2009), 368-376.

[27] S. Deng, L. Zhou, X. Wang, Solving the fuzzy bilevel linear programming with multiple followers through structured element method, Mathematical Problems in Engineering, 2014 (2014), 1-6.

[28] N. Modak, A. Biswas, A Fuzzy Programming Approach for Bilevel Stochastic Programming, Advances in Intelligence Systems and Soft Computing, 236 (2014), 125 – 136.

[29] M. Delgado, M.A. Vila, W. Voxman, On a canonical representation of fuzzy number, Fuzzy Sets and Systems, 93(1998), 125-135.

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