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The influences of mathematics beliefs on mathematics achievement through mathematics self-efficacy: A structural equation model

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Abstract: The purpose of present study was to examine the direct and indirect effects of mathematics beliefs on mathematics achievement. Specially, regarding to powerful predictor and mediator of mathematics self-efficacy. The subjects of the study were middle school students of the academic year 2013. 400 of 8th graders were selected from four public schools of Shiraz in Iran to estimate and test the hypothesized effects of mathematics beliefs on mathematics self-efficacy and mathematics achievement. A two stage Cluster Random Sampling Method was utilized. This study examined correlations between latent variables and their factors also determined coefficients between latent variables by using Measurement Model. Then the estimate of Structural Equation Model revealed that mathematics beliefs had a direct and an indirect effect on mathematics achievement. The model specified mathematics beliefs and mathematics achievement. The high goodness-of-fit indices, also acknowledged that postulated model has a good fit to the data.

Keywords: Mathematics self-efficacy, mathematics beliefs, structural equation model, measurement model, mathematics achievement

1 Introduction

As a general subject, Mathematics is required to be learned by any individuals who belong to the society. Nevertheless, the chances to acquire mathematics would determine and restrict their mathematical attainment. Indeed, mathematics is not limited to particular learners while it is known that every student has to know how to think mathematically. While instructing and learning mathematics still remains as daunting undertakings, numerous issues can be observed on the mathematics education at schools.

Studies on cognitive variables such as problem solving, ability, thinking scale, following the findings

of Coleman et al., (1966) recommend that the schools do not make a big difference while numerous surveys have been so far executed regarding the in- and out-of-school variables that may potentially influence the students' attainment. A voluminous amount of study has explored the association between the learners' individualities like their self-concepts, their attitudes of mathematics, their family backgrounds along with their motivation and consequent academic performance. On the whole, many surveys such as Papanastasiou (2002) and Suthar (2013) have established that there is a reliable outline regarding the influences of personal attributes on mathematics achievement.

According to Bandura (1986), individuals possess a self-system that enables them to exercise a measure of control over

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their thoughts, feelings and actions. In other words, student's understanding of their ability to perform school tasks or self-efficacy, predict their ability to conduct such tasks. Through self-reflection process individuals can evaluate their own experiences and thought processes. However individual interprets the result of their performance and alters their environments and their self-beliefs, which in turn informs and alters their subsequent performances.

During the last decades, the students' little capability in science, especially in mathematics, has turned into a rocketing worldwide concern. According to the recent findings, the academic attainment is accomplished through having an emphasis on learning's individual and social features. The learners' troubles with solving the mathematical tasks and efficient usage of strategies would be directly associated with the students' mathematics beliefs, self-regulation, lack of critical thinking, and problem solving ability (Moscardini, 2010; Ismail, 2009; Ismail & Awang, 2008).

After many survey findings, Ministry of education in Iran (2010), approved that the student's difficulties with mathematics low performance in secondary school directly related to their motivation, beliefs about mathematics, background in mathematics. Especially in primary and middle school the difficulties related to strategy at problem solving.

In the field of mathematics achievement, several studies used structural equation modelling or path analysis in determining the important variables that effect mathematics performance directly or indirectly via mathematics self-efficacy. Zarch & Kadivar (2006) constructed a structural model on mathematics ability and mathematics performance through mathematics self-efficacy as a mediator. The results indicated that the overall model fit the data reasonably well.

Kabiri and Kiamanesh (2004) discovered the direct and indirect impacts of mathematics attitude on mathematics achievement via mediator variables like self-efficacy and mathematics anxiety by using the path analysis model. Hailikari, Nevgi, and Komulainen (2008) found the relationships between prior knowledge, academic self-beliefs, and prior study in mathematics success in predicting the achievement in mathematics with using structural equation modelling. Kiamanesh, and Mahdavi-Hezaveh (2008) findings showed that the direct and indirect impact of mathematics beliefs on mathematics achievement are positive and significant while this study indicated that the direct and indirect effects of background education in mathematics on mathematics achievement was negative and significant.

Yusuf (2011) who examine using path analysis for variables such as self- efficacy, achievement motivation, and learning strategies with students' academic achievement. Results of Yusuf's research demonstrated that the path diagram indicated a direct and an indirect influence among the variables examined with learner's academic achievement. Particularly, the path analysis has indicated the direct impact of self-efficacy and indirect effect of achievement motivation and self-learning strategies on participants' academic accomplishment. Moreover, the direct and indirect path showed that the mediator role of self-efficacy on achievement motivation and learning strategies.

A research was conducted by Liu and Koirala (2009) who evaluated the associations between the students' self-efficacy and mathematics achievement. A correlation analysis and a linear regression analysis were applied to answer the research questions.

Moenikia and Zahed-Babelanb (2010) examined simple and multiple relationships between mathematics attitude, academic motivation and intelligence quotient with mathematics achievement. Findings of present study showed that mathematics attitude, academic motivation, and intelligence quotient can be identified and explained 33% of mathematics achievement.

The present study was designed to investigate the role of personal variables on students' mathematics achievement. To achieve this goal, two variables, i.e., mathematics self-efficacy, mathematics beliefs were measured and structural equation modelling analysis was conducted to confirm the related model on the basis of theoretical principles in mathematics education. The specific objectives of the present study was to examine the direct and indirect influence of mathematic beliefs through mathematics self-efficacy as mediator on students' mathematics achievement.

2 Hypotheses

The major hypotheses of this study of grade eight students in Iran include the following:

Hypothesis 1. Students' mathematics self-efficacy has a direct influence on their mathematics achievement.

Hypothesis 2. Students' mathematics beliefs has a direct influence on their mathematics achievement.

Hypothesis 3. Students' mathematics beliefs has a direct influence on their mathematics self-efficacy.

Hypothesis 4. Students' mathematics beliefs has indirect effects on their mathematics achievement through their effects on mathematics self-efficacy.

3 Method

3.1 Sample and data collection

The sample consisted of 400 (200 male and 200 female) eight-grade students from four schools of Shiraz in Iran. Four schools selected in from 45 middle public schools. For this purpose, two stage Cluster Random Sampling Method was used.

3.2 Measures

Mathematics achievement test. The first construct measured mathematics achievement which consisted of algebra, geometry, arithmetic, and statistics. This standard instrument was adjusted by the Ministry of Education in Shiraz for the entrance exam for grade 9 based on textbook in grade 8. This set of instrument consisted of 22 questions which were used to measure the mathematics achievement construct. Each question was measured on a 5 point scale, so the total scores would be 110. Algebra was measured using 6 questions, arithmetic was also measured using 6 questions, and 8 questions measured geometry whilst for statistics 2 questions were given. The purpose of the mathematics instrument was to measure the students' knowledge on the four main subjects based on the 8th grade textbook.

Mathematics self-efficacy scale. The second construct in this study was students' mathematics self-efficacy which consisted of mathematics behaviours in everyday life (everyday mathematics tasks) and perceptions of performance capability related to mathematics problems. These sub constructs of mathematics self-efficacy consisted of 22 items that each sub-construct consisted of 11 items which were modified based on Betz, and Hackett (2013) Likert scale instrument, middle school textbooks and the expectations from the 8th grade students. These constructs were measured based on the 9 Likert rating scale (rating from 0 as no confidence at all to 9 as complete confidence).

Mathematics belief scale. The students' mathematics beliefs were made up of three sub-constructs: Students' beliefs about mathematics, their beliefs about importance of mathematics, and their beliefs about one's ability of mathematics.

Each sub-construct consisted of 5 - 6 items adopted and adapted from Vallerand, Pelletier, Blais, Briere, Senecal, and Vallieres (1992). It needs to be asserted that the mentioned construct was measured using a 5-point Likert scale (ranging from 1 strongly disagree to 5 strongly agree).

3.3 Validity of instruments

The content and face validity of the instrument in current study were done twice before the process of data collection. The first confirmation of the instrument for the data collection process permission was done by Moallem Institution supervised by the Ministry of Education in Shiraz, Iran. A committee of experts in this Institution evaluated the items in the questionnaires. Both content validity and face validity of the questionnaire were established by the specialists in education and mathematics teaching. The first confirmation was shown by a stamp above every sheet of the instrument. After confirming the instrument by the committee of educationists in the mentioned Institution, they were sent to the Ministry of Education for procedure of data collection. The second approval was done by three experts in mathematics education. The experts investigated the instrument based on appropriateness, comprehensiveness, and clarity of each item. The researcher was advised to amend a few items in terms of language simplicity and clarity. Based on their suggestions, the questionnaires were amended.

3.4 Reliability of instrument

To test the reliability, Cronbach's Alpha was conducted. Based on the results the reliability estimates of each subscale ranged from .63 to .98. The reliability estimate of mathematics behaviors in everyday life was the lowest (α=.63) while the reliability estimate of the perceptions of performance capability in relation to mathematics problem was the highest (α=.98). The reliabilities of other variables were acceptable. The reliability estimate of mathematics self-efficacy (α=.72), beliefs about mathematics (α=.81), beliefs about importance of mathematics (α=.83), beliefs about one's ability (α=.83), mathematics beliefs (α=.91), arithmetic (α=.76), algebra (α=.77), geometry (α=.78), statistics (α=.85), mathematics achievement (α=.90) showed high reliabilities. These results suggested the scales had high internal consistency and reliability.

4 Results

4.1 Assessment of measurement models

In Measurement Model (MM), all latent constructs were entered without exogenous or endogenous assignments. The purpose of the measurement model was to test for a model fit, test for discriminant validity. The measurement model was used to examine the relationship between measured variables (indicators) and latent constructs (Byrne, 2001; Kline, 2005).

For the MM fit, data were first checked with the model chi-square goodness-of-fit, and approximate fit indices. The MM fit were examined based on Chi-square statistics, Goodness-of-Fit Index (GFI), Adjusted Goodness-of-Fit Index (AGFI), Comparative Fit Index (CFI), Incremental Fit Index (IFI), Normed Fit Index (NFI), Tucker Lewis index (TLI) and Root Mean Square Error of Approximation (RMSEA) indices (set at .05). Bentler (1990), suggested the value of CMIN/DF of < 5.0 indicates good fit of the model. Also, Chau (1997), Segars & Grover (1993) suggested that the value of GFI $> .90$ demonstrates good fit of the model. Further, Bentler (1990) declared the value of CFI $> .90$ demonstrates adequate fit of the model. In addition, according to Bentler & Bonett (1980), NFI $> .90$ indicated a reasonable fit of the model. Furthermore, Byrne (2001) suggested the RMSEA$< .08$ represent good fit of the model.

Discriminant validity refers to the extent in which a construct is truly distinct from other constructs. Discriminant validity involves relationships between a particular latent construct and other constructs of a similar nature (Brown, 2006). The discriminant validity can be tested using factor loadings at $\geq$ .5 on a factor which would indicate high validity (Hair, et al, 2010; Bryne, 2001). In addition, to test the statistical significance of factor loadings and correlations, all standardized factor loadings must be more than .5, positive, and not more than 1.0 (Bryne 2001; Hair et. al., 2010). Furthermore, according to Hair et al, (2010), if correlations between two latent constructs are greater or equal .90, it indicates violation of the discriminant validity requirements and hence, the existence of multicollinearity (Grewal et al., 2004). Based on the results in Table 4.36, the correlations among the latent constructs were not greater than .90. The correlation index among the latent variables range from.72 to .87.

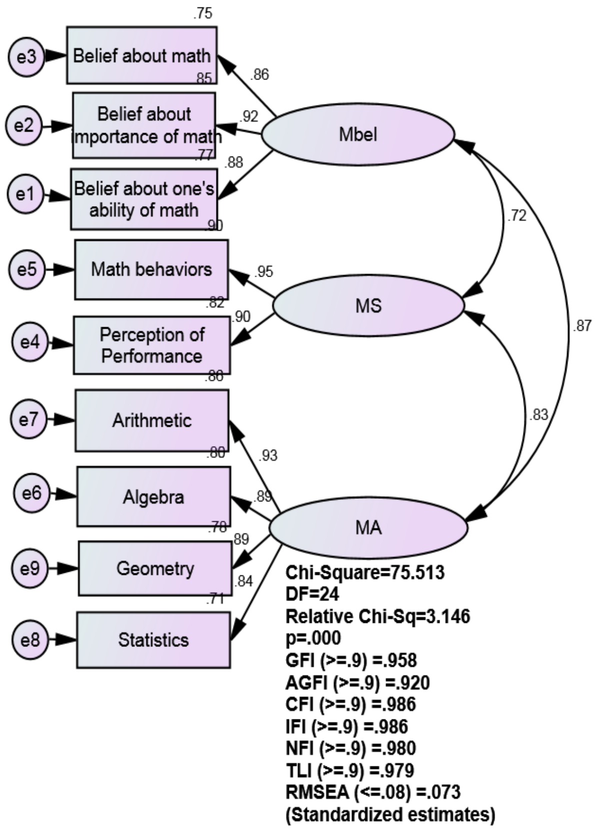

Fig. 1: Measurement model of endogenous and exogenous, Mbel: Mathematics beliefs, Ms: Mathematics self-efficacy, and MA: Mathematics achievement.

As is shown in Figure 1, the MM was fixed based on eight indices (chi-square, CFI, TLI, GFI, AGFI, IFI, NFI and RMSEA). The measurement model indicated adequate fit to the given data regard to the ratio of chi-square statistics and degrees of freedom (2.839), $P<.05$ (P=.000), CFI (.99), TLI (.98), GFI (.96), AGFI (.92), IFI (.99), NFI (.98) and RMSEA (.07). These suggested a good fit of the model to the data. Because the chi-square statistics were very sensitive to the sample size, it was more appropriate to make the judgments regarding whether the model had an acceptable fit through examining CFI, TLI, GFI, AGFI, IFI, NFI, and RMSEA fit indices (Byrne, 2001; Kline, 2005). Another important aspect of the full measurement model is that if the full measurement model fails to provide acceptable fit indices, structural models that would be tested may have worse fit indices (Kline, 2005). Also, as shown in Figure 1, all of the factor loadings and correlation coefficients were more than .5. The correlation coefficients ranged from .80 to .85 and the factor loadings ranged from .84 to .96. The values of the correlation coefficients and factor loadings indicated the measurement model was valid. The full measurement model of this study provided an acceptable fit to examine the structural models without modifying the full measurement model.

4.2 Assessment of structural model

As shown in Figure 2, the hypothesis was to analyses the full structural model that investigated the relationship among the predictor variable, mediator and criterion variable.

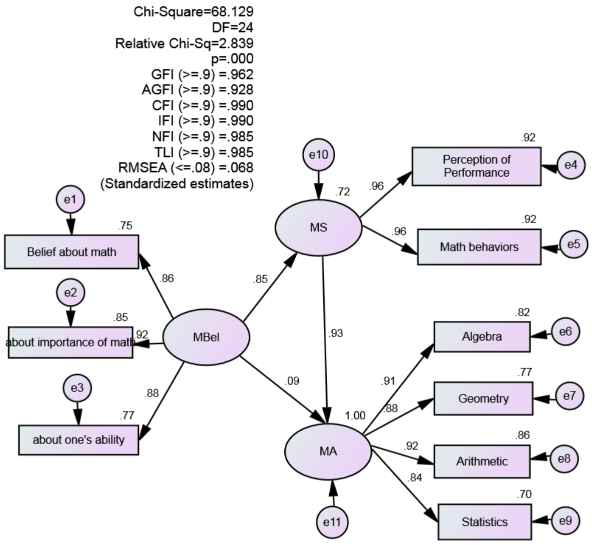

Fig. 2: Completely standardized parameter estimates for the structural equation model of mathematics achievement. Note: MS (Mathematics self-efficacy), MA (Mathematics achievement), and Mbel (Mathematics beliefs).

The model provided a good fit to the given data ($\chi 2$ [24 df, N=400] =2.428, *p*=.000, CFI=.99, TLI=.98, GFI=.96, AGFI=.92, IFI=.99, NFI=.98, and REMEA=.07).

Results presented in Table 1 (Regression Weights) indicated that there are significant relationships between predictor variable and mathematics self-efficacy as a mediator ($\beta = .85,\ p = .000 < .001$). Also, students' mathematics self-efficacy had a strong significant effect on mathematics achievement ($\beta = .85,\ p = .000 < .001$). On the other hand, there was positive relationship but insignificant between students' mathematics beliefs and their mathematics achievement ($\beta = .09,\ p = .012 > .001$). According to Kline (2005), the occurrence of obtaining coefficients with different signs in the result of a suppression criterion while controlling for other predictors is a "surprise" given the correlation between that predictor and the criterion" (Kline, 2005).

5 Discussion and conclusion

This study examined the direct and indirect relationships between students' mathematics beliefs on their mathematics achievement, direct relationships between students' mathematics beliefs on their mathematics self-efficacy, and direct relationships between students' mathematics self-efficacy and mathematics achievement by using SEM based on social

Path			Unstandardized Estimate	S.E.	C.R.	P	Standardized Estimate
MS	<—	MBel	2.471	.116	21.225	.000	.85
MA	<—	MS	.622	.028	22.539	.000	.85
MA	<—	MBel	.175	.069	2.513	.012	.09
Geo	<—	MA	.841	.030	27.659	.000	.88
Arith	<—	MA	1.027	.032	31.734	.000	.92
Alg	<—	MA	1.000				.91
Stat	<—	MA	.316	.013	24.635	.000	1.00
POP	<—	MS	1.000				.96
MB	<—	MS	2.967	.062	47.688	.000	.96
BAM	<—	MBel	1.179	.050	23.579	.000	.86
BAIM	<—	MB	1.060	.040	26.796	.000	.92
BAOAM	<—	MB	1.000				.88

Table 1: Regression Weights and Standardized Regression Weights, where *p<.001. Note: MS(Math Self-efficacy), MBel(Math Beliefs), MA(Math Achievement). Geo(Geometry). Arith(Arithmetic). Alg(Algebra). Stat(Statistics). MB(Mathematics Behaviours in everyday life). BAM(Beliefs about Math). POP(Perceptions of Performance capability related to Mathematics problems). BAIM(Beliefs about Importance of Math). BAOAM(Beliefs about One's ability of Math).

cognitive theory. The results showed, students who held strong beliefs in mathematics, had higher self-efficacy in mathematics and these led to high mathematics achievement. Also, students' mathematics beliefs had a poor and no significant direct effect on their mathematics achievement.

These results showed, students' mathematics beliefs predicted mathematics achievement through its influences on mathematics self-efficacy. In other words, these finding supported the assumption that mathematics self-efficacy might facilitate students' achievement in mathematics. Precisely, the above findings were similar to the existing literature on self- efficacy and mathematics beliefs in relation to the students' academic achievement in mathematics such as Moenikia and Zahed-Babelanb 2010; Kabiri and Kiamanesh 2004; Bandura 1977; and Zarch and Kadivar 2006.

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