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Frobenius-Euler and Frobenius-Genocchi polynomials and their differential equations

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Abstract: In the present paper, we obtain differential equations of Frobenius-Euler polynomials by using quasi-monomiality principle. Furthermore, we introduce Frobenius-Genocchi polynomials and obtain some recurrence relation and some differential equations.

Keywords: Frobenius-Euler polynomials, Frobenius-Genocchi polynomials, differential equations, recurrence relations.

1 Introduction

Appell polynomials have been an active research area since they have many applications in analytic number theory and asymptotic approximation theory. It was He and Ricci who found the differential equation of the Appell polynomials with the aid of factorization method. As a special case, they found differential equations of Bernoulli and Euler polynomials. Then, differential equations of 2D Bernoulli polynomials were found by Bretti and Ricci. Afterward, Yılmaz Yaşar and Özarslan introduced the extended 2D Bernoulli and 2D Euler polynomials and obtained the differential, integro-differential and partial differential equations. Hermite-based Appell polynomials were introduced by Khan et al. Moreover, Özarslan introduced unified Apostol-Bernoulli, Euler and Genocchi polynomials. By quasi-monomiality principle and factorization method, Srivastava, Özarslan and Yılmaz Yaşar found the differential, integro-differential and partial differential equations of Hermite-based appell polynomials. In 1935, Sheffer found the infinite order differential equations for the Appell polynomials. In 2013, Özarslan and Yılmaz Yaşar obtained a set of all finite order differential equations for the Appell polynomials. Besides, some addition theorems related with Appell polynomials were obtained by Pinter and Srivastava. Moreover, some Euler quadrature formula was obtained by Bretti and Ricci. Furthermore, a unified generating functions of the generalized Bernoulli, Euler and Genocchi polynomials were obtained by Özden, Şimşek and Srivastava. The related references are [3], [4], [11], [14], [20], [21], [22], [23], [24], [25], [30].

In the present paper, we take into consideration Frobenius-Euler and Frobenius-Genocchi polynomials and their differential equations. Let's recall some basic things about quasi-monomiality principle:

If there exist two operators $\hat{P}$ and $\hat{M}$, independent of n, acting on the polynomials as derivative and multiplicative

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operators respectively, satisfying

$$\hat{P}(P_n(x)) = nP_{n-1}(x) \text{ and } \hat{M}(P_n(x)) = P_{n+1}(x)$$

then the polynomial set $\{P_n(x)\}_{n=0}^{\infty}$ is called quasi-monomial.

In view of monomiality principle, every polynomial set is quasi-monomial [6], some new consequences were gathered for Hermite, Laguerre, Legendre and some types of Appell polynomials in [5], [9], [10], [11], [26].

Using the properties of quasi-monomiality and factorization method [11], we find the differential equations of Frobenius-Genocchi and Frobenius-Euler polynomials. First, we introduce some facts about the Appell polynomials and factorization method. It is well known that Appell polynomials are defined by the following generating relation

$$G_A(x,t) = A(t)e^{xt} = \sum_{n=0}^{\infty} R_n(x)\frac{t^n}{n!}. \tag{1}$$

Here,

$$A(t) = \sum_{k=0}^{\infty} R_k \frac{t^k}{k!}, \;\; A(0) \neq 0$$

is an analytic function at $t = 0$ and $R_k := R_k(0)$. Taking into consideration of the following summation,

$$\frac{A'(t)}{A(t)} = \sum_{n=0}^{\infty} \alpha_n \frac{t^n}{n!}$$

it is directly seen that for any $A(t)$ the derivatives of $R_n(x)$ satisfy

$$R_n'(x) = nR_{n-1}(x).$$

Taking the lowering operator as $\Phi_n := \frac{1}{n}D_x$, where $D_x := \frac{d}{dx}$, we have

$$(\Phi_1\Phi_2...\Phi_{n-1}\Phi_n)R_n(x) = R_0(x).$$

It was He and Ricci [11], who found the raising operator Ψ_n as

$$\Psi_n := (x+\alpha_0) + \sum_{k=0}^{n-1} \frac{\alpha_{n-k}}{(n-k)!}D_x^{n-k}.$$

such that

$$\Psi_n(R_n(x)) = R_{n+1}(x).$$

Thus, we have the following differential equation satisfied by Appell polynomials [11],

$$(\Phi_{n+1}\Psi_n)R_n(x) = R_n(x). \tag{2}$$

Now we focus on differential equation of Frobenius- Euler and Frobenius-Genocchi polynomials. The Frobenius-Euler polynomials are given by the following generating relation

$$\frac{(1-\lambda)}{e^t - \lambda}e^{xt} = \sum_{n=0}^{\infty} E_n^F(x;\lambda)\frac{t^n}{n!}. \tag{3}$$

The Frobenius-Euler numbers are given by

$$\frac{1-\lambda}{e^t-\lambda}=\sum_{n=0}^{\infty}E_n^F(\lambda)\frac{t^n}{n!}. \tag{4}$$

The coefficients $e_k^F(\lambda)$ which are related with Frobenius-Euler polynomials $E_k^F(x;\lambda)$ is defined as

$$e_k^F(\lambda):=-\sum_{l=0}^{k}\frac{1}{2^l}\binom{k}{l}E_{k-l}^F(\frac{1}{2};\lambda). \tag{5}$$

It is clear that

$$e_0^F=-1,\ e_1^F(\lambda)=-1-\frac{1}{\lambda-1}. \tag{6}$$

Taking $\lambda=-1$ in (3), we get usual Euler polynomials. Namely, the Euler polynomials are defined via the generating relation

$$G_E(x,t)=\frac{2e^{xt}}{e^t+1}=\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!};\ |t|<\pi. \tag{7}$$

On the other hand, the Euler numbers E_n are given by the following relation:

$$\frac{2}{e^t+e^{-t}}=\sum_{n=0}^{\infty}E_n\frac{t^n}{n!}.$$

Furthermore we have,

$$E_n(\frac{1}{2})=2^{-n}E_n.$$

The numbers e_k, which are defined (and/ or used) in [3] and [11] by

$$e_k=(-\frac{1}{2})^k\sum_{h=0}^{k}\binom{k}{h}E_{k-h} \tag{8}$$

will be appeared in our main results.

The q- analogs of Frobenius-Euler polynomials were used by Y. Şimşek, A. Bayad, V.Lokesha (see [28]) . They obtained the relation between q-Bernstein polynomials and q-Frobenius-Euler polynomials, I-functions and q-Stirling numbers. Afterward, J. Choi, D. S. Kim, T. Kim, Y.H found some identities of Frobenius-Euler numbers and polynomials (see [8]). Generalization of Apostol-type Frobenius-Euler polynomials were defined by Burak Kurt and Yılmaz Şimşek [18] via the generating relation:

$$(\frac{a^t-u}{\lambda b^t-u})^{\alpha}c^{xt}=\sum_{n=0}^{\infty}H_n^{(\alpha)}(x;u;a,b,c;\lambda)\frac{t^n}{n!},\ a,b,c\in\mathbb{R}^+,\ a\neq b,\ x\in\mathbb{R}.$$

They gave some new identities for generalized Apostol type Frobenius-Euler polynomials. Other various properties of the Frobenius-Euler and Euler polynomials were given in [1], [2] ,[8], [15], [16], [17], [19], [29]. Note that, Choi and Srivastava presented the corrected expression for a certain widely-recorded generalized Goldbach-Euler series[7].

In this paper, we mainly focus on Frobenius- Genocchi polynomials and their differential equations. As far as we have known, Frobenius-Genocchi polynomials have not been defined in literature. Let's start by defining the Frobenius-Genocchi polynomials.

Definition 1. *Frobenius-Genocchi polynomials are defined by means of the following generating relation:*

$$\frac{(1-\lambda)t}{e^t-\lambda}e^{xt} := \sum_{n=0}^{\infty} G_n^F(x;\lambda)\frac{t^n}{n!}. \tag{9}$$

Frobenius-Genocchi numbers can be obtained via the following generating relation:

$$\frac{(1-\lambda)t}{e^t-\lambda} = \sum_{n=0}^{\infty} G_n^F(\lambda)\frac{t^n}{n!}. \tag{10}$$

It is clear that

$$G_0^F(\lambda)=0,\ G_1^F(\lambda)=1. \tag{11}$$

We define the numbers $g_k(\lambda)$ by

$$g_k(\lambda) := \sum_{l=0}^{k} \frac{1}{2^l}\binom{k}{l} G_{k-l}^F(\frac{1}{2};\lambda). \tag{12}$$

By series manipulations, one gets

$$g_0=0,\ g_1=\frac{1}{2}. \tag{13}$$

Taking $\lambda=-1$ in (9), we get Genocchi polynomials:

$$\frac{2t}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} G_n(x)\frac{t^n}{n!},\ |t|<\pi.$$

Now, we present some relations between Frobenius-Euler and Frobenius-Genocchi polynomials:

$$G_n^F(x;\lambda) = nE_{n-1}^F(x;\lambda), \tag{14}$$

$$G_n^F(x;\lambda) = n\sum_{k=0}^{n-1}\binom{n-1}{k}E_{n-k-1}^F(\lambda)x^k, \tag{15}$$

$$2^{n-1}G_n^F(x+y;\lambda^2) = \sum_{k=0}^{n}\binom{n}{k}G_{n-k}^F(2x;\lambda)E_k^F(2y;-\lambda). \tag{16}$$

Note that the formulas (14), (15) and (16) can be easily proved via series manipulations.

We organize the paper as follows: In section 2, we give recurrence relation, shift operators and differential equations of the Frobenius Genocchi polynomials. In section 3, we deal with finding the recurrence relation, shift operators and differential equations of the Frobenius-Euler polynomials.

2 Recurrence Relation and Differential equations of Frobenius-Genocchi polynomials

In this section, we find recurrence relation, shift operators and differential equation of Frobenius-Genocchi polynomials via factorization method. The main result of this section is given in the following theorem:

Lemma 1. *For the Frobenius-Genocchi polynomials, we have the following recurrence relation:*

$$G_{n+1}^{F}(x;\lambda) = \frac{(n+1)}{n}[(x-\frac{1}{2(1-\lambda)})G_{n}^{F}(x;\lambda) - \frac{1}{(1-\lambda)}\sum_{k=2}^{n+1} g_k(\lambda)\binom{n+1}{k}\frac{G_{n+1-k}^{F}(x;\lambda)}{n+1}] \tag{1}$$

where $g_k(\lambda)$ is given by (12).

Proof. Differentiating both sides of (9)

$$\frac{(1-\lambda)t}{e^t-\lambda}e^{xt} := \sum_{n=0}^{\infty} G_n^F(x;\lambda)\frac{t^n}{n!}$$

with respect to t, then using Cauchy product (see [27]), (12) and (13), we get the recurrence relation

$$G_{n+1}^{F}(x;\lambda) = \frac{(n+1)}{n}[(x-\frac{1}{2(1-\lambda)})G_{n}^{F}(x;\lambda) - \frac{1}{(1-\lambda)}\sum_{k=2}^{n+1} g_k(\lambda)\binom{n+1}{k}\frac{G_{n+1-k}^{F}(x;\lambda)}{n+1}].$$

Lemma 2. *Shift operators are as follows:*

$$L_n^- := \frac{1}{n}D_x,$$

$$L_n^+ := \frac{n+1}{n}(x-\frac{1}{2(1-\lambda)}-\frac{1}{(1-\lambda)}\sum_{k=2}^{n+1}\frac{g_k(\lambda)}{k!}D_x^{k-1})$$

where $D_x = \frac{d}{dx}$.

Proof. Taking derivative with respect to x, in (9) and equating the coefficients of t^n, we obtain:

$$D_x G_n^F(x;\lambda) = nG_{n-1}^F(x;\lambda).$$

Therefore, the lowering operator is $L_n^- := \frac{1}{n}D_x$ and

$$L_n^- G_n^F(x;\lambda) = G_{n-1}^F(x;\lambda).$$

Using

$$G_{n+1-k}^F(x;\lambda) = [L_{n+2-k}^- L_{n+3-k}^- \ldots L_n^-]G_n^F(x;\lambda) = \frac{(n+1-k)!}{n!}D_x^{k-1}G_n^F(x;\lambda)$$

in (17), we get

$$L_n^+ := \frac{n+1}{n}(x-\frac{1}{2(1-\lambda)}-\frac{1}{(1-\lambda)}\sum_{k=2}^{n+1}\frac{g_k(\lambda)}{k!}D_x^{k-1}).$$

Theorem 1. *The differential equation satisfied by the Frobenius-Genocchi polynomials is given by*

$$[(x-\frac{1}{2(1-\lambda)})D_x - \frac{1}{1-\lambda}\sum_{k=2}^{n+1}\frac{g_k(\lambda)}{k!}D_x^k - (n-1)]G_n^F(x;\lambda) = 0. \tag{18}$$

Proof. Applying factorization method, ([12], [13])

$$L_{n+1}^{-}L_n^{+}G_n^F(x;\lambda) = G_n^F(x;\lambda),$$

we obtain the differential equation as

$$[(x-\frac{1}{2(1-\lambda)})D_x - \frac{1}{(1-\lambda)}\sum_{k=2}^{n+1}\frac{g_k(\lambda)}{k!}D_x^k - (n-1)]G_n^F(x;\lambda) = 0.$$

Corollary 1. *Taking $\lambda = -1$, in the above Theorem and recalling the fact that $G_n^F(x,-1) = G_n(x)$, we get the differential equation of the Genocchi polynomials:*

$$[(x-\frac{1}{4})D_x - \frac{1}{2}\sum_{k=2}^{n+1}\frac{g_k(-1)}{k!}D_x^k - (n-1)]G_n(x) = 0.$$

Lemma 3. *For Frobenius-Genocchi polynomials, we have the following recurrence relation*

$$G_{n+1}^F(x;\lambda) = \frac{(n+1)}{n}[(x-\frac{1}{1-\lambda})G_n^F(x;\lambda) + \frac{1}{1-\lambda}\sum_{k=1}^{n}\binom{n}{k}e_k^F(\lambda)G_{n-k}^F(x;\lambda)] \tag{19}$$

where $e_k^F(\lambda)$ is given by (5).

Lemma 4. *Shift operators are as follows:*

$$L_n^{-} := \frac{1}{n}D_x,$$

$$L_n^{+} = \frac{(n+1)}{n}[x - \frac{1}{1-\lambda} + \frac{1}{1-\lambda}\sum_{k=1}^{n}\frac{e_k^F(\lambda)}{k!}D_x^k].$$

Proof. Taking into consideration of Lemma 3, one can get Lemma 7.

Theorem 2. *Differential equation is given by:*

$$[(x-\frac{1}{1-\lambda})D_x + \frac{1}{1-\lambda}\sum_{k=1}^{n}\frac{e_k^F(\lambda)}{k!}D_x^{k+1} - (n-1)]G_n^F(x;\lambda) = 0 \tag{20}$$

where $e_k^F(\lambda)$ are given in (5).

Proof. Procceding in a similar manner as in the proof of Theorem 4, one can get Theorem 8 at once.

Corollary 2. *Taking $\lambda = -1$ in above theorem recalling the fact that $G_n^F(x;-1) = G_n(x)$, we get*

$$[(x-\frac{1}{2})D_x + \frac{1}{2}\sum_{k=0}^{n}\frac{e_k^F(-1)}{k!}D_x^{k+1} - (n-1)]G_n(x) = 0.$$

3 Recurrence Relation, Shift operators and Differential Equations of Frobenius-Euler Polynomials

In this section, we give the differential equations of the Frobenius-Euler polynomials via the factorization method.

Lemma 5. *For Frobenius-Euler polynomials we have the following recurrence relation:*

$$E_{n+1}^{F}(x;\lambda) = (x - \frac{1}{1-\lambda})E_n^F(x;\lambda) + \frac{1}{1-\lambda}\sum_{k=0}^{n-1}\binom{n}{k}e_{n-k}^{F}(x;\lambda)E_k^F(x;\lambda) \tag{21}$$

where $e_{n-k}^{F}(\lambda)$ are related with Frobenius-Euler numbers given in (5).

Proof. Taking derivative with respect to t, on both sides of the generating function

$$\frac{(1-\lambda)}{e^t-\lambda}e^{xt} := \sum_{n=0}^{\infty}E_n^F(x;\lambda)\frac{t^n}{n!}$$

and using (6), we get the recurrence relation

$$E_{n+1}^{F}(x;\lambda) = (x - \frac{1}{1-\lambda})E_n^F(x;\lambda) + \frac{1}{1-\lambda}\sum_{k=0}^{n-1}\binom{n}{k}E_k^F(x;\lambda)e_{n-k}^{F}(\lambda).$$

Lemma 6. *Shift operators are*

$$\begin{aligned} L_n^- &:= \frac{1}{n}D_x, \\ L_n^+ &:= (x - \frac{1}{1-\lambda}) + \frac{1}{1-\lambda}\sum_{k=0}^{n-1}\frac{e_{n-k}^F(\lambda)}{(n-k)!}D_x^{n-k}. \end{aligned}$$

Proof. Differentiating generating equation with respect to x and equating the coefficients of t^n, we get

$$D_xE_n^F(x;\lambda) = nE_{n-1}^F(x;\lambda).$$

Hence we have

$$L_n^-E_n^F(x;\lambda) = E_{n-1}^F(x;\lambda).$$

Using

$$\begin{aligned} E_k^F(x;\lambda) &= \left[L_{k+1}^-L_{k+2}^-...L_n^-\right]E_n^F(x;\lambda) \\ &= \left[\frac{1}{k+1}D_x\frac{1}{k+2}D_x...\frac{1}{n}D_x\right]E_n^F(x;\lambda) \\ &= \frac{k!}{n!}D_x^{n-k}E_n^F(x;\lambda). \end{aligned}$$

in (21), we get L_n^+ :

$$L_n^+ := x - \frac{1}{1-\lambda} + \frac{1}{1-\lambda}\sum_{k=0}^{n-1}\frac{e_{n-k}^F(\lambda)}{(n-k)!}D_x^{n-k}.$$

Theorem 3. *Differential equation is given by*

$$[(x - \frac{1}{1-\lambda})D_x + \frac{1}{1-\lambda}\sum_{k=0}^{n-1}\frac{e_{n-k}^F(\lambda)}{(n-k)!}D_x^{n-k+1} - n]E_n^F(x;\lambda) = 0. \tag{22}$$

Proof. By using factorization method ([12], [13])

$$L_{n+1}^-L_n^+E_n^F(x;\lambda) = E_n^F(x;\lambda),$$

we get the differential equation

$$[(x-\frac{1}{1-\lambda})D_x+\frac{1}{1-\lambda}\sum_{k=0}^{n-1}\frac{e_{n-k}^F(\lambda)}{(n-k)!}D_x^{n-k+1}-n]E_n^F(x;\lambda)=0.$$

Remark. (see [30])Taking $\lambda=-1$ in above theorem, using the fact that $e_{n-k}^F(-1):=e_{n-k},\ E_n^F(x;-1):=E_n(x)$, we get the following recurrence relation

$$(x-\frac{1}{2})E_n(x)+\frac{1}{2}\sum_{k=0}^{n-1}\binom{n}{k}E_k(x)e_{n-k}=E_{n+1}(x)$$

where e_{n-k} are related with the Euler numbers by (6). Shift operators are

$$L_n^-:=\frac{1}{n}D_x,$$

$$L_n^+:=(x-\frac{1}{2})+\frac{1}{2}\sum_{k=0}^{n-1}\frac{e_{n-k}}{(n-k)!}D_x^{n-k}.$$

Differential equation is given by

$$[(x-\frac{1}{2})D_x+\frac{1}{2}\sum_{k=0}^{n-1}\frac{e_{n-k}}{(n-k)!}D_x^{n-k+1}-n]E_n(x)=0.$$

References

[1] Aracı, S, Açıkgöz, M: A note on the Frobenius-Euler numbers and polynomials associated with Bernstein polynomials. Advanced Studies in Contemporary Mathematics 22 (2012), No:3, 399-406.

[2] Aracı, S, Açıkgöz, M, Park, K.H, Jolany, H: On the Unification of Two Families of Multiple Twisted Type Polynomials by using p-Adic q-integral at $q=-1$, Bulletin of the Malaysian Mathematical Sciences Society, 37(2) (2014), 543-554.

[3] Bretti,G, Ricci, P.E: Euler Polynomials and the Related Quadrature Rule, Georgian Mathematical Journal, 8 (2001), No:3, 447-453.

[4] Bretti, G, Ricci, P.E: Multidimensional extension of the Bernoulli and Appell polynomials, Taiwanese Journal of Mathematics 8 (3) (2004), 415–428.

[5] Cesarano, C: Monomiality principle and Legendre polynomails, in: G. Dattoli, H. M. Srivastava, C. Cesarano(Eds.), Advanced Special Functions and Integration Methods(Proceeding of the Melfi School on Advanced Topics in Mathematics and Physics; Melfi, 18-23 June 2000). Aracne Editrice, Rome, 2001, pp. 83-95.

[6] Cheikh, Y.B: Some Results on quasi-monomiality, Applied Mathematics and Computation 141 (2003) 63-76.

[7] Choi, J, Srivastava, H.M: Series involving the zeta functions and a family of generalized Goldbach-Euler series, Amer. Math. Monthly 121 (2014), 229-236.

[8] Choi, J, Kim, D.S, Kim, T, Kim, Y.H: A note on some identities of Frobenius-Euler numbers and polynomials, International Journal of Mathematics and Mathematical Sciences, (2012).

[9] Dattoli, G: Hermite-Bessel, Laguerre-Bessel functions: a by-product of the monomiality principle, in: D. Cocolicchio, G. Dattoli, H.M. Srivastava (Eds.), Advanced Topics in Mathematics and Physics; Melfi, 9-12 May 1999), Aracne Editrice, Rome, 2000, pp. 147-164.

[10] Dattoli, G, Torre, A, Mazzacurati,G: Quasi-monomials and isospectral problems, Nuovo Cimento B 112 (1997) 133-138.

[11] He, M.X , Ricci, P.E: Differential equation of Appell polynomials via the factorization method, Journal of Computational and Applied Mathematics 139(2), (2002), 231-237.

[12] He, M.X, Ricci, P.E: Differential equations of some classes of special functions via the factorization method, Journal of Computational Analysis and Applications 6(2004), No:3.

[13] Infeld, L, Hull,T.E: The factorization method, Rev.Mod.Phys., 23(1951), 21-68.

[14] Khan, S, Yasmin, G, Khan, R, Hassan, N.A.M: Hermite-Based Appell Polynomials, Properties and Applications, Journal of Mathematical Analysis and Applications, 351(2009) 756-764.

[15] Kim, D,S, Kim, T: Some new identities of Frobenius-Euler numbers and polynomials, Journal of Inequalities and Applications 307(2012),10 pp.

[16] Kim, D,S, Kim, T: Some identities of Frobenius-Euler polynomials arising from umbral calculus, Advances in Difference Equations 1 (2012), 196.

[17] Kim, T: Identities involving Frobenius-Euler polynomials arising from non-linear differential equations, Journal of Number Theory 132 (2012), No:12, 2854-2865.

[18] Kurt, B, Simsek, Y: On the generalized Apostol-type Frobenius-Euler polynomials, Advances in Difference Equations, 2013,1.

[19] Liu, H, Wang, W: Some Identities on the Bernoulli, Euler and Genocchi polynomials via power sums and alternate power sums, Discrete Mathematics, 309 (2009) 3346-3363.

[20] Özarslan, M. A: Unified Apostol-Bernoulli, Euler and Genocchi polynomials, Computers and Mathematics with Applications, 62(2011), 2452-2462.

[21] Özarslan, M. A, Yılmaz Yaşar, B: A set of finite order differential equations for the Appell polynomials, Journal of Computational and Applied Mathematics, 259(2014) 108-116.

[22] Özden, H, Şimşek, Y, Srivastava, H. M: A unified presentation of the generating functions of the generalized Bernoulli, Euler and Genocchi polynomials, Computers and Mathematics with Applications 60(2010), 2779-2787.

[23] Pinter, A, Srivastava, H.M: Addition Theorems for the Appell polynomials and the associated classes of polynomial expansions, Aequationes Math. 85(2013), 483-495.

[24] Sheffer, I.M: A Differential Equation for Appell Polynomial, American Mathematical Society, 1935.

[25] Srivastava, H.M, Özarslan, M.A, Yılmaz Yaşar, B: Some Families of Differential Equations Associated with the Hermite-Based Appell Polynomials and Other Classes of Hermite-Based Polynomials, Filomat, 4(2014), No:28, 695-708.

[26] Smirnov, Y, Turbiner, A : Hidden SL2-algebra of finite difference equations, Mod. Phys. Lett. A 10 (1995) 1795-1801.

[27] Srivastava,H.M, Özarslan, M.A and Kaanoğlu, C: Some families of generating functions for a certain class of three-variable polynomials, Integral Transforms and Special Functions, 21(2012) No:12, 885-896.

[28] Şimşek, Y, Bayad, A, Lokesha, V: q-Bernstein polynomials related to q-Frobenius-Euler polynomials, l-functions and q-Stirling numbers, Math. Methods Appl. Sci. 35 (2012), No: 8, 877-884.

[29] Şimşek, Y, Kim, T, Park, D.W, Ro, Y.S, Jang, L.C, Rim, S.H : An explicit formula for the multiple Frobenius-Euler numbers and polynomials, Journal of Algebra, Number Theory and Applications 4 (2004), No: 3, 519-529.

[30] Yılmaz Yaşar, B, Özarslan, M.A: Differential equations for the extended 2D Bernoulli and Euler polynomials, Advances in Difference Equations 107(2013).