Soft sets combined with interval valued intuitionistic fuzzy sets of type-2 and rough sets

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Received: 8 December 2014, Revised: 8 March 2015, Accepted: 12 March 2015
Published online: 23 March 2015

Abstract: Fuzzy set theory, rough set theory and soft set theory are all mathematical tools dealing with uncertainties. The concept of type-2 fuzzy sets was introduced by Zadeh in 1975 which was extended to interval valued intuitionistic fuzzy sets of type-2 by the authors. This paper is devoted to the discussions of the combinations of interval valued intuitionistic sets of type-2, soft sets and rough sets. Three different types of new hybrid models, namely-interval valued intuitionistic fuzzy soft sets of type-2, soft rough interval valued intuitionistic fuzzy sets of type-2 and soft interval valued intuitionistic fuzzy rough sets of type-2 are proposed and their properties are derived.

Keywords: Soft set, rough set, soft rough set, soft rough fuzzy set, soft fuzzy rough set, interval valued intuitionistic fuzzy set of type-2, interval valued intuitionistic fuzzy soft set of type-2, soft rough interval valued intuitionistic fuzzy set of type-2, soft interval valued intuitionistic fuzzy rough set of type-2.

1 Introduction

The soft set theory, initiated by Molodtso in 1999, is a completely generic mathematical tool for modeling vague concepts. In soft set theory there is no limited condition to the description of objects; so researchers can choose the form of parameters they need, which greatly simplifies the decision making process and make the process more efficient in the absence of partial information. Although many mathematical tools are available for modeling uncertainties such as probability theory, fuzzy set theory, rough set theory, interval valued mathematics, etc., but there are inherent difficulties associated with each of these techniques. Moreover all these techniques lack in the parameterization of the tools and hence they could not be applied successfully in tackling problems especially in areas like economic, environmental and social problems domains. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties and it has a unique scope for many applications in a multi-dimentional way.

Soft set theory has a rich potential for application in many directions, some of which are reported by Molodtso in his work. He successfully applied soft set theory in areas such as the smoothness of functions, game theory, operations research, Riemann integration and so on. Later on Maji et al. [14] presented some new definitions on soft sets such as subset, union, intersection and complements of soft sets and discussed in detail the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [14], Ali et al. [2] presented some new algebraic operations for soft sets and proved that certain De Morgan’s law holds in soft set theory with respect to

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these new definitions. Chen [5] presented a new definition of soft set parameterization reduction and compared this definition with the related concept of knowledge reduction in the rough set theory. Kong et al. [13] introduced the definition of normal parameter reduction into soft sets and then presented a heuristic algorithm to compute normal parameter reduction of soft sets. By amalgamating the soft sets and algebra, Aktas and Cagman [1] introduced the basic properties of soft sets, compared soft sets to the related concepts of fuzzy sets [27] and rough sets [20], pointed out that every fuzzy set and every rough set may be considered as a soft set. Jun [12] applied soft sets to the theory of BCK/BCI-algebra and introduced the concept of soft BCK/BCI-algebras. Feng et al. [7] defined soft semi rings and several related notions to establish a connection between the soft sets and semi rings. Sun et al. [22] presented the definition of soft modules and constructed some basic properties using modules and Molodtsov’s definition of soft sets. Maji et al. [15] presented the concept of the fuzzy soft set which is based on a combination of the fuzzy set and soft set models. Roy and Maji [21] presented a fuzzy soft set theoretic approach towards a decision making problem. Yang et al. [25] defined the operations on fuzzy soft sets, which are based on three fuzzy logic operations: negation, triangular norm and triangular co-norm. Xiao et al. [23] proposed a combined forecasting approach based on fuzzy soft set theory. Yang et al. [24] introduced the concept of interval valued fuzzy soft set and a decision making problem was analyzed by the interval valued fuzzy soft set. Feng et al. [8] presented an adjustable approach to fuzzy soft set based decision making and give some illustrative examples. The notion of intuitionistic fuzzy set was initiated by Atanassov [3] as a generalization of fuzzy set. Combining soft sets with intuitionistic fuzzy sets, Maji et al. [16] introduced intuitionistic fuzzy soft sets, which are rich potentials for solving decision making problems. The notion of the interval-valued intuitionistic fuzzy set was introduced by Atanassov and Gargov [4]. It is characterized by an interval-valued membership degree and an interval-valued non-membership degree. In 2010, Jiang et al. [11] introduced the concept of interval valued intuitionistic fuzzy soft sets.

The rough set theory proposed by Pawlak [20] provides a systematic method for dealing with vague concepts caused by indiscernibility in situation with incomplete information or a lack of knowledge/data. In general a fuzzy set may be viewed as a class with unsharp boundaries, but a rough set is a coarsely described crisp set [26]. Based on the equivalence relation on the universe of discourse, Dubois and Prade [6] introduced the lower and upper approximation of fuzzy sets in a Pawlak’s approximation space [20] and obtained a new notion called rough fuzzy sets which are natural extensions of rough sets. They also introduced the concept of fuzzy rough sets [6]. Feng el al. [9] provided a framework to combine rough sets and soft sets all together, which gives rise to several interesting new concepts such as soft rough sets and rough soft sets. A rough soft set is the approximation of a soft set in a Pawlak approximation space, where as a soft rough set is based on soft rough approximations in a soft approximation space. Feng[10] presented a soft rough set based multi-criteria group decision making scheme. Motivated by Dubois and Prade’s original idea about rough fuzzy set, Feng et al. [9] introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy set. By employing a fuzzy soft set to granulate the universe of discourse, Meng et al. [17] introduced a more general model called soft fuzzy rough set.

The concept of type-2 fuzzy sets was introduced by Zadeh [28] as an extension of the concept of an ordinary fuzzy sets (called type-1 fuzzy sets). Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets. These sets are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Saha and Mukherjee [19] introduced the concept of interval valued intuitionistic fuzzy set of type-2 as an extension of fuzzy sets of type-2. The aim of this paper is to provide a framework to combine interval valued intuitionistic fuzzy sets of type-2, rough sets and soft sets all together which gives rise to several interesting new concepts such as interval valued intuitionistic fuzzy soft sets of type-2, soft rough interval valued intuitionistic fuzzy sets of type-2 and soft interval valued intuitionistic fuzzy rough sets of type-2.
2 Fuzzy sets and their generalizations

This section presents a review of some fundamental notions of fuzzy sets and their generalizations.

The theory of fuzzy sets provides an appropriate framework for representing and processing vague concepts by allowing partial memberships.

Definition 2.1: [27] Let $X$ be a non empty set. Then a fuzzy set $A$ on $X$ is a set having the form

$$A = \{ (x, \mu_A(x)) : x \in X \}$$

where the function $\mu_A: X \to [0, 1]$ is called the membership function and $\mu_A(x)$ represents the degree of membership of each element $x \in X$.

Definition 2.2: [3] Let $X$ be a non empty set. Then an intuitionistic fuzzy set (IFS for short) $A$ is a set having the form

$$A = \{ (x, \mu_A(x), \gamma_A(x)) : x \in X \}$$

where the functions $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$ represents the degree of membership and the degree of non-membership respectively of each element $x \in X$ and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.3: [4] An interval valued intuitionistic fuzzy set $A$ over an universe set $U$ is defined as the object of the form

$$A = \{ <x, \mu_A(x), \gamma_A(x)> : x \in U \}$$

where $\mu_A: U \to \text{Int}([0, 1])$ and $\gamma_A: U \to \text{Int}([0, 1])$ are functions such that the condition: $\forall x \in U, 0 \leq \sup \mu_A(x) + \sup \gamma_A(x) \leq 1$ is satisfied (where $\text{Int}[0,1]$is the set of all closed sub-intervals of $[0,1]$).

We denote the class of all interval valued intuitionistic fuzzy sets on $U$ by $\text{IVIFS}^U$.

Let $A, B \in \text{IVIFS}^U$. Then

- the union of $A$ and $B$ is denoted by $A \cup B$ where $A \cup B = \{ <x, \max(\inf \mu_A(x), \inf \mu_B(x)), \max(\sup \mu_A(x), \sup \mu_B(x))>, \min(\inf \gamma_A(x), \inf \gamma_B(x)), \min(\sup \gamma_A(x), \sup \gamma_B(x))> : x \in U \}$.
- the intersection of $A$ and $B$ is denoted by $A \cap B$ where $A \cap B = \{ <x, \min(\inf \mu_A(x), \inf \mu_B(x)), \min(\sup \mu_A(x), \sup \mu_B(x))>, \max(\inf \gamma_A(x), \inf \gamma_B(x)), \max(\sup \gamma_A(x), \sup \gamma_B(x))> : x \in U \}$.

Atanassov and Gargovshow in [4] that $A \cup B$ and $A \cap B$ are again IVIFSs.

Definition 2.4: [28] A fuzzy set of type-2 is defined as a fuzzy set whose membership degree is a fuzzy set of type-1.

Thus, a fuzzy set of type-2 , $A$ on an universe $U$ is an object of the form

$$A = \{ (x, \mu_A(x)) : x \in U \}$$
where

\[ \mu_A(x) = \{(u_i, \mu_{ui}(x)) : u_i \in [0, 1]\} \]

for \( x \in U \), where \( \mu_{ui} : U \rightarrow [0, 1] \) is a function.

### 3 Soft sets and their generalizations

In this section we recall some basic notions relevant to soft sets and their generalizations.

**Definition 3.1:** [18] Let \( U \) be an universe set and \( E \) be a set of parameters. Let \( P(U) \) denotes the power set of \( U \) and \( A \subseteq E \). Then the pair \((F, A)\) is called a soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow P(U) \).

In other words, a soft set over \( U \) is a parameterized family of subsets of \( U \). For \( e \in A \), \( F(e) \) may be considered as the set of \( e \)-approximate elements of the soft set \((F, A)\).

**Definition 3.2:** [15] Let \( U \) be an universe set and \( E \) be a set of parameters. Let \( I^U \) be the set of all fuzzy subsets of \( U \) and \( A \subseteq E \). Then the pair \((F, A)\) is called a fuzzy soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow I^U \).

For \( e \in A \), \( F(e) \) is a fuzzy subset of \( U \) and is called the fuzzy value set of the parameter \( e \). Let us denote \( \mu_{F(e)}(x) \) by the membership degree that object \( x \) holds parameter \( e \), where \( e \in A \) and \( x \in U \). Then \( F(e) \) can be written as a fuzzy set such that \( F(e) = \{(x, \mu_{F(e)}(x)) : x \in U\} \).

**Definition 3.3:** [16] Let \( U \) be an universe set and \( E \) be a set of parameters. Let \( IF^U \) be the set of all intuitionistic fuzzy subsets of \( U \) and \( A \subseteq E \). Then the pair \((F, A)\) is called an intuitionistic fuzzy soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow IF^U \).

For \( e \in A \), \( F(e) \) is an intuitionistic fuzzy subset of \( U \) and is called the intuitionistic fuzzy value set of the parameter \( e \). Let us denote \( \mu_{F(e)}(x) \) by the membership degree that object \( x \) holds parameter \( e \) and \( \gamma_{F(e)}(x) \) by the membership degree that object \( x \) doesn’t hold parameter \( e \), where \( e \in A \) and \( x \in U \). Then \( F(e) \) can be written as an intuitionistic fuzzy set such that

\[ F(e) = \{(x, \mu_{F(e)}(x), \gamma_{F(e)}(x)) : x \in U\} \]

**Definition 3.4:** [11] Let \( U \) be an universe set and \( E \) be a set of parameters. Let \( IVIF^U \) be the set of all interval valued intuitionistic fuzzy sets on \( U \) and \( A \subseteq E \). Then the pair \((F, A)\) is called an interval valued intuitionistic fuzzy soft set (IVIFSS for short) over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow IVIF^U \).

For \( e \in A \), \( F(e) \) can be written as an interval valued intuitionistic fuzzy set such that

\[ F(e) = \{(x, \mu_{F(e)}(x), \gamma_{F(e)}(x)) : x \in U\} \]

where \( \mu_{F(e)}(x) \) is the interval valued fuzzy membership degree that object \( x \) holds parameter \( e \) and \( \gamma_{F(e)}(x) \) is the interval valued fuzzy membership degree that object \( x \) doesn’t hold parameter \( e \).
4 Rough set, soft rough set, rough soft set and their generalizations

This section represents a review of the notions of rough sets, soft rough sets, rough soft sets and their generalizations.

The rough set theory provides a systematic method for dealing with vague concepts caused by indiscernability in situation with incomplete information or a lack of knowledge.

**Definition 4.1:** [20] Let \( R \) be an equivalence relation on the universal set \( U \). Then the pair \( (U, R) \) is called a Pawlak approximation space. An equivalence class of \( R \) containing \( x \) will be denoted by \([x]_R\). Now for \( X \subseteq U \), the lower and upper approximation of \( X \) with respect to \( (U, R) \) are denoted by respectively \( R_- X \) and \( R_+ X \) and are defined by

\[
R_- X = \{ x \in U : [x]_R \subseteq X \},
\]

\[
R_+ X = \{ x \in U : [x]_R \cap X \neq \emptyset \}.
\]

Now if \( R_- X = R_+ X \), then \( X \) is called definable in \( (U, R) \); otherwise \( X \) is called a rough set.

Feng et al. [9] provided a framework to combine rough sets and soft sets all together, which gives rise to several interesting new concepts such as soft rough sets and rough soft sets.

**Definition 4.2:** [9] Let \( \Theta = (f, \Lambda) \) be a soft set over \( U \). The pair \( S = (U, \Theta) \) is called a soft approximation space. Based on \( S \), the operators \( apr_S \) and \( \bar{apr}_S \) are defined as:

\[
apr_S (X) = \{ u \in U : \exists a \in \Lambda (u \in f(a) \subseteq X) \},
\]

\[
\bar{apr}_S (X) = \{ u \in U : \exists a \in \Lambda (u \in f(a), f(a) \cap X \neq \emptyset) \} \quad \text{for every } X \subseteq U.
\]

The two sets \( apr_S (X) \) \& \( \bar{apr}_S (X) \) are called the lower and upper soft rough approximations of \( X \) in \( S \) respectively. If \( apr_S (X) = \bar{apr}_S (X) \), then \( X \) is said to be soft definable; otherwise \( X \) is called a soft rough set.

**Definition 4.3:** [9] Let \( (U, R) \) be a Pawlak approximation space and \( \Theta = (f, \Lambda) \) be a soft set over \( U \). Then the lower and upper rough approximations of \( \Theta \in (U, R) \) are denoted by \( R_- (\Theta) = (F_-, \Lambda) \) and \( R_+ (\Theta) = (F_+, \Lambda) \), respectively, which are soft sets over \( U \) defined by:

\[
F_- (x) = (F(x)) = \{ y \in U : [y]_R \subseteq F(x) \}
\]

and

\[
F_+ (x) = \bar{R}(F(x)) = \{ y \in U : [y]_R \cap F(x) \neq \emptyset \} \quad \text{for all } x \in U.
\]

The operators \( R_- \) and \( R_+ \) are called the lower and upper rough approximation operators on soft sets. If \( R_- (\Theta) = R_+ (\Theta) \), the soft set \( \Theta \) is said to be definable; otherwise \( \Theta \) is called a rough soft set.

Motivated by Dubois and Prade's original idea about rough fuzzy set, Feng et al. [9] introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy set.

**Definition 4.4:** [9] Let \( \Theta = (f, \Lambda) \) be a full soft set over \( U \) i.e; \( \cup_{a \in \Lambda} f(a) = U \) and the pair \( S = (U, \Theta) \) be the soft approximation space. Then for a fuzzy set \( \lambda \in \mathcal{F}^U \), the lower and upper soft rough approximations of \( \lambda \) with respect to \( S \)
are denoted by $sap_\lambda$ and $\overline{sap}_\lambda$ respectively, which are fuzzy sets in $U$ given by:

$$sap_\lambda \lambda = \{ (x, sap_\lambda(x)) : x \in U \} ,$$

$$\overline{sap}_\lambda \lambda = \{ (x, \overline{sap}_\lambda(x)) : x \in U \} ,$$

where

$$sap_\lambda(x) = \bigwedge_{a \in A} \{ \mu_\lambda(y) : \exists a \in A (\{ x, y \} \subseteq f(a)) \}$$

and

$$\overline{sap}_\lambda(x) = \bigvee_{a \in A} \{ \mu_\lambda(y) : \exists a \in A (\{ x, y \} \subseteq f(a)) \}$$

for every $x \in U$.

The operators $sap_\lambda$ and $\overline{sap}_\lambda$ are called the lower and upper soft rough approximation operators on fuzzy sets. If $sap_\lambda = \overline{sap}_\lambda$ then $\lambda$ is said to be fuzzy soft definable; otherwise $\lambda$ is called a soft rough fuzzy set.

Meng et al. [17] introduced the lower and upper soft fuzzy rough approximations of a fuzzy set by granulating the universe of discourse with the help of a fuzzy soft set and obtained a new model called soft fuzzy rough set.

**Definition 4.5:** [17] Let $\Theta = (f, A)$ be a fuzzy soft set over $U$. Then the pair $SF = (U, \Theta)$ is called a soft fuzzy approximation space. Then for a fuzzy set $\lambda \in I^U$, the lower and upper soft fuzzy rough approximations of $\lambda$ with respect to SF are denoted by $\text{Apr}_{SF} \lambda$ and $\overline{\text{Apr}}_{SF} \lambda$ respectively, which are fuzzy sets in $U$ given by:

$$\text{Apr}_{SF} \lambda = \{ (x, \text{Apr}_{SF} \lambda(x)) : x \in U \} ,$$

$$\overline{\text{Apr}}_{SF} \lambda = \{ (x, \overline{\text{Apr}}_{SF} \lambda(x)) : x \in U \} ,$$

where

$$\text{Apr}_{SF} \lambda(x) = \bigwedge_{a \in A} ((1 - f(a)(x)) \lor (1 - f(a)(y)) \lor \mu_\lambda(y)))$$

and

$$\overline{\text{Apr}}_{SF} \lambda(x) = \bigvee_{a \in A} (f(a)(x) \land (f(a)(y) \land \mu_\lambda(y)))$$

for every $x \in U$ and $\mu_\lambda(y)$ is the degree of membership of $y \in U$.

The operators $\text{Apr}_{SF}$ and $\overline{\text{Apr}}_{SF}$ are called the lower and upper soft fuzzy rough approximation operators on fuzzy sets. If $\text{Apr}_{SF} \lambda = \overline{\text{Apr}}_{SF} \lambda$, then $\lambda$ is said to be soft fuzzy definable; otherwise $\lambda$ is called a soft fuzzy rough set.

### 5 Interval valued intuitionistic fuzzy sets of type-2

The concept of interval valued intuitionistic fuzzy sets of type-2 was introduced by Saha and Mukherjee [19], as an extension of the concept of fuzzy sets of type-2.

**Definition 5.1:** [9] An *interval-valued intuitionistic fuzzy set of type-2* is defined as an interval valued intuitionistic fuzzy set whose interval valued fuzzy membership degree as well as interval valued fuzzy non-membership degree are both...
The set of all interval valued intuitionistic fuzzy sets of type-2 on an universe U is an object of the form

\[ A = \{ < x, \mu_A(x), \gamma_A(x) : x \in U \} \]

where

\[ \mu_A(x) = \{ < u_i(x), \mu_{ui}(x), \gamma_{ui}(x) : i = 1, 2, \ldots, n \} \]

and

\[ \gamma_A(x) = \{ < v_j(x), \mu_{vj}(x), \gamma_{vj}(x) : j = 1, 2, \ldots, m \} \]

for each \( x \in U \), where \( u_i(x), v_j(x) \in Int([0, 1]) \) and \( \mu_{ui}, \gamma_{ui}, \mu_{vj}, \gamma_{vj} : U \to Int([0, 1]) \) are functions such that \( x \in U, 0 \leq sup\mu_{ui}(x) + sup\gamma_{ui}(x) \leq 1 \) and \( 0 \leq sup\mu_{vj}(x) + sup\gamma_{vj}(x) \leq 1 \) for each \( i \) and \( j \).

The set of all interval valued intuitionistic fuzzy sets of type-2 on \( U \) is denoted by \( IVIFS_2(U) \).

**Definition 5.2:** An interval-valued intuitionistic fuzzy null set of type-2 over an universe \( U \) is denoted by \( \Phi_* \) and is defined by

\[ \Phi_* = \{ < x, \{ < u_i(x), [0,0], [1,1] : i = 1, \ldots, n \}, \{ < v_j(x), [0,0], [1,1] : j = 1, \ldots, m \} : x \in U \} \]

where for \( x \in U, u_i(x) = [0,0] \) and \( v_j(x) = [0,0] \) for each \( i \) and \( j \).

**Definition 5.3:** An interval-valued intuitionistic fuzzy universal set of type-2 over an universe \( U \) is denoted by \( U_* \) and is defined by

\[ U_* = \{ < x, \{ < u_i(x), [1,1], [0,0] : i = 1, \ldots, n \}, \{ < v_j(x), [1,1], [0,0] : j = 1, \ldots, m \} : x \in U \} \]

where for \( x \in U, u_i(x) = [1,1] \) and \( v_j(x) = [1,1] \) for each \( i \) and \( j \).

**Definition 5.4:** Let \( \Gamma, \Psi \in IVIFS_2(U) \) be defined by

\[ \Gamma = \{ < x, \mu_{\Gamma}(x), \gamma_{\Gamma}(x) : x \in U \} \text{ and } \Psi = \{ < x, \mu_{\Psi}(x), \gamma_{\Psi}(x) : x \in U \} \]

where for \( x \in U, \)

\[ \mu_{\Gamma}(x) = \{ < u_i(x), \mu_{ui}(x), \gamma_{ui}(x) : i = 1, \ldots, n \}, \]

\[ \gamma_{\Gamma}(x) = \{ < v_j(x), \mu_{vj}(x), \gamma_{vj}(x) : j = 1, \ldots, m \}, \]

\[ \mu_{\Psi}(x) = \{ < w_i(x), \mu_{wi}(x), \gamma_{wi}(x) : i = 1, \ldots, n \}, \]

\[ \gamma_{\Psi}(x) = \{ < t_j(x), \mu_{tj}(x), \gamma_{tj}(x) : j = 1, \ldots, m \}. \]

Then \( \Gamma \) is called a restricted subset of \( \Psi \), denoted by \( \Gamma \subseteq_2 \Psi \), if for all \( x \in U, inf u_i(x) \leq inf w_i(x), sup u_i(x) \leq sup w_i(x), inf v_j(x) \leq inf t_j(x), sup v_j(x) \leq sup t_j(x), inf \mu_{ui}(x) \leq inf \mu_{wi}(x), sup \mu_{ui}(x) \leq sup \mu_{wi}(x), inf \gamma_{ui}(x) \geq inf \gamma_{wi}(x), sup \gamma_{ui}(x) \geq sup \gamma_{wi}(x), inf \mu_{vj}(x) \leq inf \mu_{tj}(x), sup \mu_{vj}(x) \leq sup \mu_{tj}(x), inf \gamma_{vj}(x) \geq inf \gamma_{tj}(x), sup \gamma_{vj}(x) \geq sup \gamma_{tj}(x). \)

**Definition 5.5:** Let \( \Gamma, \Psi \in IVIFS_2(U) \) be defined by

\[ \Gamma = \{ < x, \mu_{\Gamma}(x), \gamma_{\Gamma}(x) : x \in U \} \text{ and } \Psi = \{ < x, \mu_{\Psi}(x), \gamma_{\Psi}(x) : x \in U \} \]

where for \( x \in U, \)

\[ \mu_{\Gamma}(x) = \{ < u_i(x), \mu_{ui}(x), \gamma_{ui}(x) : i = 1, \ldots, n \}, \]

\[ \gamma_{\Gamma}(x) = \{ < v_j(x), \mu_{vj}(x), \gamma_{vj}(x) : j = 1, \ldots, m \}, \]

\[ \mu_{\Psi}(x) = \{ < w_i(x), \mu_{wi}(x), \gamma_{wi}(x) : i = 1, \ldots, n \}, \]

\[ \gamma_{\Psi}(x) = \{ < t_j(x), \mu_{tj}(x), \gamma_{tj}(x) : j = 1, \ldots, m \}. \]

Then
Definition 6.1: Let $U$ be an universe set and $E$ be a set of parameters. Let $\mu_{al}(x) \cup \mu_{wl}(x)$, $\gamma_{al}(x) \cap \gamma_{wl}(x) >: i = 1, 2, \ldots, n$, $\{\gamma_{ij}(x) \cup \gamma_{j}(x) >: j = 1, 2, \ldots, m\} >: x \in U\}$.

Their restricted union is denoted by $\Gamma \cup_2 \Psi$ and is defined by $\Gamma \cup_2 \Psi = \{< x, \{< u_{i}(x) \cup w_{i}(x), \mu_{al}(x) \cup \mu_{wl}(x), \gamma_{al}(x) \cap \gamma_{wl}(x) >: i = 1, 2, \ldots, n\}, \{< v_{j}(x) \cup t_{j}(x), \mu_{ij}(x) \cup \mu_{j}(x), \gamma_{ij}(x) \cap \gamma_{j}(x) >: j = 1, 2, \ldots, m\} >: x \in U\}$.

Their restricted intersection is denoted by $\Gamma \cap_2 \Psi$ and is defined by $\Gamma \cap_2 \Psi = \{< x, \{< u_{i}(x) \cap w_{i}(x), \mu_{al}(x) \cap \mu_{wl}(x), \gamma_{al}(x) \cap \gamma_{wl}(x) >: i = 1, 2, \ldots, n\}, \{< v_{j}(x) \cap t_{j}(x), \mu_{ij}(x) \cap \mu_{j}(x), \gamma_{ij}(x) \cap \gamma_{j}(x) >: j = 1, 2, \ldots, m\} >: x \in U\}$.

Clearly for $\Gamma, \Psi \in IVIFS_2(U)$, we have $\Gamma \cup_2 \Psi, \Gamma \cap_2 \Psi \in IVIFS_2(U)$.

Theorem 6.6: Let $\Gamma, \Psi, \Omega \in IVIFS_2(U)$. Then

1. $\Gamma \cup_2 \Phi* = \Phi* \cup_2 \Gamma = \Gamma$.
2. $\Gamma \cap_2 \Phi* = \Phi* \cap_2 \Gamma = \Phi*$.
3. $\Gamma \cup_2 U* = U* \cup_2 \Gamma = U* \cap_2 \Gamma = \Gamma$.
4. $\Gamma \cap_2 U* = U* \cap_2 \Gamma$.
5. $\Gamma \cup_2 \Psi = \Psi \cup_2 \Gamma$.
6. $\Gamma \cap_2 \Psi = \Psi \cap_2 \Gamma$.
7. $\Gamma \cap_2 (\Psi \cap_2 \Omega) = (\Psi \cap_2 \Gamma) \cap_2 \Omega$.
8. $\Gamma \cup_2 (\Psi \cup_2 \Omega) = \Psi \cup_2 (\Gamma \cup_2 \Omega)$.
9. $\Gamma \cap_2 (\Psi \cup_2 \Omega) = (\Psi \cap_2 \Gamma) \cup_2 (\Gamma \cap_2 \Omega)$.
10. $\Gamma \cup_2 (\Psi \cap_2 \Omega) = (\Psi \cup_2 \Gamma) \cap_2 (\Gamma \cap_2 \Omega)$.

Proof:- Straight forward.

6 Interval valued intuitionistic fuzzy soft sets of type-2

In this section we present the theory of interval valued intuitionistic fuzzy sets of type-2. We also discuss some operations and properties of the interval valued intuitionistic fuzzy sets of type-2.

Definition 6.1: Let $U$ be an universe set and $E$ be a set of parameters. Let $IVIFS_2(U)$ be the set of all interval valued intuitionistic fuzzy sets of type-2 over $U$ and $\Lambda \subseteq E$. Then the pair $(f, A)$ is called an interval valued intuitionistic fuzzy soft set of type-2 over $U$, where $f$ is a mapping given by $f : A \rightarrow IVIFS_2(U)$.

Definition 6.2: Let $U$ be an universe set and $E$ be a set of parameters. Let $IVIFS_2(U)$ be the set of all interval valued intuitionistic fuzzy sets of type-2 over $U$ and $A \subseteq E$. Then the pair $(f, A)$ is called an interval valued intuitionistic fuzzy empty soft set of type-2 over $U$, denoted by $\Phi_{INF}$ where $f : A \rightarrow IVIFS_2(U)$ is defined by $f(a) = \Phi*$ for each $a \in A$.

Definition 6.3: Let $U$ be an universe set and $E$ be a set of parameters. Let $IVIFS_2(U)$ be the set of all interval valued intuitionistic fuzzy sets of type-2 over $U$ and $A \subseteq E$. Then the pair $(f, A)$ is called an interval valued intuitionistic fuzzy universal soft set of type-2 over $U$, denoted by $\Phi_{INF}$ where $f : A \rightarrow IVIFS_2(U)$ is defined by $f(a) = U*$ for each $a \in A$.

Definition 6.4: Let $U$ be an initial universe and $E$ be a set of parameters. Let $(f, A), (g, B)$ be two interval valued intuitionistic fuzzy soft sets of type-2 over $U$, where $f : A \rightarrow IVIFS_2(U)$ is defined by $f(a) = \{< x, f_{i}(a)(x), \gamma_{i}(a)(x) >: x \in U\}$, where $f_{i}(a)(x) = \{< u_{i}(a)(x), \mu_{al}(a)(x), \gamma_{al}(a)(x) >: i = 1, 2, \ldots, n\}$, $\gamma_{al}(a)(x) = \{< v_{j}(a)(x), \mu_{ij}(a)(x), \gamma_{ij}(a)(x) >: j = 1, 2, \ldots, m\}$ for $x \in U$ and $g : B \rightarrow IVIFS_2(U)$ is defined by $g(b) = \{< x, g_{i}(b)(x), \gamma_{i}(b)(x) >: x \in U\}$, where $g_{i}(b)(x) = \{< w_{i}(b)(x), \mu_{al}(b)(x), \gamma_{al}(b)(x) >: i = 1, 2, \ldots, n\}$, $\gamma_{al}(b)(x) = \{< t_{j}(b)(x), \mu_{ij}(b)(x), \gamma_{ij}(b)(x) >: j = 1, 2, \ldots, m\}$ for $x \in U$. Then,
1. Their union, denoted by \((f,A) \cup (g,B) = (h,C), say\), is an interval valued intuitionistic fuzzy soft set of type-2 over \(U\), where \(C = A \cup B\) and for \(c \in C, h : C \rightarrow IVIFS_2(U)\) is defined by

\[
h(c) = \{< x, \mu_{h(c)}(x), \gamma_{h(c)}(x) : x \in U \}, \quad \text{where}
\]

\[
\mu_{h(c)}(x) = \mu_{f(c)}(x), \quad \text{if} \quad c \in A - B
\]
\[
= \mu_{g(c)}(x), \quad \text{if} \quad c \in B - A
\]
\[
= \{< u_i(c)(x) \cup w_i(c)(x), \mu_{ui}(c)(x) \cup \mu_{wi}(c)(x), \gamma_{ui}(c)(x) \cap \gamma_{wi}(c)(x) : i = 1, 2, \ldots, n \} \quad \text{if} \quad c \in A \cap B
\]

and

\[
\gamma_{h(c)}(x) = \gamma_{f(c)}(x), \quad \text{if} \quad c \in A - B
\]
\[
= \gamma_{g(c)}(x), \quad \text{if} \quad c \in B - A
\]
\[
= \{< v_j(c)(x) \cup t_j(c)(x), \mu_{vij}(c)(x) \cup \mu_{tij}(c)(x), \gamma_{vij}(c)(x) \cap \gamma_{tij}(c)(x) : j = 1, 2, \ldots, m \} \quad \text{if} \quad c \in A \cap B.
\]

2. Their intersection, denoted by \((f,A) \cap \% (g,B) = (h,C), say\), is an interval valued intuitionistic fuzzy soft set of type-2 over \(U\), where \(C = A \cup B\) and for \(c \in C, h : C \rightarrow IVIFS_2(U)\) is defined by

\[
h(c) = \{< x, \mu_{h(c)}(x), \gamma_{h(c)}(x) : x \in U \}, \quad \text{where}
\]

\[
\mu_{h(c)}(x) = \mu_{f(c)}(x), \quad \text{if} \quad c \in A - B
\]
\[
= \mu_{g(c)}(x), \quad \text{if} \quad c \in B - A
\]
\[
= \{< u_i(c)(x) \cap w_i(c)(x), \mu_{ui}(c)(x) \cap \mu_{wi}(c)(x), \gamma_{ui}(c)(x) \cap \gamma_{wi}(c)(x) : i = 1, 2, \ldots, n \} \quad \text{if} \quad c \in A \cap B
\]

and

\[
\gamma_{h(c)}(x) = \gamma_{f(c)}(x), \quad \text{if} \quad c \in A - B
\]
\[
= \gamma_{g(c)}(x), \quad \text{if} \quad c \in B - A
\]
\[
= \{< v_j(c)(x) \cap t_j(c)(x), \mu_{vij}(c)(x) \cap \mu_{tij}(c)(x), \gamma_{vij}(c)(x) \cap \gamma_{tij}(c)(x) : j = 1, 2, \ldots, m \} \quad \text{if} \quad c \in A \cap B.
\]

**Theorem 6.5:** Let \(U\) be an universe set and \((f,A), (g,B), (h,C)\) be three interval valued intuitionistic fuzzy soft sets of type-2 over \(U\). Then

1. \((f,A) \cup \% (f,A) = (f,A)\).
2. \((f,A) \cap \% (f,A) = (f,A)\).
3. \((f,A) \cup \% \Phi_{soft} = (f,A) = \Phi_{soft} \cup \% (f,A)\), if the set of parameters for \(\Phi_{soft}\) is \(A\).
4. \((f,A) \cap \% \Phi_{soft} = \Phi_{soft} \cap \% (f,A)\), if the set of parameters for \(\Phi_{soft}\) is \(A\).
5. \((f,A) \cup \% U_{soft} = U_{soft} \cup \% (f,A)\), if the set of parameters for \(U_{soft}\) is \(A\).
6. \((f,A) \cap \% U_{soft} = U_{soft} \cap \% (f,A)\), if the set of parameters for \(U_{soft}\) is \(A\).
7. \((f,A) \cup \% (g,B) = (g,B) \cup \% (f,A)\).
8. \((f,A) \cap \% (g,B) = (g,B) \cap \% (f,A)\).
9. \((f,A) \cap \% ((g,B) \cap \% (h,C)) = ((f,A) \cap \% (g,B)) \cap \% (h,C)\).
10. \((f,A) \cup \% ((g,B) \cup \% (h,C)) = ((f,A) \cup \% (g,B)) \cup \% (h,C)\).
11. \((f,A) \cap \% ((g,B) \cap \% (h,C)) = ((f,A) \cap \% (g,B)) \cap \% ((f,A) \cap \% (h,C))\).
12. \((f,A) \cup \% ((g,B) \cap \% (h,C)) = ((f,A) \cup \% (g,B)) \cap \% ((f,A) \cup \% (h,C))\).

**Proof:** (1)-(8) are straightforward. (9) Let \(f : A \rightarrow IVIFS_2(U)\) be defined by
\[ f(a) = \{ x, \mu_{f(a)}(x), \gamma_{f(a)}(x) : x \in U \}, \text{ where for } a \in A, \]
\[ \mu_{f(a)}(x) = \{ y_i(a)(x), \mu_{i(a)}(x), \gamma_{i(a)}(x) : y_i(a)(x) \in \text{Int}([0,1]), i = 1, 2, \ldots, n \}, \]
\[ \gamma_{f(a)}(x) = \{ z_j(a)(x), \mu_{j(a)}(x), \gamma_{j(a)}(x) : z_j(a)(x) \in \text{Int}([0,1]), j = 1, 2, \ldots, m \} \text{ for } x \in U, \]

Let \( g : B \to IVIFS_2(U) \) be defined by

\[ g(b) = \{ x, \mu_{g(b)}(x), \gamma_{g(b)}(x) : x \in U \}, \text{ where for } b \in B, \]
\[ \mu_{g(b)}(x) = \{ u_i(b)(x), \mu_{ui(b)}(x), \gamma_{ui(b)}(x) : u_i(b)(x) \in \text{Int}([0,1]), i = 1, 2, \ldots, n \}, \]
\[ \gamma_{g(b)}(x) = \{ v_j(b)(x), \mu_{vj(b)}(x), \gamma_{vj(b)}(x) : v_j(b)(x) \in \text{Int}([0,1]), j = 1, 2, \ldots, m \} \text{ for } x \in U, \]

Let \( h : C \to IVIFS_2(U) \) be defined by

\[ h(c) = \{ x, \mu_{h(c)}(x), \gamma_{h(c)}(x) : x \in U \}, \text{ where for } c \in C, \]
\[ \mu_{h(c)}(x) = \{ w_i(c)(x), \mu_{wi(c)}(x), \gamma_{wi(c)}(x) : w_i(c)(x) \in \text{Int}([0,1]), i = 1, 2, \ldots, n \}, \]
\[ \gamma_{h(c)}(x) = \{ t_j(c)(x), \mu_{tj(c)}(x), \gamma_{tj(c)}(x) : t_j(c)(x) \in \text{Int}([0,1]), j = 1, 2, \ldots, m \} \text{ for } x \in U. \]

Let \( (g, B) \cap \%)((h, C) = (s, D) \) and \( D = B \cup C. \) Then for \( d \in D, \) we have,

\[ \mu_{s(d)}(x) = \mu_{g(d)}(x), \text{ if } d \in B - C, \]
\[ = \mu_{h(d)}(x), \text{ if } d \in C - B, \]
\[ = \{ u_i(d)(x) \cap w_i(d)(x), \mu_{ui(d)}(x) \cap \mu_{wi(d)}(x), \gamma_{ui(d)}(x) \cup \gamma_{wi(d)}(x) : i = 1, 2, \ldots, n \}\text{if } d \in B \cap C. \]

and

\[ \gamma_{s(d)}(x) = \gamma_{g(d)}(x), \text{ if } d \in B - C, \]
\[ = \gamma_{h(d)}(x), \text{ if } d \in C - B, \]
\[ = \{ v_j(d)(x) \cap t_j(d)(x), \mu_{vj(d)}(x) \cap \mu_{tj(d)}(x), \gamma_{vj(d)}(x) \cup \gamma_{tj(d)}(x) : j = 1, 2, \ldots, m \}\text{ if } d \in B \cap C. \]

Now \( (f, A) \cap \%(g, B) \cap \%(h, C) = (f, A) \cap \%(s, D) = (e, P), \) say, where \( P = A \cup D. \) Then for \( p \in P \) we have,

\[ \mu_{e(p)}(x) = \mu_{g(p)}(x), \text{ if } p \in B - C - A, \]
\[ = \mu_{h(p)}(x), \text{ if } p \in C - B - A, \]
\[ = \mu_{f(p)}(x), \text{ if } p \in A - B - C, \]
\[ = \{ u_i(p)(x) \cap w_i(p)(x), \mu_{ui(p)}(x) \cap \mu_{wi(p)}(x), \gamma_{ui(p)}(x) \cup \gamma_{wi(p)}(x) : i = 1, 2, \ldots, n \} \text{ if } p \in (B \cap C) - A, \]
\[ = \{ u_i(p)(x) \cap y_i(p)(x), \mu_{ui(p)}(x) \cap \mu_{yi(p)}(x), \gamma_{ui(p)}(x) \cup \gamma_{yi(p)}(x) : i = 1, 2, \ldots, n \} \text{ if } p \in (A \cap B) - C, \]
\[ = \{ u_i(p)(x) \cap w_i(p)(x), \mu_{ui(p)}(x) \cap \mu_{wi(p)}(x), \gamma_{ui(p)}(x) \cup \gamma_{wi(p)}(x) : i = 1, 2, \ldots, n \} \text{ if } p \in (A \cap C) - B, \]
\[ = \{ y_j(p)(x) \cap t_j(p)(x), \mu_{yi(p)}(x) \cap \mu_{yi(p)}(x), \gamma_{yi(p)}(x) \cup \gamma_{yi(p)}(x) : j = 1, 2, \ldots, m \} \text{ if } p \in A \cap B \cap C. \]
and

\[ \gamma_i^{(p)}(x) = \gamma_i^{(p)}(x), \text{ if } p \in B - C - A, \]
\[ = \gamma_i^{(p)}(x), \text{ if } p \in C - B - A, \]
\[ = \gamma_i^{(p)}(x), \text{ if } p \in A - B - C, \]
\[ = \{ < v_j^{(p)}(x) \cap t_j^{(p)}(x) \cap \mu_{ij^{(p)}}(x), \gamma_i^{(p)}(x) \} : j = 1, 2, \ldots, m \} \text{ if } p \in (B \cap C) - A, \]
\[ = \{ < v_j^{(p)}(x) \cap z_j^{(p)}(x) \cap \mu_{iz^{(p)}}(x), \gamma_i^{(p)}(x) \} : j = 1, 2, \ldots, m \} \text{ if } p \in (A \cap B) - C, \]
\[ = \{ < z_j^{(p)}(x) \cap t_j^{(p)}(x) \cap \mu_{ij^{(p)}}(x), \gamma_i^{(p)}(x) \} : j = 1, 2, \ldots, m \} \text{ if } p \in (A \cap C) - B, \]
\[ = \{ < z_j^{(p)}(x) \cap v_j^{(p)}(x) \cap \mu_{ij^{(p)}}(x), \gamma_i^{(p)}(x) \} : j = 1, 2, \ldots, m \} \text{ if } p \in A \cap B \cap C. \]

Let \((f, A) \cap \%(g, B) = (I, Q)\) and \(Q = A \cup B\). Then for \(q \in Q\), we have,

\[ \mu_i^{(q)}(x) = \mu_i^{(q)}(x), \text{ if } q \in A - B, \]
\[ = \mu_i^{(q)}(x), \text{ if } q \in B - A, \]
\[ = \{ < y_i^{(q)}(x) \cap u_i^{(q)}(x) \cap \mu_{wi^{(q)}}(x), \gamma_i^{(q)}(x) \} : i = 1, 2, \ldots, n \} \text{ if } q \in A \cap B, \]

and

\[ \gamma_i^{(q)}(x) = \gamma_i^{(q)}(x), \text{ if } q \in A - B, \]
\[ = \gamma_i^{(q)}(x), \text{ if } q \in B - A, \]
\[ = \{ < z_i^{(q)}(x) \cap u_i^{(q)}(x) \cap \mu_{zi^{(q)}}(x), \gamma_i^{(q)}(x) \} : j = 1, 2, \ldots, m \} \text{ if } q \in A \cap B. \]

Now \(((f, A) \cap \%((g, B)) \cap \%((h, C)) = (I, Q) \cap \%((h, C)) = (k, R)\), say, where \(R = Q \cup C\). Then for \(r \in R\) we have,

\[ \mu_i^{(r)}(x) = \mu_i^{(r)}(x), \text{ if } r \in B - C - A, \]
\[ = \mu_i^{(r)}(x), \text{ if } r \in C - B - A, \]
\[ = \mu_i^{(r)}(x), \text{ if } r \in A - B - C, \]
\[ = \{ < u_i^{(r)}(x) \cap w_i^{(r)}(x) \cap \mu_{wi^{(r)}}(x), \gamma_i^{(r)}(x) \} : i = 1, 2, \ldots, n \} \text{ if } r \in (B \cap C) - A, \]
\[ = \{ < u_i^{(r)}(x) \cap y_i^{(r)}(x) \cap \mu_{wi^{(r)}}(x) \cap \gamma_i^{(r)}(x) \} : i = 1, 2, \ldots, n \} \text{ if } r \in (A \cap B) - C, \]
\[ = \{ < y_i^{(r)}(x) \cap w_i^{(r)}(x) \cap \mu_{wi^{(r)}}(x), \gamma_i^{(r)}(x) \} : i = 1, 2, \ldots, n \} \text{ if } r \in (A \cap C) - B, \]
\[ = \{ < u_i^{(r)}(x) \cap y_i^{(r)}(x) \cap \mu_{wi^{(r)}}(x), \gamma_i^{(r)}(x) \} : i = 1, 2, \ldots, n \} \text{ if } r \in A \cap B \cap C, \]

and

\[ \gamma_i^{(r)}(x) = \gamma_i^{(r)}(x), \text{ if } r \in B - C - A, \]
\[ = \gamma_i^{(r)}(x), \text{ if } r \in C - B - A, \]
\[ = \gamma_i^{(r)}(x), \text{ if } r \in A - B - C, \]
\[ = \{ < v_j^{(r)}(x) \cap t_j^{(r)}(x) \cap \mu_{ij^{(r)}}(x) \cap \gamma_i^{(r)}(x), \gamma_j^{(r)}(x) \} : j = 1, 2, \ldots, m \} \text{ if } r \in (B \cap C) - A, \]
\[
\{< v_j(r)(x) \cap z_j(r)(x), \mu_{vjr}(r)(x) \cap \mu_{zjr}(r)(x), \gamma_{vjr}(r)(x) \cup \gamma_{zjr}(r)(x) >: j = 1,2,\ldots,m \} \text{ if } r \in (A \cap B) - C,
\]
\[
\{< z_j(r)(x) \cap t_j(r)(x), \mu_{tjr}(r)(x) \cap \mu_{zjr}(r)(x), \gamma_{tjr}(r)(x) \cup \gamma_{zjr}(r)(x) >: j = 1,2,\ldots,m \} \text{ if } r \in (A \cap C) - B,
\]
\[
\{< z_j(r)(x) \cap v_j(r)(x) \cap t_j(r)(x), \mu_{tjr}(r)(x) \cap \mu_{vrj}(r)(x) \cap \mu_{zjr}(r)(x), \gamma_{tjr}(r)(x) \cup \gamma_{vrj}(r)(x) \cup \gamma_{zjr}(r)(x) >: j = 1,2,\ldots,m \} \text{ if } r \in A \cap B \cap C.
\]

Consequently, we get \((f, A) \cap \%(g, B) \cap \%(h, C)) = ((f, A) \cap \%(g, B)) \cap \%(h, C)\).

(10)-(12) can be proved similarly.

7 Soft rough interval valued intuitionistic fuzzy sets of type-2

In this section we shall consider the lower and upper soft rough approximations of an interval valued intuitionistic fuzzy set of type-2 in a soft approximation space and obtain a new hybrid model called soft rough interval valued intuitionistic fuzzy set of type-2.

**Definition 7.1:** Let us consider an intervalvalued intuitionistic fuzzy set of type-2 \(\Gamma\) defined by \(\Gamma = \{< x, \mu_{\Gamma}(x), \gamma_{\Gamma}(x) >: x \in U \}\) where for \(x \in U\), \(\mu_{\Gamma}(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1,2,\ldots,n \}\) and \(\gamma_{\Gamma}(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1,2,\ldots,m \}\) where \(u_i(x), v_j(x) \in \text{Int}(0,1)\) and \(\mu_{ui}, \mu_{vj}, \gamma_{ui}, \gamma_{vj} : U \rightarrow \text{Int}(0,1)\) are functions such that for \(x \in U\), \(0 \leq \sup \mu_{ui}(x) + \sup \gamma_{ui}(x) \leq 1\) and \(0 \leq \sup \mu_{vj}(x) + \sup \gamma_{vj}(x) \leq 1\) for each \(i, j\).

Now let \(\Theta = (f, A)\) be a full soft set over \(U\) i.e; \(\cup_{a \in A} f(a) = U\) and \(S = (U, \Theta)\) be the soft approximation space. Then the lower and upper soft rough approximations of \(\Gamma\) with respect to \(S\) are denoted by \(\downarrow_{\text{2aps}}(\Gamma)\) and \(\uparrow_{\text{2aps}}(\Gamma)\) respectively, which are interval valued intuitionistic fuzzy sets of type-2 in \(U\) given by:

\[
\downarrow_{\text{2aps}}(\Gamma) = \{< x, < u_i(x), \bigwedge \{\text{inf} \mu_{ui}(x) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \bigwedge \{\text{sup} \mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \bigwedge \{\text{inf} \gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \bigwedge \{\text{sup} \gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\} >: i = 1,2,\ldots,n \}\}
\]
\[
\uparrow_{\text{2aps}}(\Gamma) = \{< x, < v_j(x), \bigwedge \{\text{inf} \mu_{vj}(x) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \bigwedge \{\text{sup} \mu_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \bigwedge \{\text{inf} \gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \bigwedge \{\text{sup} \gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\} >: j = 1,2,\ldots,m \}\}
\]

The operators and \(\downarrow_{\text{2aps}}\) and \(\uparrow_{\text{2aps}}\) are called the lower and upper soft rough approximation operators on interval valued intuitionistic fuzzy sets of type-2. If \(\downarrow_{\text{2aps}}(\Gamma) = \uparrow_{\text{2aps}}(\Gamma)\), then \(\Gamma\) is said to be soft definable; otherwise \(\Gamma\) is called a soft rough interval valued intuitionistic fuzzy set of type-2.

**Example 7.2:** Let \(U = \{1,2\}\) and \(\Gamma \in \text{IVIFS}_2(U)\) be defined by \(\Gamma = \{< x, \mu_{\Gamma}(x), \gamma_{\Gamma}(x) >: x \in U \}\) where for \(x \in U\), \(\mu_{\Gamma}(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1,2\}\) and \(\gamma_{\Gamma}(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1,2\}\), Let,

\[
\mu_{\Gamma}(1) = \{< [0.0,0.1], [0.3,0.4], [0.4,0.5] >, < [0.2,0.5], [0.2,0.4], [0.3,0.5] >, \}
\]
\[
\mu_{\Gamma}(2) = \{< [0.1,0.2], [0.1,0.2], [0.5,0.7] >, < [0.1,0.3], [0.2,0.4], [0.3,0.4] >, \}
\]
\[ \gamma_r(1) = \{< [0,1,0.2], [0,1,0.3], [0,3,0.4]>, < [0,0,0.1], [0,1,0.2], [0,5,0.6]> \}, \\
\gamma_r(2) = \{< [0,3,0.4], [0,6,0.7], [0,1,0.2]>, < [0,2,0.3], [0,4,0.6], [0,2,0.3]> \}. \]

Now let \( \Theta = (f,A) \) be a full soft set over \( U \), where \( A = \{a,b\} \) and \( f : A \rightarrow P(U) \) be defined as \( f(a) = \{1,2\}, f(b) = \{2\} \). Then,

\[
\downarrow_{\text{saps}}(\Gamma) = \{< 1, < [0,0,0.1], [0,1,0.2], [0,5,0.7]>, < [0,2,0.5], [0,2,0.4], [0,3,0.5]>, \\
\{< [0,1,0.2], [0,1,0.3], [0,3,0.4]>, < [0,0,0.1], [0,1,0.2], [0,5,0.6]>, < [0,1,0.2], [0,1,0.3], [0,3,0.5]>, < [0,2,0.3], [0,1,0.2], [0,5,0.6]>, \\
\} \}
\]

Since \( \downarrow_{\text{saps}}(\Gamma) \neq \uparrow_{\text{saps}}(\Gamma) \), then \( \Gamma \) a soft rough interval valued intuitionistic fuzzy set of type-2.

**Theorem 7.3:** Let \( \Theta = (f,A) \) be a full soft set over \( U \) and \( S = (U,\Theta) \) be the soft approximation space. Let \( \Gamma, \Psi \in IVIFS_2(U) \) where \( \Gamma \) and \( \Psi \) have been defined in 5.5. Then,

1. \( \downarrow_{\text{saps}}(\Phi*) = \Phi* \Rightarrow \uparrow_{\text{saps}}(\Phi*) \),
2. \( \downarrow_{\text{saps}}(U*) = U* \Rightarrow \uparrow_{\text{saps}}(U*) \),
3. \( \Gamma \subseteq \Psi \Rightarrow \downarrow_{\text{saps}}(\Gamma) \subseteq \downarrow_{\text{saps}}(\Psi) \),
4. \( \Gamma \subseteq \Psi \Rightarrow \uparrow_{\text{saps}}(\Gamma) \subseteq \uparrow_{\text{saps}}(\Psi) \),
5. \( \downarrow_{\text{saps}}(\Gamma \cap \Psi) \subseteq \downarrow_{\text{saps}}(\Gamma) \cap \downarrow_{\text{saps}}(\Psi) \),
6. \( \uparrow_{\text{saps}}(\Gamma \cap \Psi) \subseteq \uparrow_{\text{saps}}(\Gamma) \cap \uparrow_{\text{saps}}(\Psi) \),
7. \( \downarrow_{\text{saps}}(\Gamma) \cup \downarrow_{\text{saps}}(\Psi) \subseteq \downarrow_{\text{saps}}(\Gamma \cup \Psi) \),
8. \( \uparrow_{\text{saps}}(\Gamma) \cup \uparrow_{\text{saps}}(\Psi) \subseteq \uparrow_{\text{saps}}(\Gamma \cup \Psi) \).

**Proof:** (1)-(4) are straight forward. (5) We have,

\[
\downarrow_{\text{saps}}(\Gamma \cap \Psi) = \{x, \{< u_i(x) \cap \nu_j(x) \}, [\alpha \in A(\{x, y \} \subseteq f(a))], \forall \{ inf(\mu_{u_i}(y) \cap \mu_{\nu_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \\
\land \{ sup(\mu_{u_i}(y) \cap \mu_{\nu_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \forall \{ inf(\gamma_{u_i}(y) \cup \gamma_{\nu_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \\
\lor \{ sup(\gamma_{u_i}(y) \cup \gamma_{\nu_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}: i = 1,2,...,n, \\
\{< v_i(x) \cap \tau_j(x), [\alpha \in A(\{x, y \} \subseteq f(a))], \forall \{ inf(\mu_{v_i}(y) \cap \mu_{\tau_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \\
\land \{ sup(\mu_{v_i}(y) \cap \mu_{\tau_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \forall \{ inf(\gamma_{v_i}(y) \cup \gamma_{\tau_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \\
\lor \{ sup(\gamma_{v_i}(y) \cup \gamma_{\tau_j}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}: j = 1,2,...,m >: x \in U. \}
\]

Also we have,

\[
\downarrow_{\text{saps}}(\Gamma) \cap \downarrow_{\text{saps}}(\Psi) = \{x, \{< u_i(x) \}, [\alpha \in A(\{x, y \} \subseteq f(a))], \forall \{ inf(\mu_{u_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \\
\land \{ sup(\mu_{u_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \forall \{ inf(\gamma_{u_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}: i = 1,2,...,n, \\
\{< v_i(x) \}, [\alpha \in A(\{x, y \} \subseteq f(a))], \forall \{ inf(\mu_{v_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \\
\land \{ sup(\mu_{v_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \forall \{ inf(\gamma_{v_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}, \forall \{ sup(\gamma_{v_i}(y)) : \exists a \in A(\{x, y \} \subseteq f(a)) \}
\]
\[ \subseteq f(a) \] \Rightarrow j = 1, 2, \ldots, m >: x \in U \right\} \cap \{ < x, \{ < w_i(x), [\land \{ \inf_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \}] \}, \lor \{ \sup_{\gamma_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \} >: i = 1, 2, \ldots, n, \{ < t_j(x), [\land \{ \inf_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \}], [\land \{ \sup_{\gamma_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \}] >: j = 1, 2, \ldots, m >: x \in U \}

Now we have

\[ \inf_{\mu_{srl}}(y) \cap_{\mu_{srl}}(y) = \inf([\min(\inf_{\mu_{srl}}(y), \inf_{\mu_{srl}}(y)), \min(\sup_{\mu_{srl}}(y), \sup_{\mu_{srl}}(y))]) = \min(\inf_{\mu_{srl}}(y), \inf_{\mu_{srl}}(y)) \]

Therefore,

\[ \inf_{\mu_{srl}}(y) \cap_{\mu_{srl}}(y) \leq \inf_{\mu_{srl}}(y) \text{ and } \inf_{\mu_{srl}}(y) \cap_{\mu_{srl}}(y) \leq \inf_{\mu_{srl}}(y). \]

So,

\[ \land \{ \inf_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \} \leq \min(\land \{ \inf_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \} \land \{ \inf_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \}) \]

Similarly it can be shown that,

\[ \land \{ \sup_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \} \leq \min(\land \{ \sup_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \} \land \{ \sup_{\mu_{srl}}(y) : \exists a \in A(\{x, y \} \subseteq f(a)) \}) \]

Also

\[ \inf_{\gamma_{srl}}(y) \cup_{\gamma_{srl}}(y) = \inf([\max(\inf_{\gamma_{srl}}(y), \inf_{\gamma_{srl}}(y)), \max(\sup_{\gamma_{srl}}(y), \sup_{\gamma_{srl}}(y))]) = \max(\inf_{\gamma_{srl}}(y), \inf_{\gamma_{srl}}(y)) \]
Therefore, \( \inf (\gamma_{\alpha}(y) \cup \gamma_{\alpha}(y)) \geq \inf \gamma_{\alpha}(y) \), and \( \inf (\gamma_{\alpha}(y) \cup \gamma_{\alpha}(y)) \geq \inf \gamma_{\alpha}(y) \). So,

\[
\vee \{ \inf (\gamma_{\alpha}(y) \cup \gamma_{\alpha}(y)) : \exists a \in A(x, y) \leq f(a) \} \geq \max (\vee \{ \inf \gamma_{\alpha}(y) : \exists a \in A(x, y) \leq f(a) \}), \\
\vee \{ \inf \gamma_{\alpha}(y) : \exists a \in A(x, y) \leq f(a) \}.
\]

Similarly it can be shown that,

\[
\vee \{ \sup \gamma_{\alpha}(y) : \exists a \in A(x, y) \leq f(a) \} \geq \max (\vee \{ \sup \gamma_{\alpha}(y) : \exists a \in A(x, y) \leq f(a) \}), \vee \{ \sup \gamma_{\alpha}(y) : \exists a \in A(x, y) \leq f(a) \}).
\]

In a similar way we can prove the followings:

\[
\wedge \{ \inf \mu_{ij}(y) \cap \mu_{ij}(y) : \exists a \in A(x, y) \leq f(a) \} \leq \min (\wedge \{ \inf \mu_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}), \\
\wedge \{ \inf \mu_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}, \\
\wedge \{ \sup \mu_{ij}(y) \cap \mu_{ij}(y) : \exists a \in A(x, y) \leq f(a) \} \leq \min (\wedge \{ \sup \mu_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}), \\
\wedge \{ \sup \mu_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}, \\
\vee \{ \inf \gamma_{ij}(y) \cup \gamma_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}, \\
\wedge \{ \inf \gamma_{ij}(y) \cup \gamma_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}, \\
\vee \{ \sup \gamma_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}), \\
\vee \{ \sup \gamma_{ij}(y) : \exists a \in A(x, y) \leq f(a) \}).
\]

Consequently, \( \downarrow 2 \sup s(\Gamma \cap 2P) \subseteq \downarrow 2 \sup s(\Gamma) \cap 2 \downarrow 2 \sup s(P) \). (vi)-(viii)can be proved similarly.

### 8 Soft interval valued intuitionistic fuzzy rough sets of type-2

In this section we use an interval valued intuitionistic fuzzy soft set of type-2 to granulate the universe of discourse and obtain a new hybrid model called soft interval valued intuitionistic fuzzy rough set of type-2 which can be seen as an extension of soft rough interval valued intuitionistic fuzzy set of type-2.

**Definition 8.1:** Let \( \Omega = (f, A) \) be an interval valued intuitionistic fuzzy soft set of type-2 over \( U \), where \( f : A \rightarrow IVIFS_2(U) \) is defined by

\[
f(a) = \{< x, \mu_{f(a)}(x), \gamma_{f(a)}(x) : x \in U \}, \text{ where for } x \in U \\
\mu_{f(a)}(x) = \{< u_i(a)(x), \mu_{u_i(a)}(x), \gamma_{u_i(a)}(x) : i = 1, 2, \ldots, n} \text{ and} \\
\gamma_{f(a)}(x) = \{< v_j(a)(x), \mu_{v_j(a)}(x), \gamma_{v_j(a)}(x) : j = 1, 2, \ldots, m}.
\]

Then the pair \( SIVIFS_2 = (U, \Omega) \) is called a soft interval valued intuitionistic fuzzy type-2 approximation space. Let \( \Gamma \in IVIFS_2(U) \) be defined by \( \Gamma = \{< x, \mu_{\Gamma}(x), \gamma_{\Gamma}(x) : x \in U \} \) where for \( x \in U \),

\[
\mu_{\Gamma}(x) = \{< u_i(x), \mu_{u_i(x)}(x), \gamma_{u_i(x)}(x) : i = 1, 2, \ldots, n} \text{ and} \\
\gamma_{\Gamma}(x) = \{< v_j(x), \mu_{v_j(x)}(x), \gamma_{v_j(x)}(x) : j = 1, 2, \ldots, m}.
\]

Then the lower and upper soft interval valued intuitionistic fuzzy rough approximations of \( \Gamma \) w.r.t \( SIVIFS_2 \) are denoted respectively by \( \downarrow 2A\!pr_{SIVIFS_2}(\Gamma) \) and \( \uparrow 2A\!pr_{SIVIFS_2}(\Gamma) \).  
given by

\[ \downarrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)} = \{ x, \mu_{\downarrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x), \gamma_{\downarrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x) : x \in U \} \]

\[ \uparrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)} = \{ x, \mu_{\uparrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x), \gamma_{\uparrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x) : x \in U \} \]

where for \( x \in U \),

\[ \mu_{\downarrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x) = \{ u_i \ast (x), [\wedge_{e \in A}(\inf \mu_{ui}(x) \land \inf \mu_{ui}(x)), \wedge_{e \in A}(\sup \mu_{ui}(x) \land \sup \mu_{ui}(x))] \}
\]
\[ [\wedge_{e \in A}(\inf \mu_{ui}(x) \lor \inf \mu_{ui}(x)), \wedge_{e \in A}(\sup \mu_{ui}(x) \lor \sup \mu_{ui}(x))] > 1, 2, \ldots, n \}
\]

\[ \gamma_{\downarrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x) = \{ v_j \ast (x), [\wedge_{e \in A}(\inf \mu_{vj}(x) \lor \inf \mu_{vj}(x)), \wedge_{e \in A}(\sup \mu_{vj}(x) \lor \sup \mu_{vj}(x))] \}
\]
\[ [\wedge_{e \in A}(\inf \mu_{vj}(x) \lor \inf \mu_{vj}(x)), \wedge_{e \in A}(\sup \mu_{vj}(x) \lor \sup \mu_{vj}(x))] > 1, 2, \ldots, n \}
\]

\[ \mu_{\uparrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x) = \{ u_i \ast (x), [\wedge_{e \in A}(\inf \mu_{ui}(x) \lor \inf \mu_{ui}(x)), \wedge_{e \in A}(\sup \mu_{ui}(x) \lor \sup \mu_{ui}(x))] \}
\]
\[ [\wedge_{e \in A}(\inf \mu_{ui}(x) \lor \inf \mu_{ui}(x)), \wedge_{e \in A}(\sup \mu_{ui}(x) \lor \sup \mu_{ui}(x))] > 1, 2, \ldots, n \}
\]

\[ \gamma_{\uparrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)}}(x) = \{ v_j \ast (x), [\wedge_{e \in A}(\inf \mu_{vj}(x) \lor \inf \mu_{vj}(x)), \wedge_{e \in A}(\sup \mu_{vj}(x) \lor \sup \mu_{vj}(x))] \}
\]
\[ [\wedge_{e \in A}(\inf \mu_{vj}(x) \lor \inf \mu_{vj}(x)), \wedge_{e \in A}(\sup \mu_{vj}(x) \lor \sup \mu_{vj}(x))] > 1, 2, \ldots, n \}
\]

If \( \downarrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)} = \uparrow_{2\text{Apr}_{\text{SIVIF}}(\Gamma)} \), then \( \Gamma \) is called soft interval valued intuitionistic fuzzy type-2 definable; otherwise \( \Gamma \) is called a soft valued interval valued intuitionistic fuzzy rough set of type-2.

**Example 8.2:** Let \( \Omega = (f, A) \) be an interval valued intuitionistic fuzzy soft set of type-2 over \( U \), where \( U = \{1, 2\}, A = \{a, b\} \) and \( f : A \rightarrow IVIFS_2(U) \) is defined by

\[ f(a) = \{ < 1, \mu_{f(a)}(1), \gamma_{f(a)}(1) >, < 2, \mu_{f(a)}(2), \gamma_{f(a)}(2) > \} \]

and

\[ f(b) = \{ < 1, \mu_{f(b)}(1), \gamma_{f(b)}(1) >, < 2, \mu_{f(b)}(2), \gamma_{f(b)}(2) > \} \]

where

\[ \mu_{f(a)}(1) = \{ < 0, 1, 0.2, 0.4, 0.5, 0.2, 0.3, >, < 0, 3, 0.5, 0.1, 0.4, 0.5, 0.6 > \} \]
\[ \mu_{f(a)}(2) = \{ < 0, 7, 0.8, 0.1, 0.2, 0.3, 0.4, >, < 0, 5, 0.6, 0.3, 0.6, 0.2, 0.4 > \} \]
\[ \gamma_{f(a)}(1) = \{ < 0, 4, 0.5, 0.5, 0.6, 0.2, 0.3, >, < 0, 3, 0.4, 0.2, 0.6, 0.1, 0.3 > \} \]
\[ \gamma_{f(a)}(2) = \{ < 0, 3, 0.5, 0.2, 0.4, 0.2, 0.4, >, < 0, 1, 0.4, 0.5, 0.6, 0.2, 0.4 > \} \]
\[ \mu_{f(b)}(1) = \{ < 0, 7, 0.9, 0.1, 0.2, 0.7, 0.8, >, < 0, 1, 0.3, 0.4, 0.6, 0.1, 0.2 > \} \]
\[ \mu_{f(b)}(2) = \{ < 0, 5, 0.7, 0.5, 0.7, 0.1, 0.2, >, < 0, 3, 0.6, 0.2, 0.4, 0.3, 0.5 > \} \]
\[ \gamma_{f(b)}(1) = \{ < 0, 4, 0.6, 0.3, 0.4, 0.2, 0.6, >, < 0, 5, 0.6, 0.2, 0.8, 0.1, 0.2 > \} \]
\[ \gamma_{f(b)}(2) = \{ < 0, 3, 0.7, 0.1, 0.3, 0.1, 0.5, >, < 0, 2, 0.5, 0.2, 0.5, 0.3, 0.4 > \} \]

Let \( \Gamma \in IVIFS_2(U) \) be defined by \( \Gamma = \{ < 1, \mu_\Gamma(1), \gamma_\Gamma(1) >, < 2, \mu_\Gamma(2), \gamma_\Gamma(2) > \} \) where

\[ \mu_\Gamma(1) = \{ < 0, 2, 0.3, 0.4, 0.6, 0.1, 0.3 >, < 0, 2, 0.6, 0.1, 0.2, 0.6, 0.7 > \} \]
\[ \mu_\Gamma(2) = \{ < 0, 5, 0.7, 0.2, 0.4, 0.3, 0.5 >, < 0, 1, 0.2, 0.4, 0.6, 0.2, 0.4 > \} \]
Theorem 8.3: Let $\Omega = (f, A)$ be an interval valued intuitionistic fuzzy universal soft set of type-2 over $U$ and $SIVIF_2 = (U, \Omega)$ soft interval valued intuitionistic fuzzy type-2 approximation space. Then for $\Gamma, \Psi \in IVIFS_2(U)$, we have

1. $\downarrow_2pr_{SIVIF_2}(\Phi^*) = \Phi^*$
2. $\uparrow_2pr_{SIVIF_2}(U^*) = U^*$
3. $\Gamma \subseteq \Psi \Rightarrow \downarrow_2pr_{SIVIF_2}(\Gamma) \subseteq \downarrow_2pr_{SIVIF_2}(\Psi)$
4. $\Gamma \subseteq \Psi \Rightarrow \uparrow_2pr_{SIVIF_2}(\Gamma) \subseteq \uparrow_2pr_{SIVIF_2}(\Psi)$
5. $\downarrow_2pr_{SIVIF_2}(\Gamma \cap_2 \Psi) \subseteq \downarrow_2pr_{SIVIF_2}(\Gamma) \cap_2 \downarrow_2pr_{SIVIF_2}(\Psi)$
6. $\uparrow_2pr_{SIVIF_2}(\Gamma \cap_2 \Psi) \subseteq \uparrow_2pr_{SIVIF_2}(\Gamma) \cap_2 \uparrow_2pr_{SIVIF_2}(\Psi)$
7. $\downarrow_2pr_{SIVIF_2}(\Gamma \cup_2 \Psi) \subseteq \downarrow_2pr_{SIVIF_2}(\Gamma) \cup_2 \downarrow_2pr_{SIVIF_2}(\Psi)$
8. $\uparrow_2pr_{SIVIF_2}(\Gamma \cup_2 \Psi) \subseteq \uparrow_2pr_{SIVIF_2}(\Gamma) \cup_2 \uparrow_2pr_{SIVIF_2}(\Psi)$

Proof: (1)-(4) are straight forward.(5) Let $\Gamma$ and $\Psi$ be defined as follows: $\Gamma = \{ x, \mu_\Gamma(x), \gamma_\Gamma(x) : x \in U \}$ where for $x \in U$,

$$\mu_\Gamma(x) = \{ < u_i * (x), \mu_{ui} * (x), \gamma_{ui} * (x) : i = 1, 2, \ldots, n \}$$

and

$$\gamma_\Gamma(x) = \{ < v_j * (x), \mu_{vj} * (x), \gamma_{vj} * (x) : j = 1, 2, \ldots, m \}$$

and

$$\Psi = \{ x, \mu_\Psi(x), \gamma_\Psi(x) : x \in U \}$$
where for \( x \in U \),
\[
\mu_\Psi(x) = \{ < w_i * (x), \mu_{wi} * (x), \gamma_{wi} * (x) > : i = 1, 2, \ldots, n \}
\]
and
\[
\gamma_\Psi(x) = \{ < t_j * (x), \mu_{tj} * (x), \gamma_{tj} * (x) > : j = 1, 2, \ldots, m \}.
\]

Let \( f : A \rightarrow IVIFS_2(U) \) be defined by
\[
f(a) = \{ < x, \mu_{f(a)}(x), \gamma_{f(a)}(x) > : x \in U \},
\]
wherefor \( x \in U \)
\[
\mu_{f(a)}(x) = \{ < u_i(a)(x), \mu_{ai(a)}(x), \gamma_{ai(a)}(x) > : i = 1, 2, \ldots, n \}
\]
and
\[
\gamma_{f(a)}(x) = \{ < v_j(a)(x), \mu_{vj(a)}(x), \gamma_{vj(a)}(x) > : j = 1, 2, \ldots, m \}.
\]

Then
\[
\Gamma \cap_2 \Psi = \{ < x, \{ < u_i * (x) \cap w_i * (x), \mu_{ai(a)} * (x) \cap \mu_{wi} * (x), \gamma_{ai(a)} * (x) \cup \gamma_{wi} * (x) > : i = 1, 2, \ldots, n \}, \{ < v_j * (x) \cap t_j * (x), \mu_{vj(a)} * (x) \cap \mu_{tj} * (x), \gamma_{vj(a)} * (x) \cup \gamma_{tj} * (x) > : j = 1, 2, \ldots, m \} > : x \in U \}
\]

We have
\[
downarrow_{2AprsIVIFS_2} (\Gamma \cap_2 \Psi) = \{ < x, \{ < u_i * (x) \cap w_i * (x), \mu_{ai(a)} * (x) \land \mu_{ai(a)} * (x), \gamma_{ai(a)} * (x) \lor \gamma_{ai(a)} * (x) > : i = 1, 2, \ldots, n \}, \{ < v_j * (x) \cap t_j * (x), \mu_{vj(a)} * (x) \lor \mu_{vj(a)} * (x), \gamma_{vj(a)} * (x) \land \gamma_{vj(a)} * (x) > : j = 1, 2, \ldots, m \} > : x \in U \}
\]

Again we have,
\[
downarrow_{2AprsIVIFS_2}(\Gamma) \cap_2 \downarrow_{2AprsIVIFS_2}(\Psi)
\]
Similarly, we have investigated the problem of combining interval valued intuitionistic fuzzy set of type-2 with soft sets. We have also investigated some basic properties of these new hybridizations.

Now it can be shown that

\[
\begin{align*}
&\bigwedge_{a \in A}(\inf_{\gamma_j(a)}(x) \land \inf_{\mu_i}(x) \land \inf_{\mu_i}(x)), \bigwedge_{a \in A}(\sup_{\gamma_j(a)}(x) \land \sup_{\gamma_j}(x)), \\
&\bigwedge_{a \in A}(\inf_{\gamma_j(a)}(x) \lor \inf_{\gamma_j}(x), \bigwedge_{a \in A}(\sup_{\gamma_j(a)}(x) \lor \sup_{\gamma_j}(x)) \bigg) \land \bigg( \bigwedge_{i = 1}^n (\inf_{\gamma_j(a)}(x) \lor \inf_{\gamma_j}(x), \bigwedge_{a \in A}(\sup_{\gamma_j(a)}(x) \lor \sup_{\gamma_j}(x)) \bigg) \bigg)
\end{align*}
\]

Using the above results we get, \( \downarrow 2^{\mathbb{P}} \subseteq \downarrow 2^{\mathbb{P}} \subseteq 2^{\mathbb{P}} \subseteq 2^{\mathbb{P}} \). (6)-(8) can be proved similarly.

9 Conclusion

In this paper we have investigated the problem of combining interval valued intuitionistic fuzzy set of type-2 with soft sets and rough sets and we obtain three different types of hybrid models, namely-interval valued intuitionistic fuzzy soft sets of type-2, soft rough interval valued intuitionistic fuzzy sets of type-2 and soft interval valued intuitionistic fuzzy rough sets of type-2. We have also investigated some basic properties of these new hybridizations.
References