Combination of interval-valued neutrosophic soft sets and graph theory

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Abstract: In this paper, we combine the concepts of interval-valued neutrosophic soft set and graph theory. We introduce notations of interval-valued neutrosophic soft graph and complete interval-valued neutrosophic soft graph. We also present several different types operations including cartesian product, union and intersection on interval-valued neutrosophic soft graphs and investigate some properties of them.

Keywords: Interval-valued neutrosophic sets, interval-valued neutrosophic soft sets, interval-valued neutrosophic graphs, interval-valued neutrosophic soft graphs.

1 Introduction

The concept of neutrosophic set which is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information was firstly proposed by Smarandache [1]. Neutrosophic sets are generalization of the theory of fuzzy sets [2], intuitionistic fuzzy sets [3] and interval-valued intuitionistic fuzzy sets [4]. The neutrosophic sets are characterized by a truth-membership function T, an indeterminacy-membership function I and a falsity membership function F independently, which are within the real standard or nonstandard unit interval $]^-0, 1^+[$. Wang et al. [5] introduced the concept of a single-valued neutrosophic sets which is a subclass of the neutrosophic sets. They also introduced the concept of interval-valued neutrosophic sets [6] which is more sensitive than single valued neutrosophic sets in which three membership functions are independent and their value belong to the unit interval $[0, 1]$. Some more work on single valued neutrosophic sets, neutrosophic sets, interval valued neutrosophic sets and their applications may be found on [7, 8, 9, 10, 11, 12].

The concept of soft set theory which is a new mathematical tool for dealing with uncertainties was initiated by Molodtsov [13]. It has been showed that soft sets have potential applications in different fields. Maji et al. [14] presented the definition of fuzzy soft sets and investigated some properties of this notion. Thereafter many researchers have applied this concept on different branches of mathematics. The concept of interval-valued fuzzy soft set has been introduced by Yang et al. [15]. The definition of neutrosophic soft sets was firstly given by Maji [16]. He also discussed many operations such as union, intersection and complement on neutrosophic soft sets. The concept of interval-valued neutrosophic soft set which is a generalization of fuzzy soft sets, interval-valued fuzzy soft sets and neutrosophic soft sets were given by Deli [17].

Graph theory was firstly introduced by Euler [18]. Since then graph theory has become the most important part of combinatorial mathematics. A graph is used to create a relationship between a given set of elements. Each element can be represented by a vertex and the relationship between them can be represented by an edge. The concept of graph has been applied to different algebraic structures in mathematics.

In this paper, primarily, we give the concepts of soft set, neutrosophic set, neutrosophic soft set, interval-valued neutrosophic soft set, graph and interval-valued neutrosophic graph. After that we introduce the notion of interval-valued

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neutrosophic soft graph and give some related examples. We also present several different types operations including cartesian product, union and intersection on interval-valued neutrosophic soft graphs and investigate some properties of them.

2 Preliminaries

Definition 1. [1] *A neutrosophic set A on the universe of discourse U is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in U\}$, where the functions $T, I, F : U \to]^-0, 1^+[$ define a truth-membership function, an indeterminacy-membership function and a falsity-membership function of an element $x \in U$ for a set A, respectively, with the condition $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$. The neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. We consider the neutrosophic set which takes the value from the subset of $[0,1]$. The family of all neutrosphic sets on U is denoted by $\mathcal{N}(U)$.*

Definition 2. [16] *Let U be an inital universe set, E be a set of parameters and $A \subseteq E$. If F is a mapping given by $F : A \to \mathcal{N}(U)$, then a pair (F, A) is called a neutrosophic soft set over U.*

Definition 3. [17] *Let U be an inital universe set. Then an interval-valued neutrosophic set A over U can be represented by the set $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in U\}$, where*

$$T_A(x) = [T_A(x)^-, T_A(x)^+]$$
$$I_A(x) = [I_A(x)^-, I_A(x)^+]$$
$$F_A(x) = [F_A(x)^-, F_A(x)^+]$$

and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for each point x in U. The family of all interval-valued neutrosphic sets on U is denoted by $IV\mathcal{N}(U)$.

Definition 4. [17] *Let U be an initial universe set and $A \subseteq E$ be a set of parameters. If F is a mapping given by $F : A \to IV\mathcal{N}(U)$, then a pair (F, A) is called an interval-valued neutrosophic soft set over U.*

Definition 5. [18] *A graph G^* consists of set of finite objects $V = \{v_1, v_2, v_3, \ldots, v_n\}$ called vertices and other set $E = \{e_1, e_2, e_3, \ldots, e_n\}$ whose elements are called edges. Usually a graph is denoted as $G^* = (V, E)$.*

Definition 6. [19] *An interval-valued neutrosophic graph of a graph $G^* = (V, E)$ is given by a pair $G = (A, B)$, where $A = < [T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+] >$ is an interval-valued neutrosophic set on V and $B = < [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_A^+] >$ is an interval-valued neutrosophic relation on E such that*

$$T_B^-(v_i, v_j) \le min\{T_A^-(v_i), T_A^-(v_j)\}, \quad T_B^+(v_i, v_j) \le min\{T_A^+(v_i), T_A^+(v_j)\}$$
$$I_B^-(v_i, v_j) \ge max\{I_A^-(v_i), I_A^-(v_j)\}, \quad I_B^+(v_i, v_j) \ge max\{I_A^+(v_i), I_A^-(v_j)\}$$
$$F_B^-(v_i, v_j) \ge max\{F_A^-(v_i), F_A^-(v_j)\}, \quad F_B^+(v_i, v_j) \ge max\{F_A^+(v_i), F_A^+(v_j)\}$$

for all $(v_i, v_j) \in E$.

3 Interval-valued neutrosophic soft graphs

Definition 7. *An interval-valued neutrosophic soft graph is given by ordered 4-tuple $G = (G^*, K, M, A)$ such that*

   (i) *$G^* = (V, E)$ is a simple graph*
   (ii) *A is a nonempty set of parameters*
  (iii) *(K, A) is an interval-valued neutrosophic soft set over V*
  (iv) *(M, A) is an interval-valued neutrosophic soft set over E*
   (v) *$(K(e), M(e))$ is an interval-valued neutrosophic graph of G^* for all $e \in A$. That is*

$$T_{M(e)}^-(x, y) \le min\{T_{K(e)}^-(x), T_{K(e)}^-(y)\}, \quad T_{M(e)}^+(xy) \le min\{T_{K(e)}^+(x), T_{K(e)}^+(y)\}$$

$$I_{M(e)}^-(x,y) \geq max\{I_{K(e)}^-(x), I_{K(e)}^-(y)\}, \quad I_{M(e)}^+(xy) \geq max\{I_{K(e)}^+(x), I_{K(e)}^+(y)\}$$

$$F_{M(e)}^-(x,y) \geq max\{F_{K(e)}^-(x), F_{K(e)}^-(y)\}, \quad F_{M(e)}^+(xy) \geq max\{F_{K(e)}^+(x), F_{K(e)}^+(y)\}$$

such that $0 \leq T_{M(e)}(x,y) + I_{M(e)}(x,y) + F_{M(e)}(x,y) \leq 3$ *for all* $e \in A$ *and* $x,y \in V$. *The interval-valued neutrosophic graph* $(K(e), M(e))$ *is denoted by* $H(e)$ *for convenience. An interval-valued neutrosophic soft graph is a parameterized family of interval-valued neutrosophic graphs.*

Example 1. Consider a simple graph $G^* = (V, E)$ such that $V = \{v_1, v_2, v_3\}$ and $E = \{v_1v_2, v_2v_3, v_3v_1\}$. Let $A = \{e_1, e_2\}$ be a set of parameters and let (K, A) be an interval-valued neutrosophic soft set over V with its approximate function $K : A \rightarrow IV\mathcal{N}(V)$ defined by

$$K(e_1) = \{v_1|([0.2,0.6],[0.1,0.3],[0.2,0.3]), v_2|([0.4,0.5],[0.6,0.7],[0.3,0.4], v_3|([0.2,0.3],[0.2,0.4],[0.3,0.5])\}$$
$$K(e_2) = \{v_1|([0.3,0.4],[0.4,0.5],[0.2,0.7]), v_2|([0.2,0.4],[0.3,0.4],[0.4,0.5]), v_3|([0.2,0.3],[0.5,0.6],[0.4,0.8])\}$$

Now let (M, A) be an interval-valued neutrosophic soft set over E with its approximate function $M : A \rightarrow IV\mathcal{N}(E)$ defined by

$$M(e_1) = \{v_1v_2|([0.1,0.4],[0.7,0.8],[0.4,0.5])\}$$
$$M(e_2) = \{v_1v_2|([0.1,0.2],[0.6,0.7],[0.5,0.8]), v_2v_3|([0.1,0.2],[0.6,0.7],[0.6,0.9]), v_3v_1|([0.1,0.3],[0.6,0.7],[0.5,0.8])\}$$

It is clearly seen that $H(e_1) = (K(e_1), M(e_1))$, $H(e_2) = (K(e_2), M(e_2))$ are interval-valued neutrosophic graphs corresponding to the parameters e_1 and e_2. Hence $G = (G^*, K, M, A)$ is an interval-valued neutrosophic soft graph of G^* as shown in Figure 1.
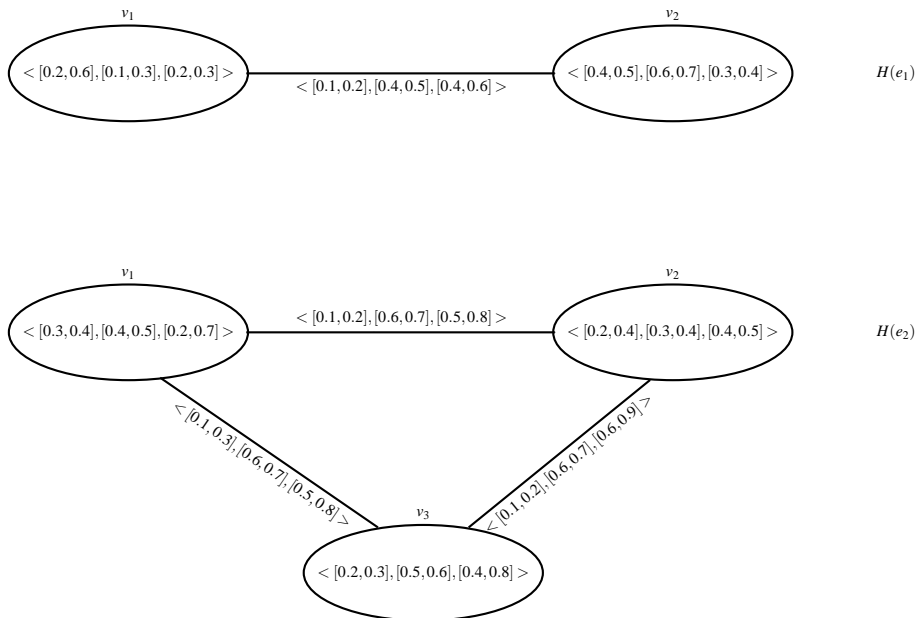

Fig. 1: Interval-valued neutrosophic soft graph G.

Definition 8. *Let* $G_1 = (G^*, K_1, M_1, A)$ *and* $G_2 = (G^*, K_2, M_2, B)$ *be two interval-valued neutrosophic soft graphs of* G^*. *Then* G_1 *is called an interval-valued neutrosophic soft subgraph of* G_2 *if*

  (i) $A \subseteq B$
  (ii) $H_1(e) = (K_1(e), M_1(e))$ *is an interval-valued neutrosophic subgraph of* $H_2(e) = (K_2(e), M_2(e))$ *for all* $e \in A$.

Example 2. Consider the simple graph $G^* = (V, E)$ as taken in Example 1.
Now let $B = \{e_1\}$ be a parameter set, (K_1, B) be an interval-valued neutrosophic soft set over V and (M_1, B) be an interval-valued neutrosophic soft set on E defined by

$$K_1(e_1) = \{v_1 | ([0.1, 0.4], [0.3, 0.5], [0.4, 0.6]), v_2 | ([0.3, 0.4], [0.7, 0.8], [0.5, 0.7], v_3 | ([0.1, 0.3], [0.4, 0.5], [0.6, 0.7])\}$$
$$M_1(e_1) = \{v_1 v_2 | ([0.1, 0.3], [0.8, 0.9], [0.7, 0.9]), v_2 v_3 | ([0.1, 0.2], [0.8, 0.9], [0.7, 0.8]), v_3 v_1 | ([0.1, 0.2], [0.5, 0.6], [0.7, 0.8])\}$$

It is clearly seen that $H_1(e_1) = (K_1(e_1), M_1(e_1))$ is interval-valued neutrosophic graphs corresponding to the parameters e_1. Also $G_1 = (G^*, K_1, M_1, B)$ is an interval-valued neutrosophic soft graph as shown in Figure 2. Hence G_1 is an interval-valued neutrosophic soft subgraph of G.
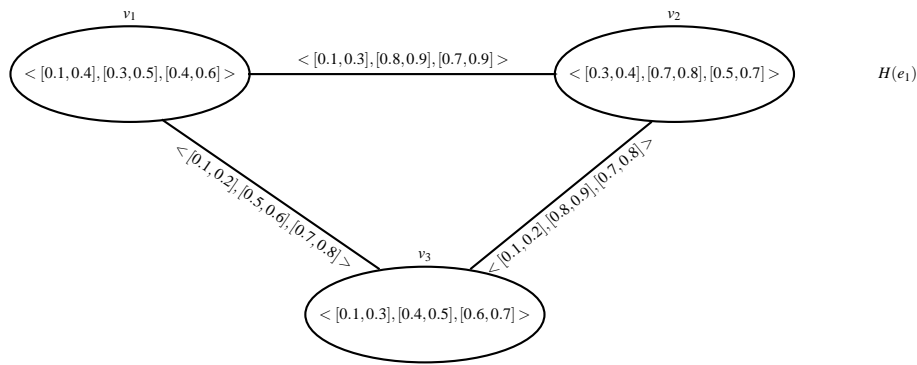

Fig. 2: Interval-valued neutrosophic soft graph G_1

Definition 9. *An interval-valued neutrosophic soft graph $G = (G^*, K, M, A)$ is called a complete interval-valued neutrosophic soft graph if*

$$T^-_{M(e)}(x, y) = min\{T^-_{K(e)}(x), T^-_{K(e)}(y)\}$$
$$T^+_{M(e)}(x, y) = min\{T^+_{K(e)}(x), T^+_{K(e)}(y)\}$$
$$I^-_{M(e)}(x, y) = max\{I^-_{K(e)}(x), I^-_{K(e)}(y)\}$$
$$I^+_{M(e)}(x, y) = max\{I^+_{K(e)}(x), I^+_{K(e)}(y)\}$$
$$F^-_{M(e)}(x, y) = max\{F^-_{K(e)}(x), F^-_{K(e)}(y)\}$$
$$F^+_{M(e)}(x, y) = max\{F^+_{K(e)}(x), F^+_{K(e)}(y)\}$$

for all $e \in A$ and $xy \in E$.

Example 3. Consider a simple graph $G^* = (V, E)$ such that

$$V = \{v_1, v_2, v_3, v_4\} \text{ and } E = \{v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_1, v_1 v_3, v_2 v_4\}.$$

Let $A = \{e_1, e_2\}$ be a set of parameters and (K, A) be an interval-valued neutrosophic soft sets over V with its approximation function $K : A \to IV\mathcal{N}(V)$ defined by

$$K(e_1) = \{v_1 | ([0.2, 0.3], [0.1, 0.5], [0.5, 0.6]), v_2 | ([0.1, 0.4], [0.3, 0.4], [0.4, 0.7]),$$
$$v_3 | ([0.1, 0.2], [0.1, 0.3], [0.5, 0.7]), v_4 | ([0.4, 0.5], [0.3, 0.5], [0.6, 0.8])\}$$
$$K(e_2) = \{v_1 | ([0.3, 0.4], [0.2, 0.5], [0.7, 0.9]), v_2 | ([0.1, 0.2], [0.3, 0.4], [0.4, 0.7]),$$
$$v_3 | ([0.2, 0.4], [0.3, 0.4], [0.5, 0.7])\}, v_4 | ([0.5, 0.7], [0.2, 0.3], [0.4, 0.6])$$

Let (M,A) be an interval-valued neutrosophic soft sets over E with its approximation function $K : A \rightarrow IV\mathcal{N}(E)$ defined by

$$M(e_1) = \{v_1v_2|([0.1,0.3],[0.3,0.5],[0.5,0.7]), v_2v_3|([0.1,0.2],[0.3,0.4],[0.5,0.7]), v_3v_1|([0.1,0.2],[0.1,0.5],[0.5,0.7])\}$$
$$M(e_2) = \{v_1v_2|([0.1,0.2],[0.3,0.5],[0.7,0.9]), v_2v_3|([0.1,0.2],[0.3,0.4],[0.5,0.7]), v_3v_4|([0.2,0.4],[0.3,0.4],[0.5,0.7]),$$
$$v_4v_1|([0.3,0.4],[0.2,0.5],[0.7,0.9]), v_1v_3|([0.2,0.4],[0.3,0.5],[0.7,0.9]), v_2v_4|([0.1,0.2],[0.3,0.4],[0.4,0.7])\}$$

It is easy to see that $H(e_1)$ and $H(e_2)$ are interval valued neutrosophic graphs of G^* corresponding to the parameters e_1, e_2 and e_3 respectively. Hence $G = (G^*, K, M, A)$ is a complete interval-valued neutrosophic soft graph of G^*.

Definition 10. *Let $G_1 = (G_1^*, K_1, M_1, A)$ and $G_2 = (G_2^*, K_2, M_2, B)$ be two interval valued neutrosophic soft graphs of simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The cartesian product of G_1 and G_2 is denoted by $G_1 \times G_2 = (G^*, K, M, A \times B)$, where $G^* = (V_1 \times V_2, E_1 \times E_2)$, and is defined by*

$$
\begin{cases}
\begin{cases}
(T_{K_1(e_1)}^- \times T_{K_2(e_2)}^-)(x_1, x_2) = min\{T_{K_1(e_1)}^-(x_1), T_{K_2(e_2)}^-(x_2)\} \\
(T_{K_1(e_1)}^+ \times T_{K_2(e_2)}^+)(x_1, x_2) = min\{T_{K_1(e_1)}^+(x_1), T_{K_2(e_2)}^+(x_2)\} \\
(I_{K_1(e_1)}^- \times I_{K_2(e_2)}^-)(x_1, x_2) = max\{I_{K_1(e_1)}^-(x_1), I_{K_2(e_2)}^-(x_2)\} \\
(I_{K_1(e_1)}^+ \times I_{K_2(e_2)}^+)(x_1, x_2) = max\{I_{K_1(e_1)}^+(x_1), I_{K_2(e_2)}^+(x_2)\} \\
(F_{K_1(e_1)}^- \times F_{K_2(e_2)}^-)(x_1, x_2) = max\{F_{K_1(e_1)}^-(x_1), F_{K_2(e_2)}^-(x_2)\} \\
(F_{K_1(e_1)}^+ \times F_{K_2(e_2)}^+)(x_1, x_2) = max\{F_{K_1(e_1)}^+(x_1), F_{K_2(e_2)}^+(x_2)\}
\end{cases} \\
\text{for all} \quad (x_1, x_2) \in V_1 \times V_2. \\
\begin{cases}
(T_{M_1(e_1)}^- \times T_{M_2(e_2)}^-)((x, x_2)(x, y_2)) = min\{T_{K_1(e_1)}^-(x), T_{M_2(e_2)}^-(x_2 y_2)\} \\
(T_{M_1(e_1)}^+ \times T_{M_2(e_2)}^+)((x, x_2)(x, y_2)) = min\{T_{K_1(e_1)}^+(x), T_{M_2(e_2)}^+(x_2 y_2)\} \\
(I_{M_1(e_1)}^- \times I_{M_2(e_2)}^-)((x, x_2)(x, y_2)) = max\{I_{K_1(e_1)}^-(x), I_{M_2(e_2)}^-(x_2 y_2)\} \\
(I_{M_1(e_1)}^+ \times I_{M_2(e_2)}^+)((x, x_2)(x, y_2)) = max\{I_{K_1(e_1)}^+(x), I_{M_2(e_2)}^+(x_2 y_2)\} \\
(F_{M_1(e_1)}^- \times F_{M_2(e_2)}^-)((x, x_2)(x, y_2)) = max\{F_{K_1(e_1)}^-(x), F_{M_2(e_2)}^-(x_2 y_2)\} \\
(F_{M_1(e_1)}^+ \times F_{M_2(e_2)}^+)((x, x_2)(x, y_2)) = max\{F_{K_1(e_1)}^+(x), F_{M_2(e_2)}^+(x_2 y_2)\}
\end{cases} \\
\text{for all} \quad x \in V_1 \quad \text{and} \quad (x_2, y_2) \in E_2. \\
\begin{cases}
(T_{M_1(e_1)}^- \times T_{M_2(e_2)}^-)((x_1, z)(y_1, z)) = min\{T_{M_1(e_1)}^-(x_1 y_1), T_{K_2(e_2)}^-(z)\} \\
(T_{M_1(e_1)}^+ \times T_{M_2(e_2)}^+)((x_1, z)(y_1, z)) = min\{T_{M_1(e_1)}^+(x_1 y_1), T_{K_2(e_2)}^+(z)\} \\
(I_{M_1(e_1)}^- \times I_{M_2(e_2)}^-)((x_1, z)(y_1, z)) = max\{I_{M_1(e_1)}^-(x_1 y_1), I_{K_2(e_2)}^-(z)\} \\
(I_{M_1(e_1)}^+ \times I_{M_2(e_2)}^+)((x_1, z)(y_1, z)) = max\{I_{M_1(e_1)}^+(x_1 y_1), I_{K_2(e_2)}^+(z)\} \\
(F_{M_1(e_1)}^- \times F_{M_2(e_2)}^-)((x_1, z)(y_1, z)) = max\{F_{M_1(e_1)}^-(x_1 y_1), F_{K_2(e_2)}^-(z)\} \\
(F_{M_1(e_1)}^+ \times F_{M_2(e_2)}^+)((x_1, z)(y_1, z)) = max\{F_{M_1(e_1)}^+(x_1 y_1), F_{K_2(e_2)}^+(z)\}
\end{cases} \\
\text{for all} \quad z \in V_2 \quad \text{and} \quad (x_1, y_1) \in E_1.
\end{cases}
$$

for all $e_1 \in A$ and $e_2 \in B$.

Example 4. Consider two graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ such that

$$V_1 = \{u_1, u_2\}, E_1 = \{u_1u_2\} \text{ and } V_2 = \{v_1, v_2, v_3\}, E_2 = \{v_1v_2, v_2v_3\}$$

Let $A = \{e_1\}$ be a set of parameters, and let (K_1, A) and (M_1, A) be two interval-valued neutrosophic soft sets over V_1 and E_1, respectively, defined by

$$K_1(e_1) = \{u_1|([0.3,0.5],[0.1,0.2],[0.4,0.6]), u_2|([0.5,0.6],[0.2,0.4],[0.1,0.3])\}$$
$$M_1(e_1) = \{u_1u_2|([0.2,0.3],[0.3,0.5],[0.6,0.7])\}$$

Now let $B = \{e_2\}$ be a set of parameters, and let (K_2, B) and (M_2, B) be two interval-valued neutrosophic soft sets over V_2 and E_2, respectively, defined by

$$K_2(e_2) = \{v_1|([0.4,0.5],[0.3,0.4],[0.6,0.7]), v_2|([0.3,0.6],[0.2,0.3],[0.1,0.4]), v_3|([0.2,0.5],[0.4,0.6],[0.3,0.5])\}$$
$$M_2(e_2) = \{v_1v_2|([0.1,0.2],[0.4,0.5],[0.7,0.8]), v_2v_3|([0.1,0.4],[0.6,0.7],[0.8,0.9])\}$$

It is easy to see that, $H(e_1) = (K_1(e_1), M_1(e_1))$ and $H(e_2) = (K_2(e_2), M_2(e_2))$ are interval-valued neutrosophic graphs. Hence $G_1 = (G_1^*, K_1, M_1, A)$ and $G_2 = (G_2^*, K_2, M_2, B)$ are interval-valued neutrosophic soft graphs of G_1^* and G_2^*, respectively, as shown in Figure 3.
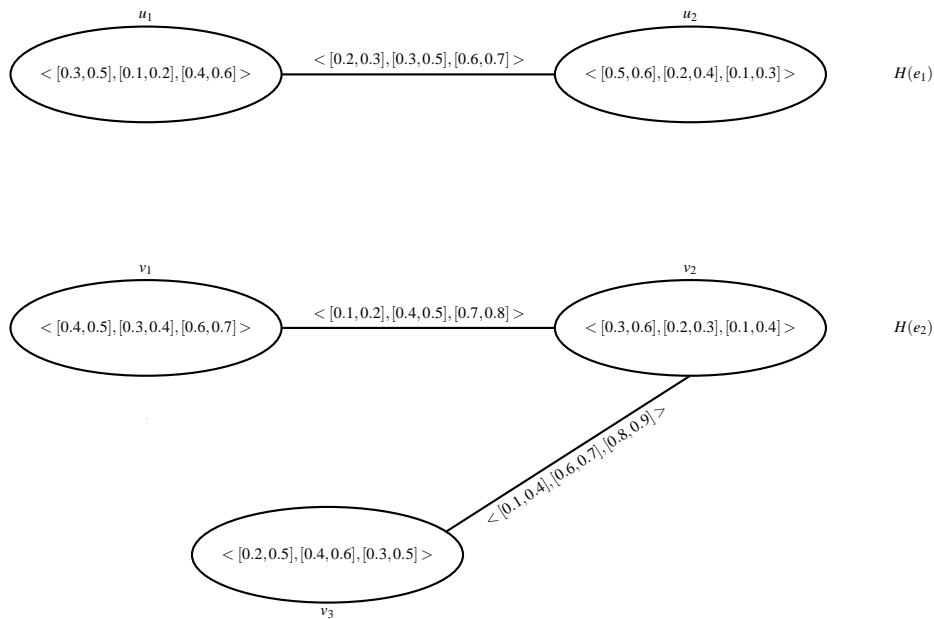

Fig. 3: Interval-valued neutrosophic soft graph G_1 and G_2

The cartesian product of G_1 and G_2 is as shown in Figure 4.

Theorem 1. *If G_1 and G_2 are two interval-valued neutrosophic soft graphs, then so is $G_1 \times G_2$.*

Proof. Let $G_1 = (G_1^*, K_1, M_1, A)$ and $G_2 = (G_2^*, K_2, M_2, B)$ be two interval-valued neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. From Definition 10, for all $e_1 \in A$ and $e_2 \in B$, there are three cases.

Case (i) If $x_1 \in V_1$ and $x_2 \in V_2$, then

$$(T_{K_1(e_1)}^- \times T_{K_2(e_2)}^-)(x_1, x_2) = \min(T_{K_1(e_1)}^-(x_1), T_{K_2(e_2)}^-(x_2)) \leq \min[(T_{K_1(e_1)}^- \times T_{K_2(e_2)}^-)(x_1), (T_{K_1(e_1)}^- \times T_{K_2(e_2)}^-)(x_2)]$$

Similarly, we can show that $(T_{K_1(e_1)}^+ \times T_{K_2(e_2)}^+)(x_1, x_2) \leq \min[(T_{K_1(e_1)}^+ \times T_{K_2(e_2)}^+)(x_1), (T_{K_1(e_1)}^+ \times T_{K_2(e_2)}^+)(x_2)]$

$$(I_{K_1(e_1)}^- \times I_{K_2(e_2)}^-)(x_1, x_2) = \max(I_{K_1(e_1)}^-(x_1), I_{K_2(e_2)}^-(x_2)) \geq \max[(I_{K_1(e_1)}^- \times I_{K_2(e_2)}^-)(x_1), (I_{K_1(e_1)}^- \times I_{K_2(e_2)}^-)(x_2)]$$

Similarly, we can show that $(I_{K_1(e_1)}^+ \times I_{K_2(e_2)}^+)(x_1, x_2) \geq \max[(I_{K_1(e_1)}^+ \times I_{K_2(e_2)}^+)(x_1), (I_{K_1(e_1)}^+ \times I_{K_2(e_2)}^+)(x_2)]$

$$(F_{K_1(e_1)}^- \times F_{K_2(e_2)}^-)(x_1, x_2) = \max(F_{K_1(e_1)}^-(x_1), F_{K_2(e_2)}^-(x_2)) \geq \max[(F_{K_1(e_1)}^- \times F_{K_2(e_2)}^-)(x_1), (F_{K_1(e_1)}^- \times F_{K_2(e_2)}^-)(x_2)]$$
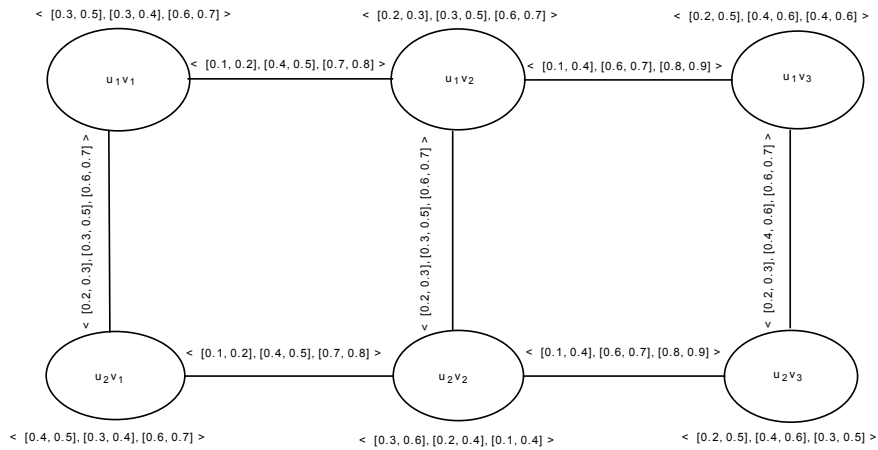
Fig. 4: Cartesian product of G_1 and G_2

Similarly, we can show that $(F_{K_1(e_1)}^+ \times F_{K_2(e_2)}^+)_{(x_1,x_2)} \geq \max[(F_{K_1(e_1)}^+ \times F_{K_2(e_2)}^+)_{(x_1)}, (F_{K_1(e_1)}^+ \times F_{K_2(e_2)}^+)_{(x_2)}]$

Case(ii) If $x \in V_1$ and $(x_2, y_2) \in E_2$, then

$$
\begin{aligned}
(T_{M_1(e_1)}^- \times T_{M_2(e_2)}^-)_{((x,x_2)(x,y_2))} &= \min(T_{K_1(e_1)}^-(x), T_{M_2(e_2)}^-(x_2,y_2)) \\
&\leq \min[T_{K_1(e_1)}^-(x), \min(T_{K_2(e_2)}^-(x_2), T_{K_2(e_2)}^-(y_2))] \\
&= \min[\min(T_{K_1(e_1)}^-(x), T_{K_2(e_2)}^-(x_2)), \min(T_{K_1(e_1)}^-(x), T_{K_2(e_2)}^-(y_2))] \\
&= \min[(T_{K_1(e_1)}^- \times T_{K_2(e_2)}^-)_{(x,x_2)}, (T_{K_1(e_1)}^- \times T_{K_2(e_2)}^-)_{(x,y_2)}]
\end{aligned}
$$

Similarly, we can show that $(T_{M_1(e_1)}^+ \times T_{M_2(e_2)}^+)_{((x,x_2)(x,y_2))} \leq \min[(T_{K_1(e_1)}^+ \times T_{K_2(e_2)}^+)_{(x,x_2)}, (T_{K_1(e_1)}^+ \times T_{K_2(e_2)}^+)_{(x,y_2)}]$

$$
\begin{aligned}
(I_{M_1(e_1)}^- \times I_{M_2(e_2)}^-)_{((x,x_2)(x,y_2))} &= \max(I_{K_1(e_1)}^-(x), I_{M_2(e_2)}^-(x_2,y_2)) \\
&\geq \max[I_{K_1(e_1)}^-(x), \max(I_{K_2(e_2)}^-(x_2), I_{K_2(e_2)}^-(y_2))] \\
&= \max[\max(I_{K_1(e_1)}^-(x), I_{K_2(e_2)}^-(x_2)), \max(I_{K_1(e_1)}^-(x), I_{K_2(e_2)}^-(y_2))] \\
&= \max[(I_{K_1(e_1)}^- \times I_{K_2(e_2)}^-)_{(x,x_2)}, (I_{K_1(e_1)}^- \times I_{K_2(e_2)}^-)_{(x,y_2)}]
\end{aligned}
$$

Similarly, we can show that $(I_{M_1(e_1)}^+ \times I_{M_2(e_2)}^+)_{((x,x_2)(x,y_2))} \geq \max[(I_{K_1(e_1)}^+ \times I_{K_2(e_2)}^+)_{(x,x_2)}, (I_{K_1(e_1)}^+ \times I_{K_2(e_2)}^+)_{(x,y_2)}]$

$$
\begin{aligned}
(F_{M_1(e_1)}^- \times F_{M_2(e_2)}^-)_{((x,x_2)(x,y_2))} &= \max(F_{K_1(e_1)}^-(x), F_{M_2(e_2)}^-(x_2,y_2)) \\
&\geq \max[I_{K_1(e_1)}^-(x), \max(F_{K_2(e_2)}^-(x_2), F_{K_2(e_2)}^-(y_2))] \\
&= \max[\max(F_{K_1(e_1)}^-(x), F_{K_2(e_2)}^-(x_2)), \max(F_{K_1(e_1)}^-(x), F_{K_2(e_2)}^-(y_2))] \\
&= \max[(F_{K_1(e_1)}^- \times F_{K_2(e_2)}^-)_{(x,x_2)}, (F_{K_1(e_1)}^- \times F_{K_2(e_2)}^-)_{(x,y_2)}]
\end{aligned}
$$

Similarly, we can show that $(F_{M_1(e_1)}^+ \times F_{M_2(e_2)}^+)_{((x,x_2)(x,y_2))} \geq \max[(F_{K_1(e_1)}^+ \times F_{K_2(e_2)}^+)_{(x,x_2)}, (F_{K_1(e_1)}^+ \times F_{K_2(e_2)}^+)_{(x,y_2)}]$

Case (iii) If $x \in V_2$ and $(x_1, y_1) \in E_1$, then it can be shown in a similar way to case (ii).

Definition 11. *Let $G_1 = (G_1^*, K_1, M_1, A)$ and $G_2 = (G_2^*, K_2, M_2, B)$ be two interval-valued neutrosophic soft graphs of simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The union of G_1 and G_2 is denoted by $G_1 \cup G_2 = (G^*, K, M, A \cup B)$, where $G^* = (V_1 \cap V_2, E_1 \cap E_2)$, and is defined by*

$$\begin{cases}
\begin{cases}
(T^-_{K_1(e)} \cup T^-_{K_2(e)})(x) = T^-_{K_1(e)}(x) & if \quad e \in A\backslash B \\
(T^-_{K_1(e)} \cup T^-_{K_2(e)})(x) = T^-_{K_2(e)}(x) & if \quad e \in B\backslash A \\
(T^-_{K_1(e)} \cup T^-_{K_2(e)})(x) = max(T^-_{K_1(e)}(x), T^-_{K_2(e)}(x)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(T^+_{K_1(e)} \cup T^+_{K_2(e)})(x) = T^+_{K_1(e)}(x) & if \quad e \in A\backslash B \\
(T^+_{K_1(e)} \cup T^+_{K_2(e)})(x) = T^+_{K_2(e)}(x) & if \quad e \in B\backslash A \\
(T^+_{K_1(e)} \cup T^+_{K_2(e)})(x) = max(T^+_{K_1(e)}(x), T^+_{K_2(e)}(x)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(I^-_{K_1(e)} \cup I^-_{K_2(e)})(x) = I^-_{K_1(e)}(x) & if \quad e \in A\backslash B \\
(I^-_{K_1(e)} \cup I^-_{K_2(e)})(x) = I^-_{K_2(e)}(x) & if \quad e \in B\backslash A \\
(I^-_{K_1(e)} \cup I^-_{K_2(e)})(x) = min(I^-_{K_1(e)}(x), I^-_{K_2(e)}(x)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(I^+_{K_1(e)} \cup I^+_{K_2(e)})(x) = I^+_{K_1(e)}(x) & if \quad e \in A\backslash B \\
(I^+_{K_1(e)} \cup I^+_{K_2(e)})(x) = I^+_{K_2(e)}(x) & if \quad e \in B\backslash A \\
(I^+_{K_1(e)} \cup I^+_{K_2(e)})(x) = min(I^+_{K_1(e)}(x), I^+_{K_2(e)}(x)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(F^-_{K_1(e)} \cup F^-_{K_2(e)})(x) = F^-_{K_1(e)}(x) & if \quad e \in A\backslash B \\
(F^-_{K_1(e)} \cup F^-_{K_2(e)})(x) = F^-_{K_2(e)}(x) & if \quad e \in B\backslash A \\
(F^-_{K_1(e)} \cup F^-_{K_2(e)})(x) = min(F^-_{K_1(e)}(x), F^-_{K_2(e)}(x)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(F^+_{K_1(e)} \cup F^+_{K_2(e)})(x) = F^+_{K_1(e)}(x) & if \quad e \in A\backslash B \\
(F^+_{K_1(e)} \cup F^+_{K_2(e)})(x) = F^+_{K_2(e)}(x) & if \quad e \in B\backslash A \\
(F^+_{K_1(e)} \cup F^+_{K_2(e)})(x) = min(F^+_{K_1(e)}(x), F^+_{K_2(e)}(x)) & if \quad e \in A\cap B
\end{cases} \\
for\ all \quad x \in V. \\
\begin{cases}
(T^-_{M_1(e)} \cup T^-_{M_2(e)})(x,y) = T^-_{M_1(e)}(x,y) & if \quad e \in A\backslash B \\
(T^-_{M_1(e)} \cup T^-_{M_2(e)})(x,y) = T^-_{M_2(e)}(x,y) & if \quad e \in B\backslash A \\
(T^-_{M_1(e)} \cup T^-_{M_2(e)})(x,y) = max(T^-_{M_1(e)}(x,y), T^-_{M_2(e)}(x,y)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(T^+_{M_1(e)} \cup T^+_{M_2(e)})(x,y) = T^+_{M_1(e)}(x,y) & if \quad e \in A\backslash B \\
(T^+_{M_1(e)} \cup T^+_{M_2(e)})(x,y) = T^+_{M_2(e)}(x,y) & if \quad e \in B\backslash A \\
(T^+_{M_1(e)} \cup T^+_{M_2(e)})(x,y) = max(T^+_{M_1(e)}(x,y), T^+_{M_2(e)}(x,y)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(I^-_{M_1(e)} \cup I^-_{M_2(e)})(x,y) = I^-_{M_1(e)}(x,y) & if \quad e \in A\backslash B \\
(I^-_{M_1(e)} \cup I^-_{M_2(e)})(x,y) = I^-_{M_2(e)}(x,y) & if \quad e \in B\backslash A \\
(I^-_{M_1(e)} \cup I^-_{M_2(e)})(x,y) = min(I^-_{M_1(e)}(x,y), I^-_{M_2(e)}(x,y)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(I^+_{M_1(e)} \cup I^+_{M_2(e)})(x,y) = I^+_{M_1(e)}(x,y) & if \quad e \in A\backslash B \\
(I^+_{M_1(e)} \cup I^+_{M_2(e)})(x,y) = I^+_{M_2(e)}(x,y) & if \quad e \in B\backslash A \\
(I^+_{M_1(e)} \cup I^+_{M_2(e)})(x,y) = min(I^+_{M_1(e)}(x,y), I^+_{M_2(e)}(x,y)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(F^-_{M_1(e)} \cup F^-_{M_2(e)})(x,y) = F^-_{M_1(e)}(x,y) & if \quad e \in A\backslash B \\
(F^-_{M_1(e)} \cup F^-_{M_2(e)})(x,y) = F^-_{M_2(e)}(x,y) & if \quad e \in B\backslash A \\
(F^-_{M_1(e)} \cup F^-_{M_2(e)})(x,y) = min(F^-_{M_1(e)}(x,y), F^-_{M_2(e)}(x,y)) & if \quad e \in A\cap B
\end{cases} \\
\begin{cases}
(F^+_{M_1(e)} \cup F^+_{M_2(e)})(x,y) = F^+_{M_1(e)}(x,y) & if \quad e \in A\backslash B \\
(F^+_{M_1(e)} \cup F^+_{M_2(e)})(x,y) = F^+_{M_2(e)}(x,y) & if \quad e \in B\backslash A \\
(F^+_{M_1(e)} \cup F^+_{M_2(e)})(x,y) = min(F^+_{M_1(e)}(x,y), F^+_{M_2(e)}(x,y)) & if \quad e \in A\cap B
\end{cases} \\
for\ all \quad (x,y) \in E.
\end{cases}$$

Example 5. Consider two graphs $G^*_1 = (V_1, E_1)$ and $G^*_2 = (V_2, E_2)$ such that $V_1 = \{u_1, u_2, u_3\}$, $E_1 = \{u_1u_2, u_1u_3, u_2u_3\}$ and $V_2 = \{v_1, v_2\}$, $E_2 = \{u_1, u_3, v_1v_2\}$. Let $A = \{e_1, e_2\}$ be a set of parameters and let (K_1, A) and (M_1, A) be two interval-valued

neutrosophic soft sets over V_1 and E_1 respectively, defined by

$K_1(e_1) = \{u_1|([0.4,0.6],[0.2,0.3],[0.1,0.3]),u_2|([0.4,0.7],[0.2,0.4],[0.1,0.3])\}$

$M_1(e_1) = \{u_1u_2|([0.3,0.4],[0.4,0.5],[0.3,0.5])\}$

$K_1(e_2) = \{u_1|([0.3,0.5],[0.2,0.3],[0.3,0.4]),u_2|([0.2,0.3],[0.2,0.3],[0.1,0.4]),u_3|([0.1,0.3],[0.2,0.4],[0.3,0.5])\}$

$M_1(e_2) = \{u_1u_3|([0.1,0.2],[0.3,0.5],[0.4,0.6]),u_2u_3|([0.1,0.3],[0.4,0.5],[0.4,0.5])\}$

Now let $B = \{e_2, e_3\}$ be a set of parameters and let (K_2, A) and (M_2, A) be two interval-valued neutrosophic soft sets over V_2 and E_2 respectively, defined by

$$K_2(e_2) = \{v_1|([0.2,0.3],[0.1,0.2],[0.3,0.5]),v_2|([0.2,0.4],[0.1,0.3],[0.2,0.5])\}$$

$$M_2(e_2) = \{v_1v_2|([0.1,0.2],[0.4,0.5],[0.4,0.6])\}$$

$$K_2(e_3) = \{v_1|([0.3,0.5],[0.2,0.3],[0.3,0.4]),v_2|([0.2,0.3],[0.1,0.3],[0.4,0.5]\}$$

$$M_2(e_3) = \{v_1v_2|([0.1,0.2],[0.3,0.4],[0.4,0.5]\}$$

It is easy to see that $H_1(e_1) = (K_1(e_1), M_1(e_1))$, $H_1(e_2) = (K_1(e_2), M_1(e_2))$, $H_2(e_2) = (K_2(e_2), M_2(e_2))$ and $H_2(e_3) = (K_2(e_3), M_2(e_3))$ are interval valued neutrosophic graphs. Hence $G_1 = (G_1^*, K_1, M_1, A)$ and $G_2 = (G_2^*, K_2, M_2, B)$ are interval-valued neutrosophic soft graphs of G_1^* and G_2^* respectively as shown in Figure 5 and Figure 6. From Definition 12, $G_1 \cup G_2 = (G^*, K, M, A \cup B)$, where $A \cup B = \{e_1, e_2, e_3\}$. Also $H(e_1) = H_1(e_1)$,
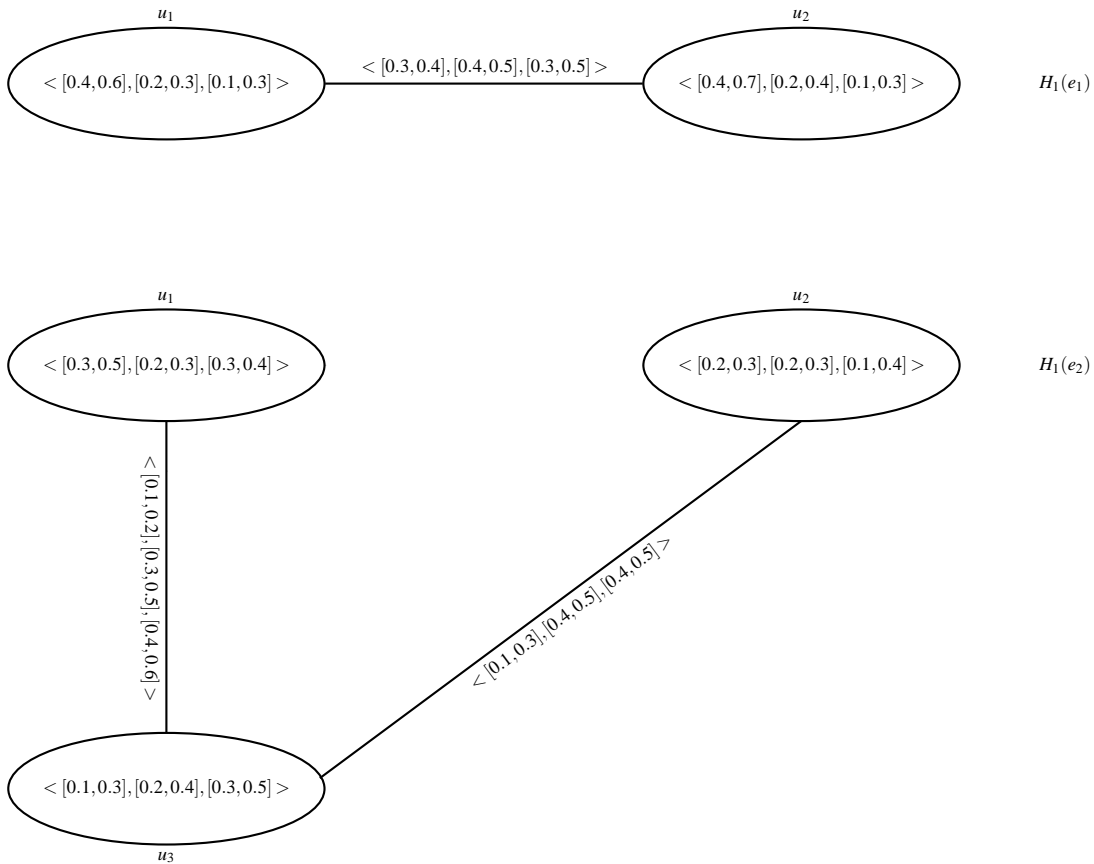

Fig. 5: Interval-valued neutrosophic soft graph G_1

$H(e_2) = H_1(e_2) \cup H_2(e_2)$, and $H(e_3) = H_2(e_3)$ are interval-valued neutrosophic graphs corresponding to the parameters
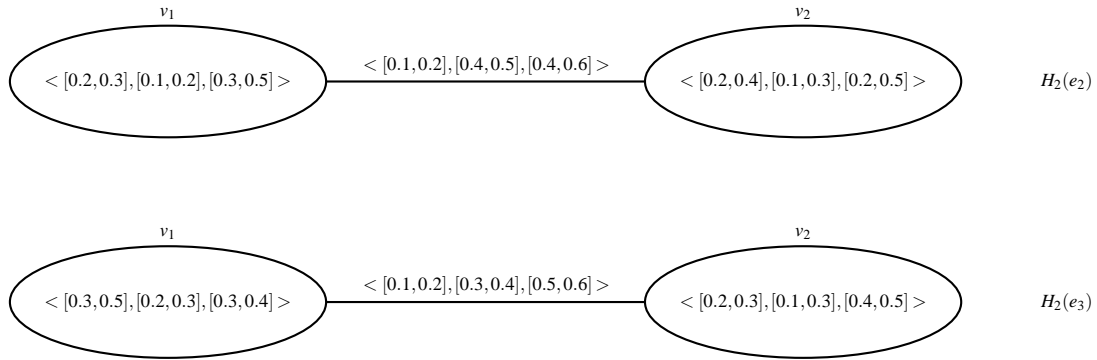
v_1

$< [0.2, 0.3], [0.1, 0.2], [0.3, 0.5] >$ $< [0.1, 0.2], [0.4, 0.5], [0.4, 0.6] >$ v_2 $< [0.2, 0.4], [0.1, 0.3], [0.2, 0.5] >$ $H_2(e_2)$

v_1

$< [0.3, 0.5], [0.2, 0.3], [0.3, 0.4] >$ $< [0.1, 0.2], [0.3, 0.4], [0.5, 0.6] >$ v_2 $< [0.2, 0.3], [0.1, 0.3], [0.4, 0.5] >$ $H_2(e_3)$

Fig. 6: Interval-valued neutrosophic soft graph G_2

e_1, e_2 and e_3. Hence, the union of G_1 and G_2 corresponding to the parameter e_2 is interval-valued neutrosophic soft graph as shown in Figure 7. The figures of $H(e_1)$ and $H(e_2)$ can be drawn similarly.
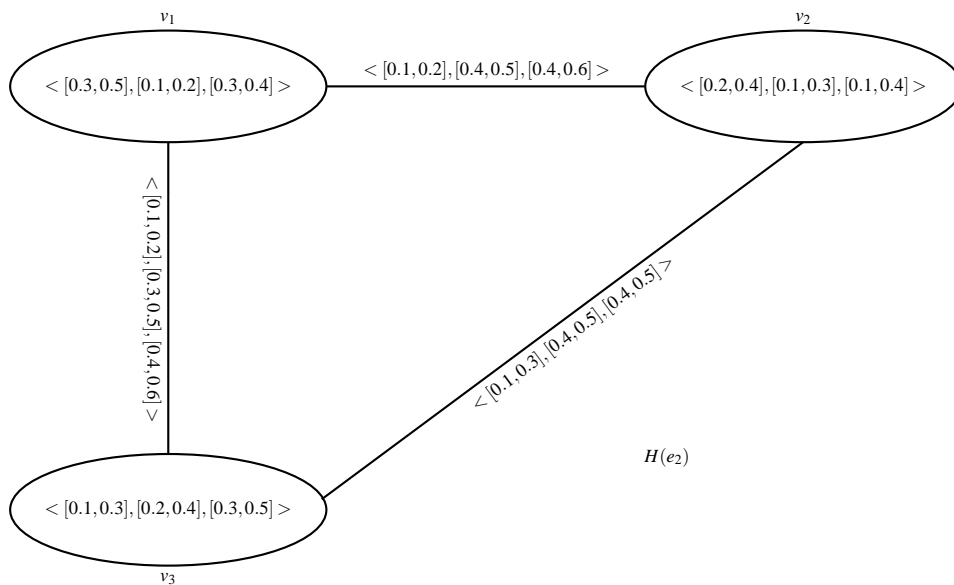

v_1 v_2

$< [0.3, 0.5], [0.1, 0.2], [0.3, 0.4] >$ $< [0.1, 0.2], [0.4, 0.5], [0.4, 0.6] >$ $< [0.2, 0.4], [0.1, 0.3], [0.1, 0.4] >$

$< [0.1, 0.2], [0.3, 0.5], [0.4, 0.6] >$

$< [0.1, 0.3], [0.4, 0.5], [0.4, 0.5] >$

$H(e_2)$

$< [0.1, 0.3], [0.2, 0.4], [0.3, 0.5] >$

v_3

Fig. 7: The union of G_1 and G_2 corresponding to the parameter e_2

Theorem 2. *If G_1 and G_2 are two interval-valued neutrosophic soft graphs, then so is $G_1 \cup G_2$.*

Proof. By using Definition 3.10, it can be shown in a similar way to proof of Theorem 1.

Definition 12. *Let $G_1 = (G_1^*, K_1, M_1, A)$ and $G_2 = (G_2^*, K_2, M_2, B)$ be two interval-valued neutrosophic soft graphs of simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The intersection of G_1 and G_2 is denoted by $G_1 \cap G_2 =$*

$(G^*, K, M, A \cup B)$, *where* $G^* = (V_1 \cap V_2, E_1 \cap E_2)$, *and is defined by*

$$
\begin{cases}
\begin{cases}
(T^-_{K_1(e)} \cap T^-_{K_2(e)})(x) = T^-_{K_1(e)}(x) & if \quad e \in A \backslash B \\
(T^-_{K_1(e)} \cap T^-_{K_2(e)})(x) = T^-_{K_2(e)}(x) & if \quad e \in B \backslash A \\
(T^-_{K_1(e)} \cap T^-_{K_2(e)})(x) = min(T^-_{K_1(e)}(x), T^-_{K_2(e)}(x)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(T^+_{K_1(e)} \cap T^+_{K_2(e)})(x) = T^+_{K_1(e)}(x) & if \quad e \in A \backslash B \\
(T^+_{K_1(e)} \cap T^+_{K_2(e)})(x) = T^+_{K_2(e)}(x) & if \quad e \in B \backslash A \\
(T^+_{K_1(e)} \cap T^+_{K_2(e)})(x) = min(T^+_{K_1(e)}(x), T^+_{K_2(e)}(x)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(I^-_{K_1(e)} \cap I^-_{K_2(e)})(x) = I^-_{K_1(e)}(x) & if \quad e \in A \backslash B \\
(I^-_{K_1(e)} \cap I^-_{K_2(e)})(x) = I^-_{K_2(e)}(x) & if \quad e \in B \backslash A \\
(I^-_{K_1(e)} \cap I^-_{K_2(e)})(x) = max(I^-_{K_1(e)}(x), I^-_{K_2(e)}(x)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(I^+_{K_1(e)} \cap I^+_{K_2(e)})(x) = I^+_{K_1(e)}(x) & if \quad e \in A \backslash B \\
(I^+_{K_1(e)} \cap I^+_{K_2(e)})(x) = I^+_{K_2(e)}(x) & if \quad e \in B \backslash A \\
(I^+_{K_1(e)} \cap I^+_{K_2(e)})(x) = max(I^+_{K_1(e)}(x), I^+_{K_2(e)}(x)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(F^-_{K_1(e)} \cap F^-_{K_2(e)})(x) = F^-_{K_1(e)}(x) & if \quad e \in A \backslash B \\
(F^-_{K_1(e)} \cap F^-_{K_2(e)})(x) = F^-_{K_2(e)}(x) & if \quad e \in B \backslash A \\
(F^-_{K_1(e)} \cap F^-_{K_2(e)})(x) = max(F^-_{K_1(e)}(x), F^-_{K_2(e)}(x)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(F^+_{K_1(e)} \cap F^+_{K_2(e)})(x) = F^+_{K_1(e)}(x) & if \quad e \in A \backslash B \\
(F^+_{K_1(e)} \cap F^+_{K_2(e)})(x) = F^+_{K_2(e)}(x) & if \quad e \in B \backslash A \\
(F^+_{K_1(e)} \cap F^+_{K_2(e)})(x) = max(F^+_{K_1(e)}(x), F^+_{K_2(e)}(x)) & if \quad e \in A \cap B
\end{cases} \\
for \ all \quad x \in V. \\
\begin{cases}
(T^-_{M_1(e)} \cap T^-_{M_2(e)})(x,y) = T^-_{M_1(e)}(x,y) & if \quad e \in A \backslash B \\
(T^-_{M_1(e)} \cap T^-_{M_2(e)})(x,y) = T^-_{M_2(e)}(x,y) & if \quad e \in B \backslash A \\
(T^-_{M_1(e)} \cap T^-_{M_2(e)})(x,y) = min(T^-_{M_1(e)}(x,y), T^-_{M_2(e)}(x,y)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(T^+_{M_1(e)} \cap T^+_{M_2(e)})(x,y) = T^+_{M_1(e)}(x,y) & if \quad e \in A \backslash B \\
(T^+_{M_1(e)} \cap T^+_{M_2(e)})(x,y) = T^+_{M_2(e)}(x,y) & if \quad e \in B \backslash A \\
(T^+_{M_1(e)} \cap T^+_{M_2(e)})(x,y) = min(T^+_{M_1(e)}(x,y), T^+_{M_2(e)}(x,y)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(I^-_{M_1(e)} \cap I^-_{M_2(e)})(x,y) = I^-_{M_1(e)}(x,y) & if \quad e \in A \backslash B \\
(I^-_{M_1(e)} \cap I^-_{M_2(e)})(x,y) = I^-_{M_2(e)}(x,y) & if \quad e \in B \backslash A \\
(I^-_{M_1(e)} \cap I^-_{M_2(e)})(x,y) = max(I^-_{M_1(e)}(x,y), I^-_{M_2(e)}(x,y)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(I^+_{M_1(e)} \cap I^+_{M_2(e)})(x,y) = I^+_{M_1(e)}(x,y) & if \quad e \in A \backslash B \\
(I^+_{M_1(e)} \cap I^+_{M_2(e)})(x,y) = I^+_{M_2(e)}(x,y) & if \quad e \in B \backslash A \\
(I^+_{M_1(e)} \cap I^+_{M_2(e)})(x,y) = max(I^+_{M_1(e)}(x,y), I^+_{M_2(e)}(x,y)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(F^-_{M_1(e)} \cap F^-_{M_2(e)})(x,y) = F^-_{M_1(e)}(x,y) & if \quad e \in A \backslash B \\
(F^-_{M_1(e)} \cap F^-_{M_2(e)})(x,y) = F^-_{M_2(e)}(x,y) & if \quad e \in B \backslash A \\
(F^-_{M_1(e)} \cap F^-_{M_2(e)})(x,y) = max(F^-_{M_1(e)}(x,y), F^-_{M_2(e)}(x,y)) & if \quad e \in A \cap B
\end{cases} \\
\begin{cases}
(F^+_{M_1(e)} \cap F^+_{M_2(e)})(x,y) = F^+_{M_1(e)}(x,y) & if \quad e \in A \backslash B \\
(F^+_{M_1(e)} \cap F^+_{M_2(e)})(x,y) = F^+_{M_2(e)}(x,y) & if \quad e \in B \backslash A \\
(F^+_{M_1(e)} \cap F^+_{M_2(e)})(x,y) = max(F^+_{M_1(e)}(x,y), F^+_{M_2(e)}(x,y)) & if \quad e \in A \cap B
\end{cases} \\
for \ all \quad (x,y) \in E.
\end{cases}
$$

Example 6. Let consider the Example 5. From Definition 11, $G_1 \cap G_2 = (G^*, K, M, A \cup B)$, where $A \cup B = \{e_1, e_2, e_3\}$. Also $H(e_1) = H_1(e_1)$, $H(e_2) = H_1(e_2) \cap H_2(e_2)$, and $H(e_3) = H_2(e_3)$ are interval-valued neutrosophic graphs corresponding to the parameters e_1, e_2 and e_3. Hence, the intersection of G_1 and G_2 corresponding to the parameter e_2 is interval-valued neutrosophic soft graph as shown in Figure 8. The figures of $H(e_1)$ and $H(e_2)$ can be drawn similarly.

$$H(e_2) = H_1(e_2) \cap H_2(e_2) = \{v_1|([0.2, 0.3], [0.2, 0.3], [0.3, 0.5]), v_2|([0.2, 0.3], [0.2, 0.3], [0.2, 0.5])\}.$$
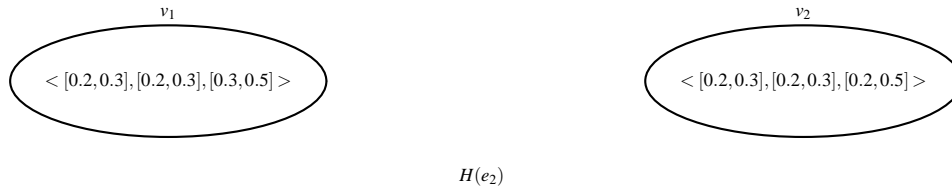
v_1

$< [0.2, 0.3], [0.2, 0.3], [0.3, 0.5] >$

v_2

$< [0.2, 0.3], [0.2, 0.3], [0.2, 0.5] >$

$H(e_2)$

Fig. 8: The intersection of G_1 and G_2 corresponding to the parameter e_2

Theorem 3. *If G_1 and G_2 are two interval-valued neutrosophic soft graphs, then so is $G_1 \cap G_2$.*

Proof. By using Definition 12, it can be shown in a similar way to proof of Theorem 1.

4 Conclusion

Graph theory is an extremely useful mathematical tool to solve the complicated problems in different fields. The interval-valued neutrosophic soft sets constitute a generalization of interval-valued fuzzy soft set theory. The interval-valued neutrosophic soft models give more sensitive, flexibility and conformity to the systems as compared to the interval-valued fuzzy soft models. We applied the concept of interval-valued neutrosophic soft sets to graph structures and describe method of their construction. We also defined cartesian product, union and intersection on interval-valued neutrosophic soft graphs and gave some of their properties. We want to make, in near future, some algorithm and models using these results.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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