On the eccentric adjacency index of graphs

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Abstract: In this paper, we present some bounds for the eccentric adjacency index. Also we will calculate the eccentric adjacency index of some thorny graphs.

Keywords: Eccentric-adjacency index, Zagreb indices, Thorny graphs.

1 Introduction

Let G be a n vertices graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex u in a graph G, denoted by $d(u)$, is the number of proper edges incident on u. A graph in which every vertex has degree k is k regular.

The distance between two vertices in a graph is the length of the shortest walk between them. The eccentricity of a vertex u in a graph G, denoted by $e_G(u)$, is the maximum distance from u to any vertex. The radius of a connected graph is the minimum eccentricity. The diameter of a connected graph is the maximum eccentricity. Diameter and radius of a graph is denoted by $D = D(G)$ and $r = r(G)$, respectively. The total eccentricity [1] of a graph is denoted by $\zeta(G)$ and is equal to sum of eccentricities of all the vertices of the graph. The total eccentricity is

$$\zeta(G) = \sum_{u \in V(G)} e_G(u),$$

where $e_G(u)$ is the eccentricity of the vertex u. A central vertex in a graph is a vertex whose eccentricity equals the radius of the graph. A graph in which every vertex has same eccentricity is self centered graph. A pendant vertex is a vertex of degree one.

As usual, star graph, complete graph, compete bipartite graph and cycle on n vertices is denoted S_n, K_n, $K_{m,n}$ and C_n, respectively. The minimum vertex degree and the maximum vertex degree in a graph is denoted by Δ and δ, respectively. For other undefined notations and terminology from graph theory, the readers are referred to [7].

Gutman and Trinajstic[9,10] introduced Zagreb indices. The first and second Zagreb indices of G are denoted by $M_1(G)$ and $M_2(G)$, respectively and defined as:

$$M_1(G) = \sum_{u \in V(G)} d_G^2(u) \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v).$$

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The eccentric connectivity index of a graph G is defined as [19]

$$\xi^c(G) = \sum_{u \in V(G)} d(u)e(u).$$

For more results of eccentric connectivity index, we refer our reader to [16,19,22].

In 2001, Gupta et al.[8] proposed a topological descriptor that adjacency-cut distance based. Eccentric adjacency index (also known as Ediz eccentric connectivity index see [4]) is denoted as $\xi^{ad}(G)$ and is defined as :

$$\xi^{ad}(G) = \sum_{u \in V(G)} \frac{S(u)}{e(u)},$$

where S_u is sum degrees of vertices adjacent to vertex u. Gupta et al.[8] investigated the power of eccentric adjacent index in the estimation of anti-HIV activity. The accuracy of prediction $\xi^{ad}(G)$ is more than ninety percent. Thus $\xi^{ad}(G)$ proposes a vast potential for QSAR/QSPR studies.

Ediz [4] investigated some mathematical properties of Ediz eccentric conenctivity index. Sharafdini et al.[18] investigated the eccentric adjacency index of several infinite class of fullerenes. Wu et al.[21] studied the Ediz eccentric connectivity index of some molecular structures. Farahani [5,6] determined Ediz eccentric connectivity index of molecular graph circumcoronene series of benzenoid H_k. Malik [14] computed eccentric adjacent index of the join and corona products of graphs.

In this paper we propose to investigate some mathematical properties of this novel connectivity index. In this paper, we will give some bounds for the eccentric-adjacency index. Also we will present eccentric-adjacency index of thorn graphs obtained some well-defined graphs.

2 Main results

In this section, we give some bounds for the eccentric-adjacency index. After that we will obtain eccentric-adjacency indices of thorny star graphs, complete graphs, bipartite complete graphs and cycles.

We first give the following lemma which will be used in this paper.

Lemma 1. *Let G be a graph. Then*

$$\sum_{u \in V(G)} S(u) = M_1(G).$$

Proof. When calculating this sum, we can see that each of the degree of $u \in V(G)$ vertex is calculated in this sum up to degree of its. So we get the following the result.

$$\sum_{u \in V(G)} S(u) = \sum_{u \in V(G)} d(u).d(u) = \sum_{u \in V(G)} d^2(u) = M_1(G).$$

Theorem 1. *We have for any graph*

$$\frac{M_1(G)}{D} \leq \xi^{ad}(G) \leq \frac{M_1(G)}{r}. \tag{1}$$

Furthermore, the equality in each inequality is attained if and only if G is self centered graph.

Proof. Since G be a simple connected graph of order n, diameter D and radius r, then By Lemma 1 and the fact $r \leq e(u) \leq D$, we have

$$\xi^{ad}(G) = \sum_{u \in V(G)} \frac{S(u)}{e(u)} \leq \sum_{u \in V(G)} \frac{S(u)}{r} = \frac{1}{r} \sum_{u \in V(G)} S(u) = \frac{M_1(G)}{r}.$$

Now suppose that equality holds in (1). Thus we have $e(u) = r$ for any $u \in V(G)$. Hence the equality holds if and only if G is self centered graph. Similarly, we get

$$\xi^{ad}(G) \geq \frac{M_1(G)}{D}.$$

The equality holds if and only if G is self centered graph.

Lemma 2. [12] *Let G be a graph with n vertices and m edges. Then*

$$M_1(\bar{G}) = M_1(G) + n(n-1)^2 - 4m(n-1).$$

Corollary 1. *Let G be a connected graph with $n \geq 4$ vertices for which the complement $\bar{G}$ is also connected. Then*

$$\xi^{ad}(G) + \xi^{ad}(\bar{G}) \leq M_1(G) + \frac{n(n-1)^2}{2} - 2m(n-1),$$

with equality holds if and only if G and $\bar{G}$ are self centered graphs with radius two.

Proof. Let $M_1(G)$ and $M_1(\bar{G})$ be the first Zagreb indices of G and $\bar{G}$,respectively . Since both G and $\bar{G}$ are connected, each has radius at least two, and then by Lemma 2 and Theorem 1,

$$\xi^{ad}(G) + \xi^{ad}(\bar{G}) \leq \frac{M_1(G) + M_1(\bar{G})}{2} = M_1(G) + \frac{n(n-1)^2}{2} - 2m(n-1),$$

with equality holds if and only both if G and $\bar{G}$ are self centered, and $r(G) = r(\bar{G}) = 2$.

Remark. By some basic elementary calculations, one may see that the bounds (1) are better than the results [4, Theo. 1 and Theo. 2].

Lemma 3. *Radon Inequality)[17]. For every real numbers $p > 0, x_k \geq 0, a_k > 0,$ for $1 \leq k \leq n$, the following inequality holds true:*

$$\sum_{k=1}^{n} \frac{x_k^{p+1}}{a_k^p} \geq \frac{(\sum_{k=1}^{n} x_k)^{p+1}}{(\sum_{k=1}^{n} a_k)^p}. \tag{2}$$

The equality holds if and only if $\dfrac{x_1}{a_1} = \dfrac{x_2}{a_2} = \cdots = \dfrac{x_n}{a_n}.$

Theorem 2. *We have for any graph G*

$$\xi^{ad}(G) \geq \frac{M_1(G) + \delta^2 n(n-1)}{\zeta(G)}, \tag{3}$$

and the equality is attained if and only if G is a regular self centered graph.

Proof. Using (2) we get

$$\xi^{ad}(G) = \sum_{u \in V(G)} \frac{(\sqrt{S(u)})^2}{e(u)} \geq \frac{(\sum_{u \in V(G)} \sqrt{S(u)})^2}{\sum_{u \in V(G)} e(u)} = \frac{M_1(G) + 2 \sum_{1 \leq i < j \leq n} \sqrt{S(i)S(j)}}{\zeta(G)}.$$

Since any for $u \in V(G)$ $S(u) \geq \delta^2$, we have

$$\xi^{ad}(G) \geq \frac{M_1(G) + \delta^2 n(n-1)}{\zeta(G)}$$

By Lemma 2, In (3) equality holds if and only if G is a regular self centered graph.

Remark. By some basic elementary calculations, one may see that the bounds (3) are better than the results [4, Theo. 2].

In theorem 2, by applying $e(u) \leq n - d(u)$ [22] for all of $u \in V(G)$ we get the following result.

Corollary 2. *et G be a simple connected graph with n vertices and δ minimum degrees. Then*

$$\xi^{ad}(G) \geq \frac{M_1(G) + \delta^2 n(n-1)}{n^2 - 2m},$$

with equality holds if and only if G is a regular self centered graph.

Lemma 4. *(Chebyshev's inequality)[13] Let $a_1 \geq a_2 \geq \cdots a_n$ and $b_1 \geq b_2 \geq \cdots b_n$ be real numbers. Then*

$$n \sum_{i=1}^{n} a_i b_i \geq (\sum_{i=1}^{n} a_i)(\sum_{i=1}^{n} b_i),$$

with equality holding if and only if $a_1 = a_2 = \cdots = a_n$ and $b_1 = b_2 = \cdots = b_n$.

Theorem 3. *For any connected graph G on n vertices*

$$\xi^{ad}(G) \geq \frac{nM_1(G)}{\zeta(G)}. \tag{4}$$

The equality holds if and only if G regular self centered graph.

Proof. By Lemma 4, for $i = 1, 2, \cdots, n$ by taking $a_i = S(i)$ and $b_i = e(i)$ we get

$$n \sum_{i=1}^{n} \frac{S(i)}{e(i)} \geq (\sum_{i=1}^{n} S(i))(\sum_{i=1}^{n} \frac{1}{e(i)}). \tag{5}$$

Also by aritmetic-harmonic mean inequality, we have

$$\sum_{i=1}^{n} \frac{1}{e(i)} \geq \frac{n^2}{\sum\limits_{i=1}^{n} e(i)} = \frac{n^2}{\zeta(G)}. \tag{6}$$

As a result by (5) and (6), we have

$$\xi^{ad}(G) \geq \frac{nM_1(G)}{\zeta(G)},$$

with equality holds if and only if G regular self centered graph.

Corollary 3. *Let G be a connected graph of order n. We have $e(u) \leq n - d_{uv}$ for $u \in V(G)$. Then by Theorem 3 we get*

$$\xi^{ad}(G) \geq \frac{nM_1(G)}{n^2 - 2m}.$$

Remark. By some basic elementary calculations, one may see that the bounds in (4) are better than the results [4, Theo. 3]. Also, if *G* regular self centered graph, (4) and [4, Theorem 3] bounds are the same.

Proposition 1. *Let W_n and B_n denote the graphs of the pyramid and the bipyramid with n-gonal base, $n \geq 3$. Then*

$$\xi^{ad}(W_n) = \frac{n^2 + 12n}{2} \quad and \quad \xi^{ad}(B_n) = n^2 + 8n.$$

Let $K_n - ke$ be the graph obtained from K_n by deleting k independent edges for $0 \leq k \leq \lfloor \frac{n}{2} \rfloor$.

Theorem 4. *Let G be a connected graph.. Let k be the number of vertices with eccentricity 1 in graph G. Then*

$$\xi^{ad}(G) \leq \frac{M_1(G) + k(2m - n + 1)}{2},$$

with equality if and only if $G \cong K_k \vee (K_{n-k} - \frac{n-k}{2}e)$, where $n - k$ is even.

Proof. $T = \{v_0, v_1, ..., v_k\}$ be the set of vertices with eccentricity 1. Then $e(u) \geq 2$, $d(u) \leq n - 2$ for any $u \in V(G) \setminus T$. By the definition eccentric-adjacency index, we have

$$\xi^{ad}(G) = \sum_{v_i \in T} \frac{S(v_i)}{e(i)} + \sum_{u \in V(G) \setminus T} \frac{S(u)}{e(u)} \leq [2m - (n-1)]k + \frac{\sum\limits_{u \in V(G) \setminus T} S(u)}{2}.$$

Since $\sum\limits_{u \in V(G) \setminus T} S(u) + \sum\limits_{v_i \in T} S(v_i) = \sum\limits_{u \in V(G)} S(u)$, we have $\sum\limits_{u \in V(G) \setminus T} S(u) = M_1(G) - \sum\limits_{v_i \in T} S(v_i)$. So we get

$$\xi^{ad}(G) \leq \frac{M_1(G) + k(2m - n + 1)}{2}.$$

The above equality holds if and only if $G \cong K_k \vee (K_{n-k} - \frac{n-k}{2}e)$, where $n - k$ is even.

Theorem 5. *Let G be a simple connected graph on n vertices with m edges. Let*

$$a = \lfloor \frac{2n - 1 - \sqrt{(2n-1)^2 - 8m}}{2} \rfloor$$

be the largest integer satisfying that $x^2 - (2n - 1)x + 2m \geq 0$. Then

$$\xi^{ad}(G) \leq \frac{M_1(G) + a(2m - n + 1)}{2},$$

with equality if and only if $G \cong K_a \vee (K_{n-a} - \frac{n-a}{2}e)$, where $n - a$ is even.

Proof. Let be k is the number of vertices with eccentricity 1 in graph G. Then degree of these vertices be $n - 1$. Since $2m = \sum_{u \in V(G)} \geq k(n-1) + k(n-k)$, so implies that $k \leq a$. Therefore by Theorem 4, we get

$$\xi^{ad}(G) \leq \frac{M_1(G) + k(2m - n + 1)}{2} \leq \frac{M_1(G) + a(2m - n + 1)}{2},$$

with equality if and only if $G \cong K_a \vee (K_{n-a} - \frac{n-a}{2}e)$, where $n - a$ is even.

Theorem 6. *Let G be a simple connected graph. Then*

$$E(G) \leq M_1(G) \tag{7}$$

The equality holds if and only if G is complete graph with n vertices.

Proof. We have

$$\xi^{ad}(G) = \sum_{u \in V(G)} \frac{S(u)}{e(u)} \leq \sum_{u \in V(G)} S(u) = M_1(G).$$

Suppose that equality holds in (7). Then we have $e(v) = r$ for any $v \in V(G)$. Hence we get $G \cong K_n$. Conversely, if $G \cong K_n$, we have $E(K_n) = M_1(K_n) = n(n-1)^2$. Proof is completed.

Theorem 7. *Let G be a simple connected graph. Then*

$$\xi^{ad}(G) \geq \frac{kn^2}{\zeta(G)},$$

where $k = min(S(u), u \in V(G))$.

Proof. We have

$$\xi^{ad}(G) = \sum_{u \in V(G)} \frac{S(u)}{e(u)} \geq \sum_{u \in V(G)} \frac{k}{e(u)}. \tag{8}$$

Also by aritmetic-harmonic mean inequality, we get

$$\sum_{u \in V(G)} \frac{1}{e(u)} \geq \frac{n^2}{\sum_{u \in V(G)} e(u)} = \frac{n^2}{\zeta(G)} \tag{9}$$

By (8) and (9), we have

$$\xi^{ad}(G) \geq \frac{kn^2}{\zeta(G)}.$$

Definition 1. [11]. *Let $p_1, p_2, \cdots, p_n$ be non-negative integers. The thorn graph of the graph G, with parameters $p_1, p_2, \cdots, p_n$, is obtained by attaching p_i new vertices of degree one to the vertex u_i of the graph G, $i = 1, 2, \cdots, n$. The thorn graph of the graph G will be denoted by G^*.*

Recently, various studies on thorn graphs have done by some researchers [2,3,15,20]. Now, we will examine the eccentric adjacency indices of some specific thorn graphs.

Theorem 8. *Let S_n and S_n^* denote star graph and its thorn graph with n vertices, respectively. Then*

$$\xi^{ad}(S_n^*) = \frac{13}{12}A_1 + \frac{1}{4}A_2 + \frac{(n-1)(2n+1)}{6} + \frac{8n-15}{4}p_1 + \frac{1}{12}p_1^2,$$

where p_1 is the number of pendant vertices added to the central vertex of S_n. Also $A_1 = \sum_{i=1}^{n} p_i$, $A_2 = \sum_{i=1}^{n} p_i^2$.

Proof. Let S_n^* be the thorny star graph. Then we have $d(u_1) = n - 1 + p_1$, $d(u_i) = 1 + p_i, i = 2, 3, ..., n$, $e(u_1) = 2$, $e(u_i) = 3$, $i = 2, 3, ..., n$, $d(u_{ij}) = 1$, $e(u_{ij}) = 4$, for $i = 2, 3, ..., n, j = 2, 3, ..., p_i$, $e(u_{1j}) = 3$, for $j = 1, 2, ..., p_1$. So, we get the following

result

$$\xi^{ad}(S_n^*) = \frac{S(u_1)}{e(u_1)} + \sum_{i=2}^{n} \frac{S(u_i)}{e(u_i)} + \sum_{j=1}^{p_1} \frac{S(u_{1j})}{e(u_{1j})} + \sum_{i=2}^{n} \sum_{j=1}^{p_1} \frac{S(u_{ij})}{e(u_{ij})}$$

$$= \frac{p_1}{2} + \sum_{i=2}^{n} \frac{p_i+1}{2} + \sum_{i=2}^{n} \frac{p_1+p_i+n-1}{3} + \sum_{j=1}^{p_1} \frac{p_1+n-1}{3} + \sum_{i=2}^{n} \sum_{j=1}^{p_i} \frac{(p_i+1)p_i}{4}$$

$$= \frac{13}{12}A_1 + \frac{1}{4}A_2 + \frac{(n-1)(2n+1)}{6} + \frac{8n-15}{4}p_1 + \frac{1}{12}p_1^2$$

Theorem 9. *The eccentric-adjacency index of thorny cycle C_n^* is given by*

$$\xi^{ad}(C_n^*) = \frac{2A_1+4n}{\lfloor \frac{n}{2} \rfloor + 1} + \frac{2A_1+A_2}{\lfloor \frac{n}{2} \rfloor + 2},$$

where $A_1 = \sum_{i=1}^{n} p_i$ and $A_2 = \sum_{i=1}^{n} p_i^2$.

Proof. Let C_n^* be the thorny cycle graph. Then we have $d(u_i) = p_i+2$, $d(u_{ij}) = 1$, $e(u_i) = \lfloor \frac{n}{2} \rfloor + 1$ and $e(u_{ij}) = \lfloor \frac{n}{2} \rfloor + 2$ for $i = 1,2,...,n; j = 1,2,...,p_i$. Thus we get

$$\xi^{ad}(C_n^*) = \sum_{i=1}^{n} \frac{S(u_i)}{e(u_i)} + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{S(u_{ij})}{e(u_{ij})} = \sum_{i=1}^{n} \frac{2p_i+4}{\lfloor \frac{n}{2} \rfloor + 1} + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{(p_i+2)p_i}{\lfloor \frac{n}{2} \rfloor + 2} = \frac{2A_1+4n}{\lfloor \frac{n}{2} \rfloor + 1} + \frac{2A_1+A_2}{\lfloor \frac{n}{2} \rfloor + 2}.$$

Theorem 10. *The eccentric-adjacency index of thorny complete graph K_n^* is given by*

$$\xi^{ad}(K_n^*) = \frac{5n-2}{3}A_1 + \frac{1}{3}A_2 + \frac{n(n-1)^2}{2}$$

where $A_1 = \sum_{i=1}^{n} p_i$, $A_2 = \sum_{i=1}^{n} p_i^2$.

Proof. Let K_n^* be the thorny complete graph. Let the vertices of K_n are denoted by u_i. Also, let the vetices u_i in K_n are denoted by u_{ij}, $i = 1,2,...,n; j = 1,2,...,p_i$. Therefore we have $d(u_i) = p_i+n-1$, $d(u_{ij}) = 1$, $e(u_i) = 2$, $e(u_{ij}) = 3$ for $i = 1,2,...,n; j = 1,2,...,p_i$. Thus the eccentric-adjacency index of K_n^* is given by

$$\xi^{ad}(K_n^*) = \sum_{i=1}^{n} \frac{S(u_i)}{e(u_i)} + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{S(u_{ij})}{e(u_{ij})} = \sum_{i=1}^{n} \frac{A_1+(n-1)^2}{2} + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{p_i+n-1}{3} = \frac{5n-2}{3}A_1 + \frac{1}{3}A_2 + \frac{n(n-1)^2}{2}.$$

Let $K_{m,n}$ be a complete bipartite graph with vertex set $\{u_1, u_2,...,u_m, v_1v_2,...,v_n\}$. Let $K_{m,n}^*$ be thorny complete bipartite graph obtained from $K_{m,n}$ by attaching pendant vertices p_i and p_k^* to u_i and v_k. Let the newly attached pendant vertices are denoted by u_{ij} and v_{kl}, where $i = 1,2,...,m; j = 1,2,...,p_i; k = 1,2,...,n; l = 1,2,...,p_i^*$.

Theorem 11. *The eccentric-adjacency index of thorny cycle $K_{m,n}^*$ is given by*

$$\xi^{ad}(K_{m,n}^*) = \frac{7n+4}{12}A_1 + \frac{7m+4}{12}A_1^* + \frac{A_2+A_2^*}{4} + \frac{nm(n+m)}{3},$$

where $A_1 = \sum_{i=1}^{m} p_i$, $A_1^* = \sum_{i=1}^{n} p_i^*$, $A_2 = \sum_{i=1}^{m} p_i^2$ and $A_2^* = \sum_{i=1}^{n} (p_i^*)^2$.

Proof. The degree and eccentricity of the vertices of $K_{m,n}^*$ are given by $d(u_i) = p_i + n$, $d(v_k) = p_k^* + m$, $d(u_{ij}) = d(v_{kl}) = 1$, $e(u_i) = 3$, $e(v_k) = 3$, $e(u_{ij}) = 4$, $e(v_{kl}) = 4$ for $i = 1, 2, ..., m$; $j = 1, 2, ..., p_i$; $k = 1, 2, ..., n$; $l = 1, 2, ..., p_i^*$. Then we have

$$\xi^{ad}(K_{m,n}^*) = \sum_{i=1}^m \frac{p_i + A_1^* + nm}{3} + \sum_{i=1}^n \frac{p_i^* + A_1 + nm}{3} + \sum_{i=1}^m \sum_{j=1}^{p_i} \frac{p_i + n}{4} + \sum_{i=1}^m \sum_{j=1}^{p_i^*} \frac{p_i^* + m}{4}$$

$$= \frac{A_1 + mA_1^* + nm^2}{3} + \frac{A_1^* + nA_1 + n^2 m}{3} + \frac{A_2 + nA_1}{4} + \frac{A_2^* + mA_1^*}{4}$$

$$= \frac{7n+4}{12} A_1 + \frac{7m+4}{12} A_1^* + \frac{A_2 + A_2^*}{4} + \frac{nm(n+m)}{3}.$$

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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